Politechnika Poznańska
Wydział Budownictwa Architektury
i inżynierii Środowiska

Ćwiczenie nr 2

OBLICZANIE PRZEMIESZCZEŃ UKŁADÓW STATYCZNIE WYZNACZALNYCH Z ZASTOSOWANIEM RÓWNANIA PRACY WIRTUALNEJ

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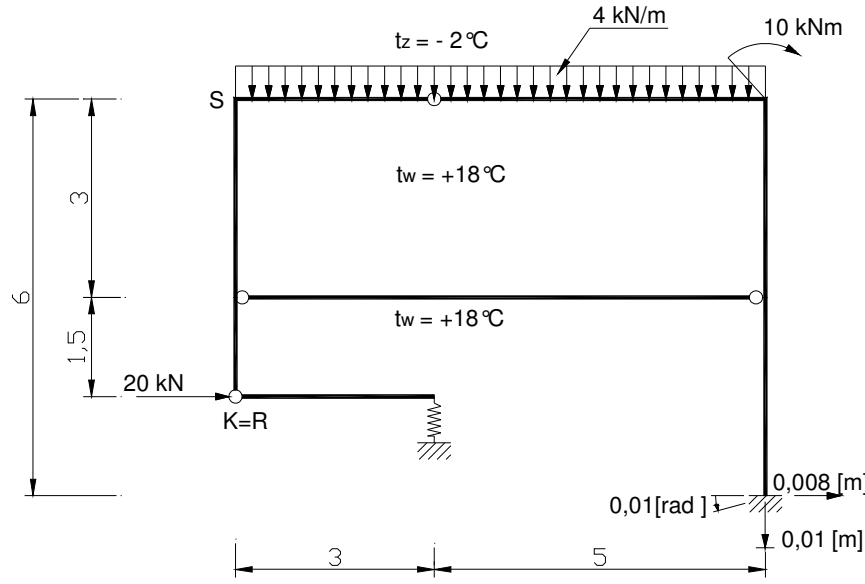
Rok studiów: III
Semestr: V

Dane wyjściowe do projektu:

