

ĆWICZENIE NR 2

METODA SIŁ

Termin oddania: 16.06.2024

Data	Uwagi sprawdzającego	Podpis

Dla zadanej **BELKI (9)** należy:

1. Korzystając z metody sił obliczyć siły przekrojowe (M, N, T) od zadanego obciążenia i wykonać kontrolę kinematyczną.
2. Zaprojektować przekrój z profilu dwuteowego (IN, IPE, HEB, HEA).
3. Obliczyć **obrót przekroju w punkcie "B"** korzystając z równania pracy wirtualnej i twierdzenia redukcyjnego.

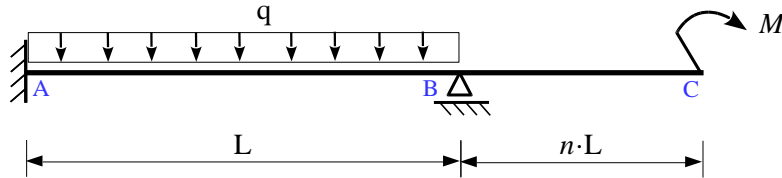
Dla zadanej **RAMY (3)** należy:

1. Przyjąć wstępnie przekroje I_1 i I_2 (różne) z profili dwuteowych (IN, IPE, HEB, HEA).
2. Korzystając z metody sił obliczyć siły przekrojowe (M, N, T) od zadanego obciążenia i wykonać kontrolę kinematyczną.
3. Sprawdzić naprężenia w obu grupach przekrojów I_1 i I_2 oraz sformułować wnioski (w przypadku niespełnienia warunku nośności, obliczeń nie trzeba powtarzać).
4. Obliczyć **przesunięcie poziome punktu "C"** korzystając z równania pracy wirtualnej i twierdzenia redukcyjnego.

We wszystkich obliczeniach przyjmij: $E = 210 \text{ GPa}$, $\sigma_{\text{dop}} = 235 \text{ MPa}$.

DANE DLA BELKI:

Nr schematu: **9**

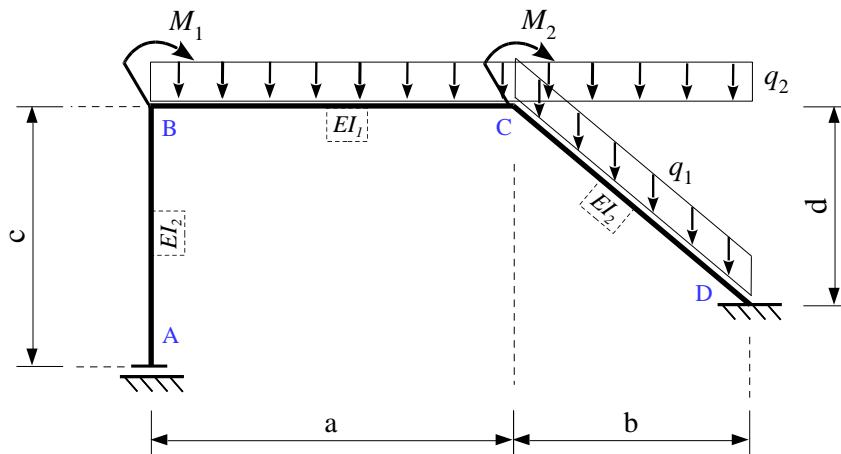


Dane:

L [m]	n	q [kN/m]	M [kNm]	P [kN]
6,5	0,6	7,0	16,0	-

DANE DLA RAMY:

Nr schematu: **3**

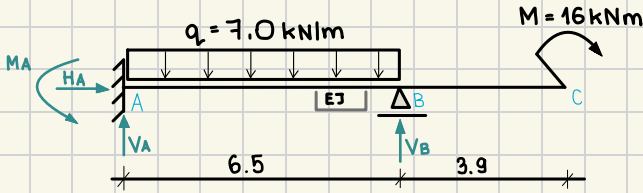


Dane:

a [m]	b [m]	c [m]	d [m]	q ₁ [kN/m]	q ₂ [kN/m]	M ₁ [kNm]	M ₂ [kNm]
4,5	4,5	4,8	4,8	13,5	-	14,0	-

Uwaga! Wartość dodatnia oznacza zwrot obciążenia zgodny z podanym na schemacie, ujemna – przeciwny.

