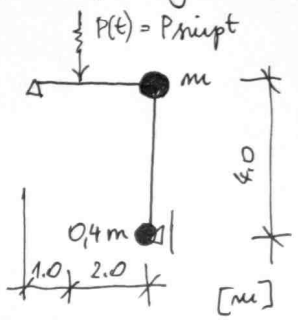
(B) KOLOKWIUM NR 2 4.06.2008

ZAD 1. Wyznaczyć częstość(ci) kołową(e) drgań własnych oraz amplitudę(y) drgań wymuszonych; równania ruchu zapisać (i rozwiązać :)

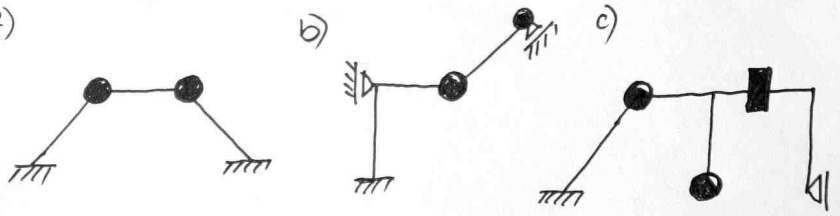
a) przez współczynniki δ_{ik}

b) przez współczynniki r_{ik}

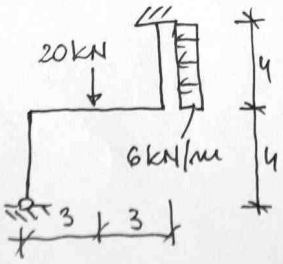


$m = 300$ kg
$EJ = const$
$E = 205$ GPa
$J = 1450$ cm^4
$P = 12$ kN
$p = 30$ Hz

ZAD. 2. Wyznaczyć SSD (zaznaczyć q_i na schemacie)



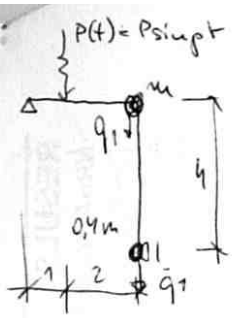
ZAD. 3. Wyznaczyć wektor $\underline{P} = \underline{P}_w - \underline{R}_0$. Zastosować redukcję statyczną na poziomie pręta, uwzględnić warunki podparcia układu



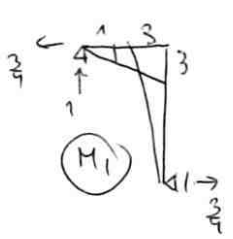
$$\underline{R}_0 = \underline{T}^T \cdot \underline{\tilde{R}}_0$$

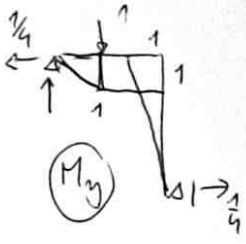
$$\underline{T} = \begin{bmatrix} \underline{C} & \underline{0} \\ \underline{0} & \underline{C} \end{bmatrix} \quad \underline{C} = \begin{bmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1a



$m = 300\,kg$
$E = 205\,GPa$
$J = 1450\,cm^4$
$P = 12\,kN$
$p = 30\,Hz$
$EI = 2972500\,Nm^2$





$$\delta_{11} = \frac{1}{EI}\left[\frac{1}{2}\cdot 3\cdot 3\cdot\frac{2}{3}\cdot 3 + \frac{1}{2}\cdot 4\cdot 3\cdot\frac{2}{3}\cdot 3\right) = \frac{9+12}{EI} = \frac{21}{EI}$$

$$\delta_{1y} = \frac{1}{EI}\left[\frac{1}{2}\cdot 1\cdot 1\cdot\frac{2}{3}\cdot 1 + 2\cdot 1\cdot\frac{1}{2}\cdot 4 + \frac{1}{2}\cdot 4\cdot 3\cdot\frac{2}{3}\cdot 1\right] = \frac{1}{EI}\left[\frac{1}{3}+4+4\right] = \frac{8,1(3)}{EJ}$$

DRG. WŁASNE:

$q_1 = \delta_{11}(-1{,}4\,m\ddot{q}_1)$ $\quad q_1 = A_1\sin\omega t$ $\quad \ddot{q}_1 = -A_1\omega^2\sin\omega t$

$A_1(1 - 1{,}4\,m\omega^2\delta_{11}) = 0$

$1 - \delta_{11}\cdot 1{,}4\,m\omega^2 = 0$

$$\omega = \sqrt{\frac{1}{\delta_{11}\cdot 1{,}4\,m}} = \sqrt{\frac{2972500}{21\cdot 1{,}4\cdot 300}} = 18{,}358\ \frac{rad}{s}$$