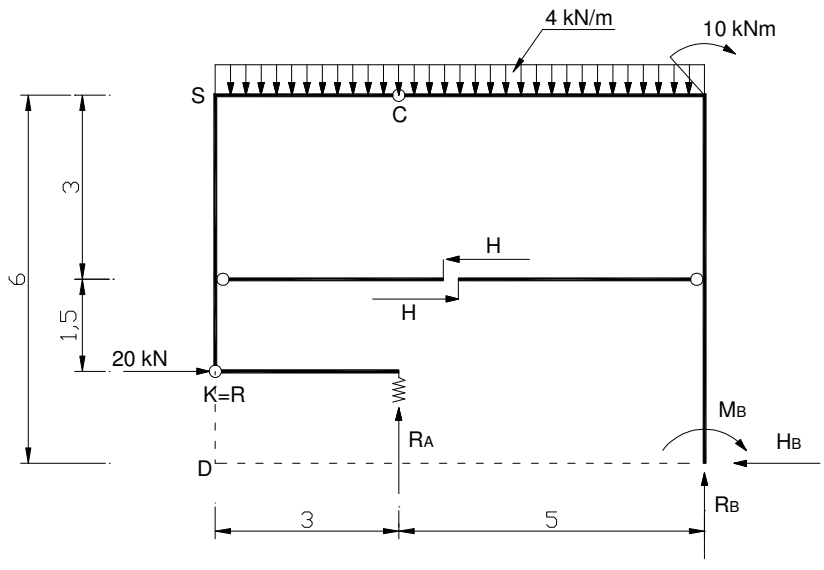
Δ_1=0,01 m
Δ_2=0,008 m
φ = 0,01 rad

$t_w = +18^0C$
$t_z = -2^0C$
$t_m = +4^0C$

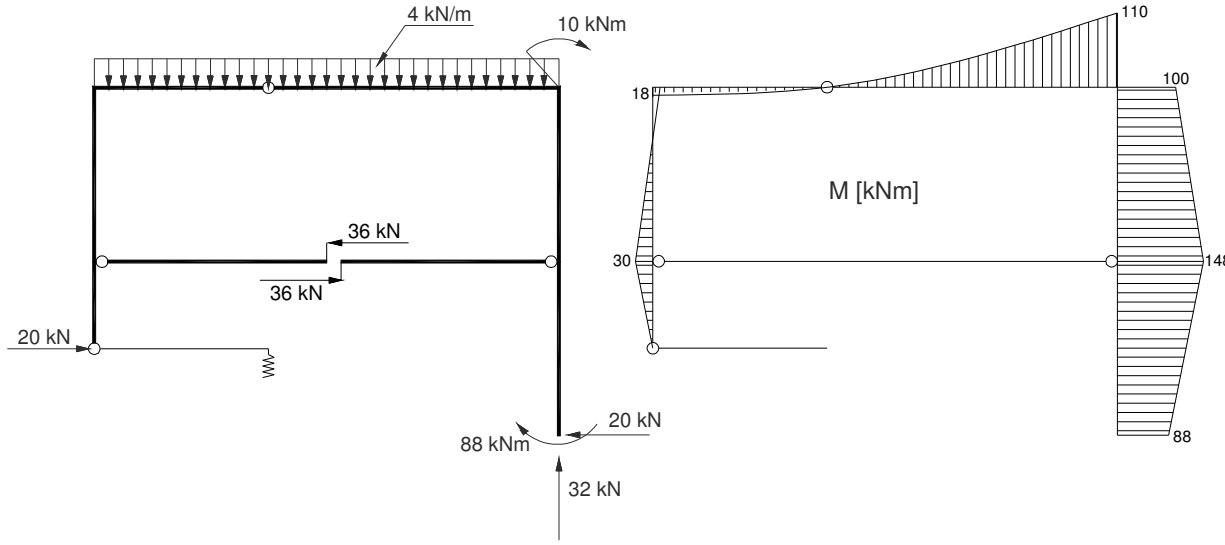


$$\overline{1,0}\cdot\delta=\sum\int_S\frac{\overline{M}M}{EI}ds+\sum\int_S\frac{\overline{N}N}{EA}ds+\sum\int_S\frac{\overline{T}T}{GA}\chi ds+\sum_i\overline{R}_iR_i\frac{1}{k}-\sum_n\overline{R}_n\Delta_n+\sum\int_S\overline{M}\frac{\alpha\Delta t}{h}ds+\sum\int_S\overline{N}\alpha t_0 ds$$

1. Wyznaczanie reakcji podporowych:



$\Sigma M_K^P=0$
$-R_A\cdot 3=0$
$\underline{R_A=0}$
$\Sigma M_B=0$
$20\cdot 1,5-4\cdot 8\cdot 4+10+M_B=0$
$\underline{M_B=88\ kNm}$
$\Sigma M_C^L=0$
$-20\cdot 4,5-4\cdot 3\cdot 1,5+H\cdot 3=0$
$\underline{H = 36\ kN}$
$\Sigma M_D=0$
$-R_B\cdot 8+20\cdot 1,5+4\cdot 8\cdot 4+10+88=0$
$\underline{R_B=32\ kN}$
$\Sigma M_C^P=0$
$4\cdot 5\cdot 2,5+10+88-36\cdot 3-32\cdot 5+H_B\cdot 6=0$
$\underline{H_B=20\ kN}$



2. Projektowanie przekroju:

$$1{,}2 \cdot \sigma_{max} \le \sigma_{dop} = 200MPa$$

$$1{,}2 \cdot \frac{M_{max}}{W} \le 20kN/cm^2$$

M_{max} =148 kNm

$$W \ge \frac{14800kNcm}{20\,kN/cm^2} \cdot 1{,}2 = 888cm^3$$

Przyjmuję przekrój z dwuteownika I340:

W=923,53cm^3
I = 15700 cm^4
A= 86,8 cm^2

$$\frac{14800kNcm}{923{,}53cm^3} \cdot 1{,}2 = 19{,}23\,kN/cm^2$$

$$\sigma_{max} = 192{,}3MPa < \sigma_{dop}$$

Przekrój ściągu o przekroju kołowym:

$$1{,}2 \cdot \frac{N}{A} \le \sigma_{dop} = 200MPa$$

N = 36 kN

$$A \ge 1{,}2\frac{36kN}{20\,kN/cm^2} = 2{,}16\ cm^2$$

$$\pi \cdot R^2 \ge 2{,}16cm^2$$

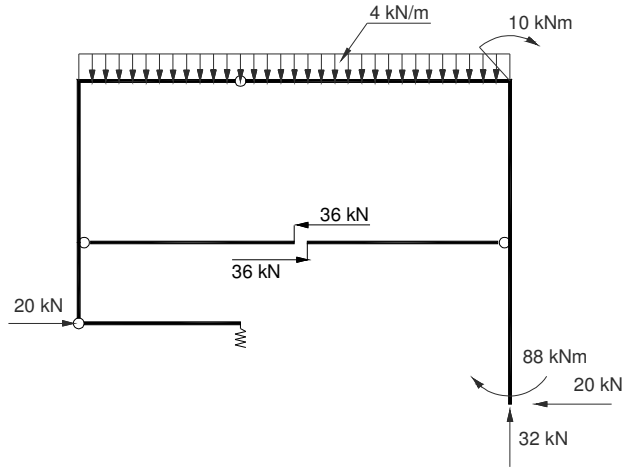
$$R \ge \sqrt{\frac{2{,}16}{\pi}} = 0{,}83cm$$

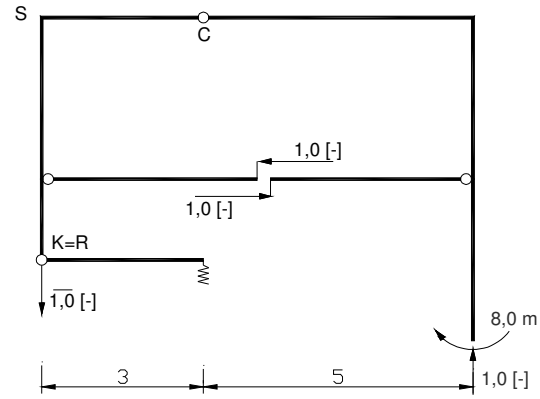
Przyjmuje przekrój ø20mm

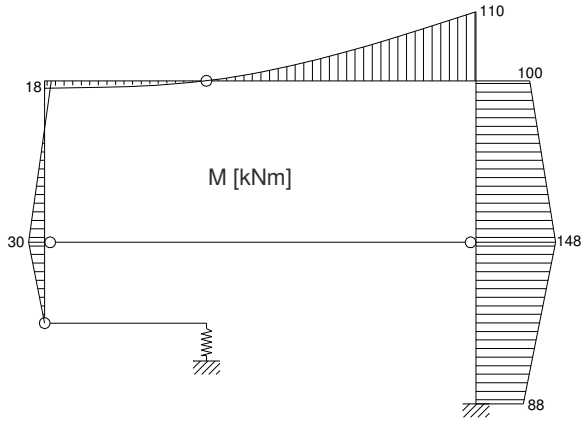
$$1{,}2 \cdot \frac{36}{\pi \cdot 1^2} = 13{,}75\,kN/cm^2 \le \sigma_{dop}$$

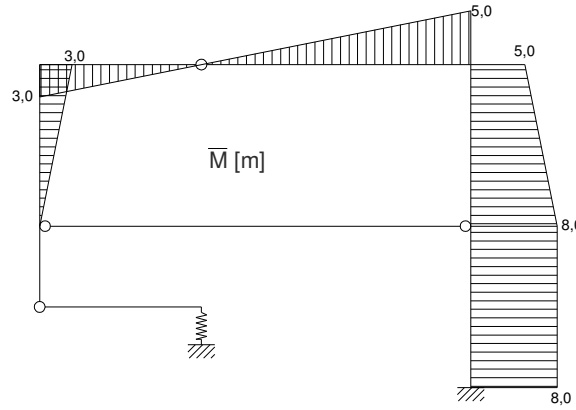
I = 0,79 cm^4
F = 3,14 cm^2

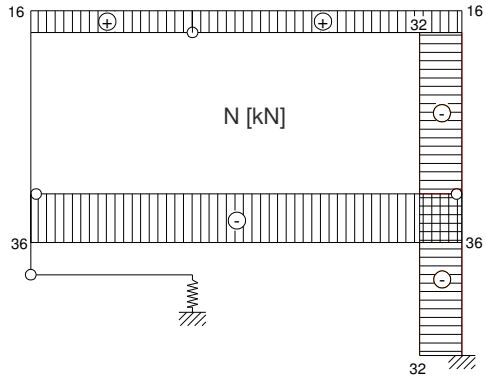
3. Obliczenie przemieszczenia pionowego punktu K (wpływ M, N, T)