DRG. WYMUSZONE

$q_1 = \delta_{11}(-1{,}4\,m\ddot{q}_1) + \delta_{1y}\cdot P(t)$

$q_1 = A_1\sin pt$ $\quad p = 30\cdot 2\pi\ \left[\frac{rad}{s}\right]$

$\ddot{q}_1 = -A_1 p^2\sin pt$

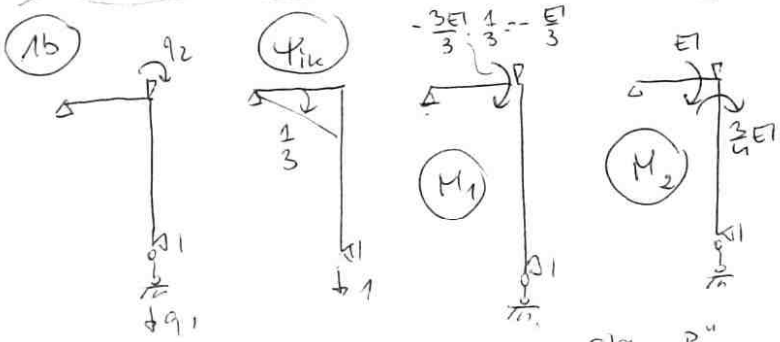
$A_1(1 - 1{,}4\,m p^2\delta_{11}) = \delta_{1y}\cdot P(t)$

$$A_1\left(1 - 1{,}4\cdot 300\cdot(30\cdot 2\pi)^2\cdot\frac{21}{EI}\right) = \frac{8,1(3)}{EI}\cdot 12000$$

$$A_1 = \frac{8,1(3)\cdot 12000}{2972500}\cdot\frac{1}{1 - \frac{21\cdot 1{,}4}{2972500}\cdot 300\cdot(30\cdot 2\pi)^2} =$$

$$= -0{,}000322157\ m$$

1b



$r_{11}\cdot\bar{1} + \left(-\frac{EI}{3}\right)\cdot\frac{1}{3} = 0 \rightarrow r_{11} = \frac{EI}{9}$

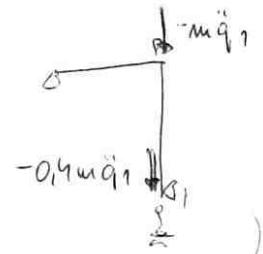
$r_{12}\cdot\bar{1} + EI\cdot\frac{1}{3} = 0 \rightarrow r_{12} = -\frac{EI}{3}$

$r_{21} = -\frac{EI}{3}$

$r_{22} = EI + \frac{3}{4}EI = \frac{7}{4}EI$

Stan „P"

DRG. WŁASNE

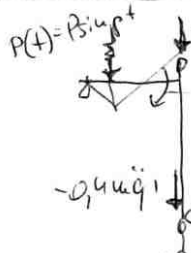


$r_{1P} = 1{,}4\,m\ddot{q}_1$

$r_{2P} = 0$

Stan „P"

DRG. WYMUSZONE



$-\frac{1}{2}P(t)\cdot 1\cdot\left(1 - \frac{1}{9}\right) = -\frac{4}{9}P(t)$

$r_{1P}\cdot\bar{1} - 1{,}4\,m\ddot{q}_1\cdot\bar{1} + P(t)\cdot\frac{1}{3} + \frac{4}{9}P(t)\cdot\frac{1}{3} = 0$

$r_{1P} = 1{,}4\,m\ddot{q}_1 - \frac{13}{27}P(t)$

DRG. WŁASNE:

$$\begin{cases}\frac{EI}{9}q_1 - \frac{EI}{3}q_2 + 1{,}4\,m\ddot{q}_1 = 0\\ -\frac{EI}{3}q_1 + \frac{7}{4}q_2 + 0 = 0\end{cases}$$

$q_2 = \frac{EI}{3}q_1\cdot\frac{4}{7EI} = \frac{4}{21}q_1$

$\frac{EI}{9}\cdot q_1 - \frac{EI}{3}\cdot\frac{4}{21}q_1 + 1{,}4\,m\ddot{q}_1 = 0$

$\frac{EI}{21}q_1 + 1{,}4\,m\ddot{q}_1 = 0 \quad /\cdot\frac{21}{EI}$

$q_1 + \frac{EI}{21}\cdot 1{,}4\,m\ddot{q}_1 = 0$

δ_{11} $\quad q_1 = \delta_{11}(-1{,}4\,m\ddot{q}_1)$ j/w

DRG. WYMUSZONE

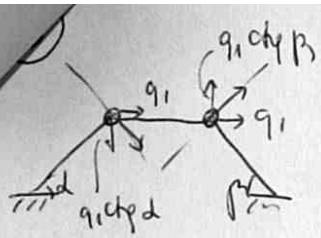
$$\begin{cases}\frac{EI}{9}q_1 - \frac{EI}{3}q_2 + 1{,}4\,m\ddot{q}_1 - \frac{13}{27}P(t) = 0\\ -\frac{EI}{3}q_1 + \frac{7}{4}EIq_2 + \frac{4}{9}P(t) = 0 \rightarrow q_2 = \frac{4}{21}q_1 - \frac{16}{63}\frac{P(t)}{EI}\end{cases}$$