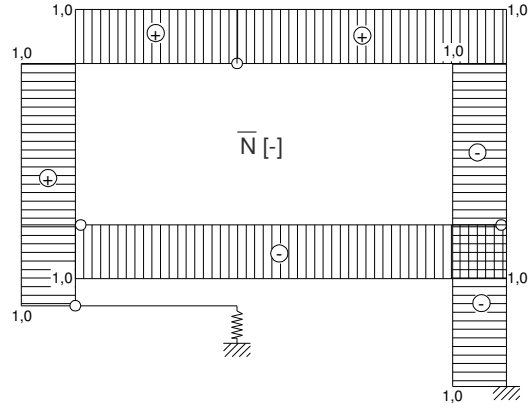


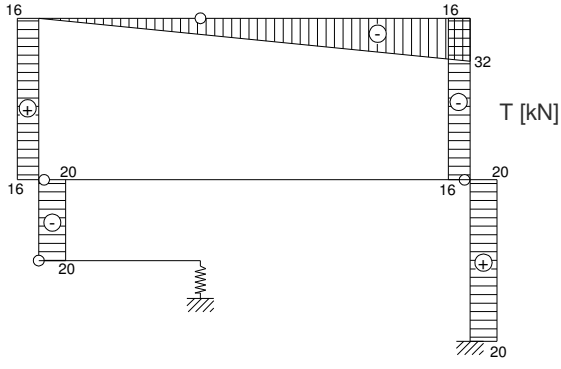


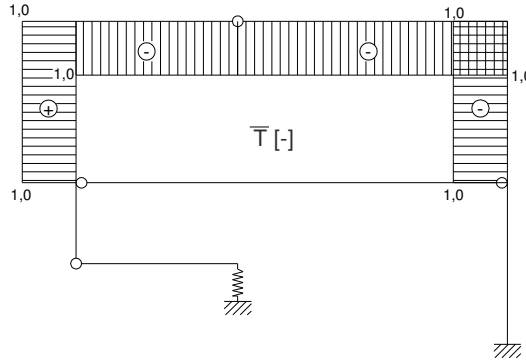












$$\overline{1,0}\,V_K=\frac{1}{EI}\left[\frac{1}{2}\cdot3\cdot30\cdot\frac{1}{3}\cdot3+\frac{1}{2}\cdot3\cdot18\cdot\frac{2}{3}\cdot3+\frac{2}{3}\cdot\frac{4\cdot3^2}{8}\cdot\frac{1}{2}\cdot3+\frac{1}{2}\cdot5\cdot90\cdot\frac{2}{3}\cdot5-\frac{2}{3}\cdot\frac{4\cdot5^2}{8}\cdot\frac{1}{2}\cdot5+\frac{3}{6}\cdot(2\cdot100\cdot5+\right.$$

$$\left.+2\cdot148\cdot8+100\cdot8+148\cdot5)+\frac{3}{6}\cdot(2\cdot148\cdot8+2\cdot88\cdot8+148\cdot8+88\cdot8)\right]+\frac{1}{EA}\cdot[8\cdot16\cdot1+6\cdot(-32)\cdot(-1)]+$$

$$+\frac{1}{EA_1}[8\cdot(-36)\cdot(-1)]+\frac{\chi}{GA}\cdot[3\cdot16\cdot1+\frac{1}{2}\cdot8\cdot(-32)\cdot(-1)+3\cdot(-16)\cdot(-1)]$$

$$\chi=\frac{A}{A_{śr}}\qquad\Rightarrow\qquad\chi=\frac{86,8}{(34-2\cdot1,83)\cdot1,22}=2,35$$

$$G=\frac{E}{2(1+\nu)}\qquad\Rightarrow\qquad G=\frac{20500}{2(1+0,3)}=7884,62\text{kN/cm}^2$$

$EI=205\cdot10^6\cdot15700\cdot10^{-8}=32185\ \text{kNm}^2$ $\qquad EA_1=20500\cdot3,14=64370\ \text{kN}$

$EA=20500\cdot86,8=1779400\ \text{kN}$ $\qquad GA=7884,62\cdot86,8=684385,02\text{kN}$

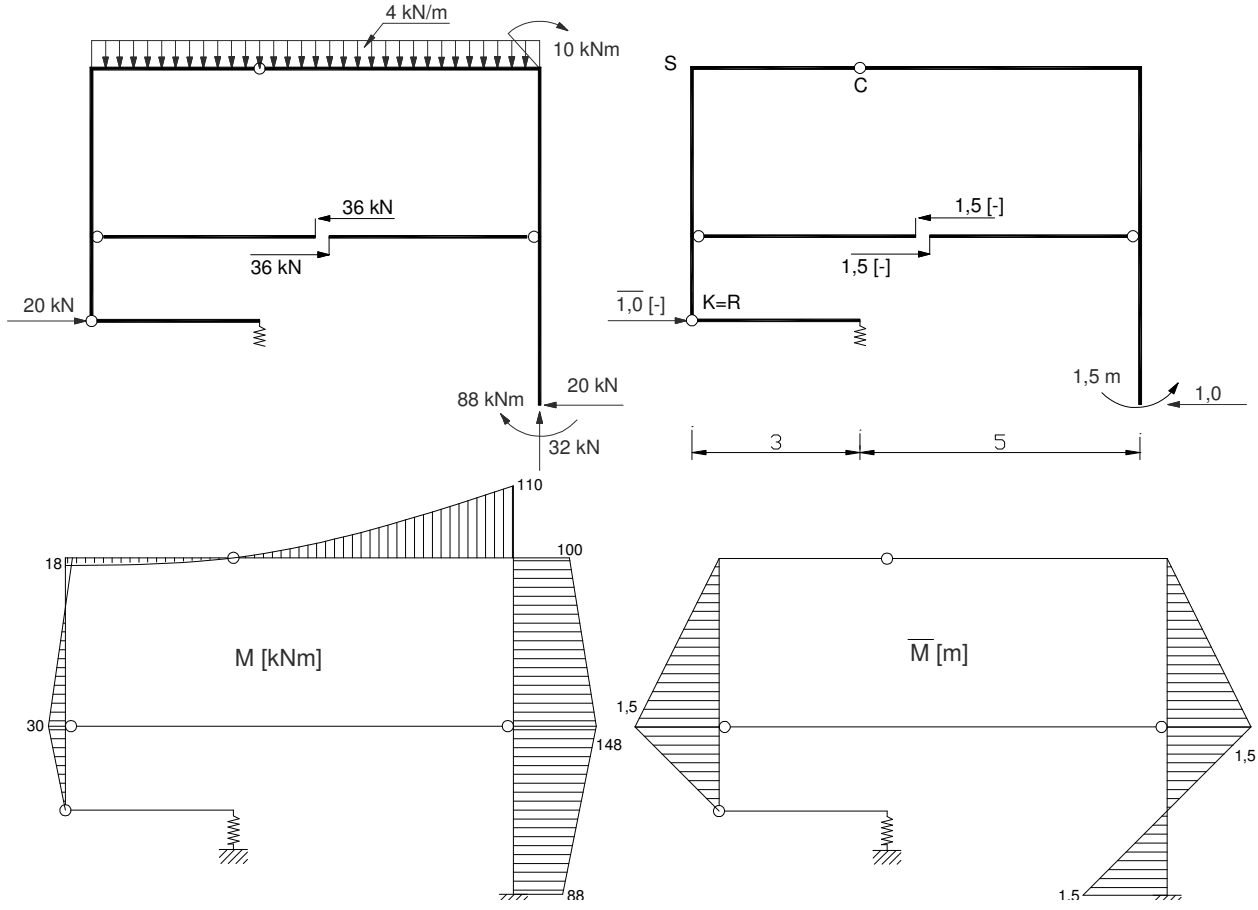
$$V_K=\frac{1}{EI}\cdot6118,67+\frac{1}{EA}\cdot320+\frac{1}{EA_1}\cdot288+\frac{\chi}{GA}\cdot224=$$