$\frac{EI}{9}\cdot q_1 - \frac{EI}{3}\left(\frac{4}{21}q_1 - \frac{16}{63}\frac{P(t)}{EI}\right) + 1{,}4\,m\ddot{q}_1 - \frac{13}{27}P(t) = 0$

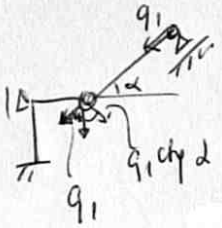
$\frac{EI}{21}q_1 + 1{,}4\,m\ddot{q}_1 - \frac{95}{189}P(t) = 0 \quad /\cdot\frac{21}{EI}$

$q_1 = \underbrace{\frac{21}{EI}(1{,}4\,m\ddot{q}_1)}_{\delta_{11}} + \underbrace{\frac{1375}{189}}_{\delta_{1y}}\cdot P(t)$

$q_1 = \delta_{11}(-1{,}4\,m\ddot{q}_1) + \delta_{1y}\cdot P(t)$ j/w



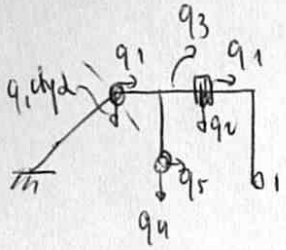
SSD = 1



$\varphi_2 = q_1$

$x = q_1 \cdot \mathrm{ctg}\,\alpha$

SSD = 1



SSD = 5

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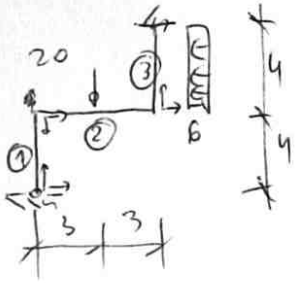
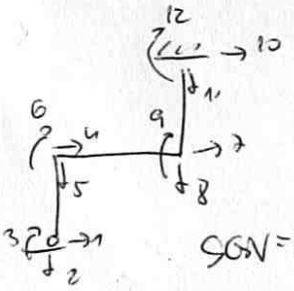
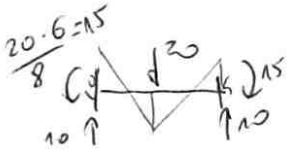
③

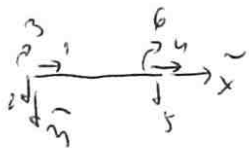
$$\tilde{R}_0^1 = \underline{R}_0^1 = \mathrm{col}(0,0,0,0,0,0)$$



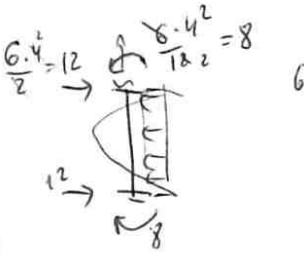


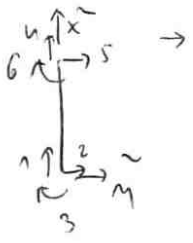
SSV = 12





$$\tilde{R}_0^2 = \begin{bmatrix} 0 \\ -10 \\ -15 \\ 0 \\ -10 \\ 15 \end{bmatrix} = \underline{R}_0^2$$





$\alpha = -90°$

$$\tilde{R}_0^3 = \begin{bmatrix} 0 \\ 12 \\ 8 \\ 0 \\ 12 \\ -8 \end{bmatrix}$$

$$T = \begin{bmatrix} c & s & 0 & 0 & 0 & 0 \\ -s & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & s & 0 \\ 0 & 0 & 0 & -s & c & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\underline{R}_0^3 = \begin{bmatrix} 0 & 1 & 0 & & & \\ -1 & 0 & 0 & & \underline{0} & \\ 0 & 0 & 1 & & & \\ & & & 0 & 1 & 0 \\ & \underline{0} & & -1 & 0 & 0 \\ & & & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 12 \\ 8 \\ 0 \\ 12 \\ -8 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 8 \\ 12 \\ 0 \\ -8 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 & & & \\ 1 & 0 & 0 & & \underline{0} & \\ 0 & 0 & 1 & & & \\ & & & 0 & -1 & 0 \\ & \underline{0} & & 1 & 0 & 0 \\ & & & 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 & 0 \\ 0 & -10 \\ 0 & -15 \\ & 0 & 12 \\ & -10 & 0 \\ & 15 & 8 \\ & & 12 \\ & & 0 \\ & & -8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 10 \\ 15 \\ -12 \\ 10 \\ -23 \\ -12 \\ 0 \\ 8 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 \\ 10 \\ 15 \\ -12 \\ 10 \\ -23 \end{bmatrix}$$