$$=\frac{6118,67}{32185}+\frac{320}{1779400}+\frac{288}{64370}+\frac{2,35\cdot224}{684385,02}=0,1901+0,00018+0,00447+0,00077=\mathbf{0,19552m=19,55cm}$$

$$\left[\frac{\text{kNm}^3}{\text{kNm}^2}\right]+\left[\frac{\text{kNm}}{\text{kN}}\right]+\left[\frac{\text{kNm}}{\text{kN}}\right]+\left[\frac{\text{kNm}}{\text{kN}}\right]\quad=[\text{m}]$$

4. Przemieszczenie wypadkowe punktu K (wpływ M)

Wypadkowe przemieszczenie punktu K wyznaczam jako wypadkową przemieszczenia poziomego i pionowego. Ponieważ przemieszczenie pionowe zostało wyznaczone w punkcie 3, pozostaje tylko do wyznaczenie przemieszczenie poziome.



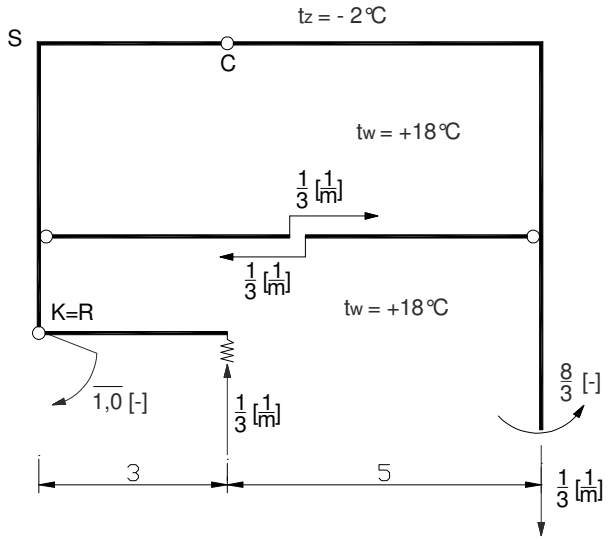
$$\overline{1,0}\cdot V_K=\frac{1}{EI}\left[\frac{1}{2}\cdot 3\cdot 30\cdot\frac{1}{3}\cdot 3+\frac{1}{2}\cdot 3\cdot 18\cdot\frac{2}{3}\cdot 3+\frac{2}{3}\cdot\frac{4\cdot 3^2}{8}\cdot\frac{1}{2}\cdot 3+\frac{1}{2}\cdot 5\cdot 90\cdot\frac{2}{3}\cdot 5-\frac{2}{3}\cdot\frac{4\cdot 5^2}{8}\cdot\frac{1}{2}\cdot 5+\right.$$
$$\left.+\frac{3}{6}\cdot(2\cdot 100\cdot 5+2\cdot 148\cdot 8+100\cdot 8+148\cdot 5)+\frac{3}{6}\cdot(2\cdot 148\cdot 8+2\cdot 88\cdot 8+148\cdot 8+88\cdot 8)\right]+\frac{1}{EA_1}[8\cdot(-36)\cdot(-1)]=$$
$$=\frac{1}{EI}\cdot 6118{,}67+\frac{1}{EA_1}\cdot 288=0{,}1901+0{,}00447=\mathbf{0{,}19457\ m}=\mathbf{19{,}46cm}\qquad\left[\frac{kNm}{kN}\right]=[m]$$

$$\overline{1,0}\cdot H_K=\frac{1}{EI}\left[\frac{1}{2}\cdot 1{,}5\cdot 30\cdot\frac{2}{3}\cdot 1{,}5+\frac{1}{2}\cdot 3\cdot 30\cdot\frac{2}{3}\cdot 1{,}5+\frac{3}{6}\cdot(2\cdot 100\cdot 0+2\cdot 148\cdot 1{,}5+100\cdot 1{,}5+148\cdot 0)+\right.$$
$$\left.+\frac{3}{6}\cdot(2\cdot 148\cdot 1{,}5-2\cdot 88\cdot 1{,}5-148\cdot 1{,}5+88\cdot 1{,}5)\right]+\frac{1}{EA_1}[8\cdot(-36)\cdot(-1{,}5)]=$$
$$=\frac{1}{EI}\cdot 409{,}5+\frac{1}{EA_1}\cdot 432=0{,}0127+0{,}0067=\mathbf{0{,}0194m}=\mathbf{1{,}94cm}\qquad\left[\frac{kNm}{kN}\right]=[m]$$

$\delta_K=[(V_K)^2+(H_K)^2]^{0,5}$

δ_K = 19,56 cm

5. Obrót przekroju K (wpływ zmian temperatury)



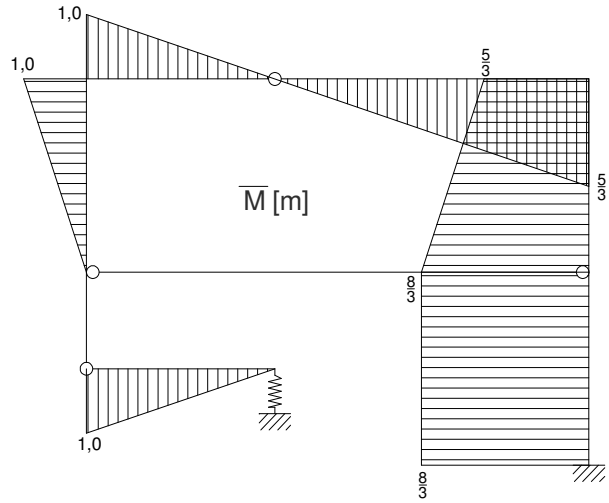
$|\Delta t| = t_w - t_z = 18-(-2) = 20^oC$

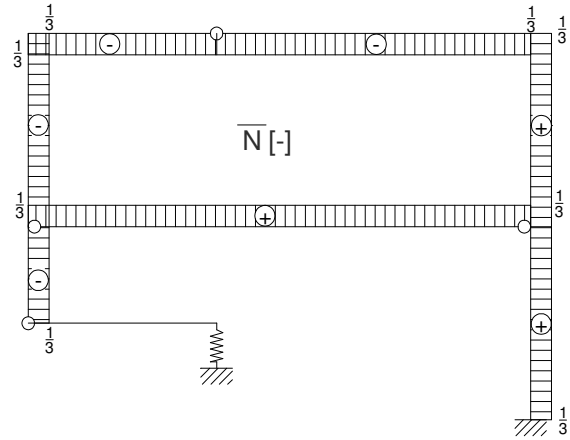
$t_o = t_{śr} - t_m = \frac{t_z + t_w}{2} - t_m$

$t_0 = (-2+18)/2-4 = 4^oC$

– dla ściągu:

$t_{01} = (18+18)/2-4 = 14^oC$





$$\overline{1,0}\varphi_K = \frac{1}{EI}\left[\frac{1}{2}\cdot 3\cdot 30\cdot\frac{1}{3}\cdot 1 - \frac{1}{2}\cdot 3\cdot 18\cdot\frac{2}{3}\cdot 1 + \frac{2}{3}\cdot\frac{4\cdot 3^2}{8}\cdot\frac{1}{2}\cdot 1 - \frac{1}{2}\cdot 5\cdot 90\cdot\frac{2}{3}\cdot\frac{5}{3} + \frac{2}{3}\cdot\frac{4\cdot 5^2}{8}\cdot\frac{1}{2}\cdot\frac{5}{3} +\right.$$

$$+\frac{3}{6}\cdot(-2\cdot 100\cdot\frac{5}{3} - 2\cdot 148\cdot\frac{8}{3} - 100\cdot\frac{8}{3} - 148\cdot\frac{5}{3}) + \frac{3}{6}\cdot(-2\cdot 148\cdot\frac{8}{3} - 2\cdot 88\cdot\frac{8}{3} - 148\cdot\frac{8}{3} - 88\cdot\frac{8}{3}) +$$

$$\left.+\frac{\alpha|\Delta t|}{h}\cdot\left[-\frac{1}{2}\cdot 3\cdot 1 - \frac{1}{2}\cdot 3\cdot 1 - \frac{1}{2}\cdot 3\cdot 1 + \frac{1}{2}\cdot 5\cdot\frac{5}{3} + \frac{1}{2}\cdot 3\cdot(\frac{5}{3}+\frac{8}{3}) + 3\cdot\frac{8}{3}\right] + \alpha t_0[4{,}5\cdot(-\frac{1}{3}) + 8\cdot(-\frac{1}{3}) + 6\cdot\frac{1}{3}] + \alpha t_{01}[8\cdot\frac{1}{3}]\right] =$$

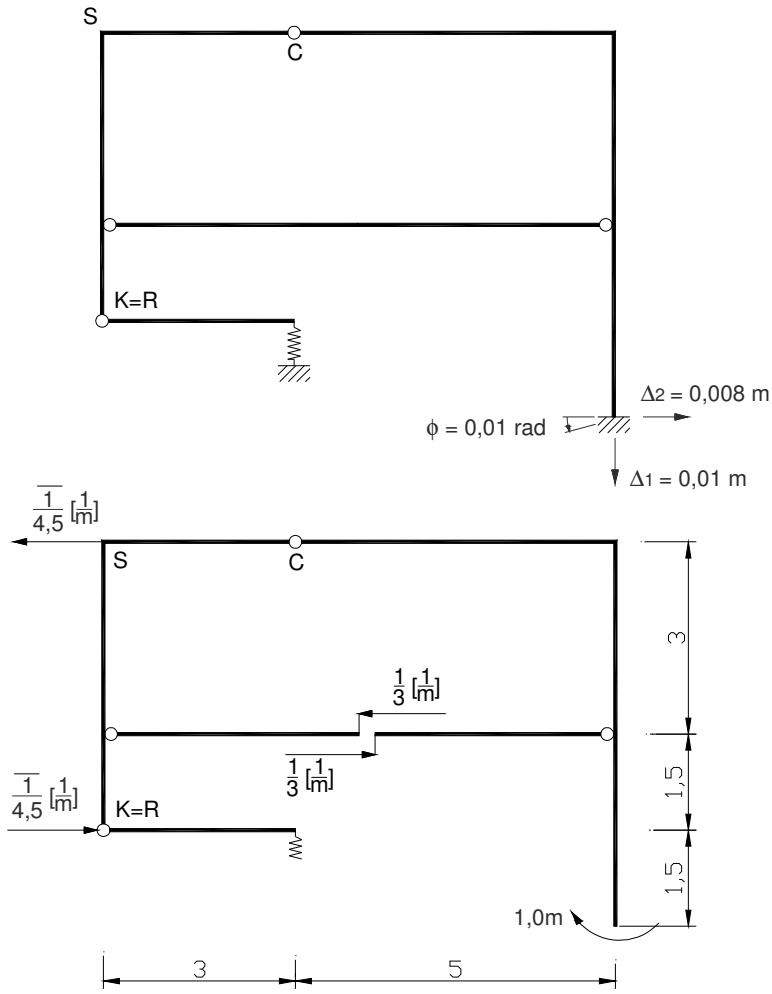
$$= \frac{1}{EI}\cdot 1858{,}56 + \frac{\alpha|\Delta t|}{h}\cdot 14{,}17 + \alpha t_0\cdot 2{,}17 + \alpha t_{01}\cdot 2{,}67$$

$$\alpha = 1{,}2\cdot 10^{-5}\,\frac{1}{^\circ C}$$

$$\varphi_K = \frac{1858{,}56}{32185} + \frac{1{,}2\cdot 10^{-5}\cdot 20}{0{,}34}\,14{,}17 + 1{,}2\cdot 10^{-5}\cdot 4\cdot 2{,}17 + 1{,}2\cdot 10^{-5}\cdot 14\cdot 2{,}67$$

$$= 0{,}05775 + 0{,}01000 + 0{,}00010 + 0{,}00045 = \mathbf{0{,}0683rad} \quad \Rightarrow \quad \varphi_K = \mathbf{3{,}9^o}$$

6. Obrót cięciwy R,S (od osiadania podpór)



$$\varphi_{R,S} = -\sum_n \overline{R_n}\Delta_n = -(-1\cdot 0{,}01) = \mathbf{0{,}01\ rad} \quad \Rightarrow \quad \varphi_{R,S} = \mathbf{0{,}57^o}$$