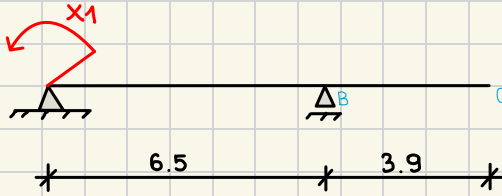
Sara Jackowiak, projekt nr 2



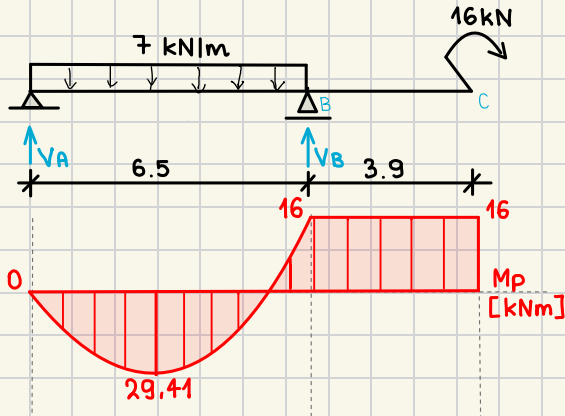
SSN = LR - LRR - LPW	
4 - 3 - 0 = 1	
SSN = 1	

układ równań kanonicznych $\delta_{11}x_1 + \Delta_1p = 0$

układ zastępczy



Stan P, $x_1 = 0$



$$\sum M_A = 0 \Rightarrow -6,5V_B + 7 \cdot 6,5 \cdot \frac{1}{2} \cdot 6,5 + 16 = 0$$

$$-6,5V_B + 163,875 = 0 \quad V_B = 25,21 \text{ kN}$$

$$\sum Y = 0 \Rightarrow 6,5 \cdot V_A + 25,21 = 0$$

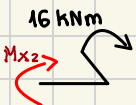
$$V_A = 20,29 \text{ kN}$$

$$M_{x1} = 20,29x - 7x \cdot \frac{x}{2} = 20,29x - 3,5x^2$$

$$M(0) = 0 \text{ kNm} \quad M(6,5) = -16 \text{ kNm}$$

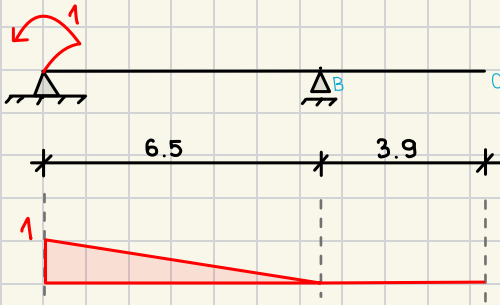
$$20,29 - 7x = 0 \Rightarrow x = 2,8986 \text{ m}$$

$$M(2,8986) = 29,41 \text{ kNm}$$



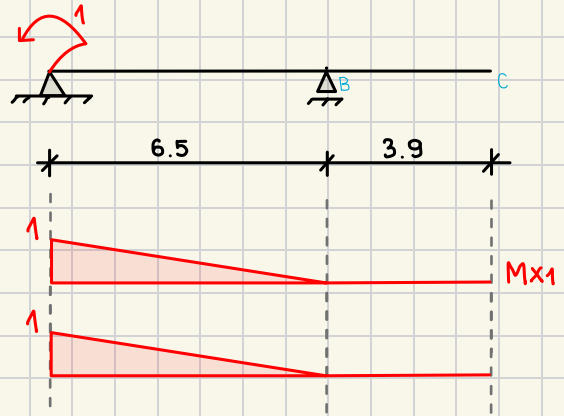
$$M_{x2} = -16 \text{ kNm}$$

Start $P=0, x_1 = 1$

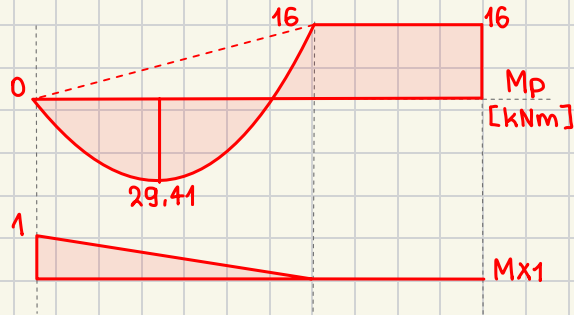


$$\delta_{11} = \int \frac{M_{x1} \cdot M_{x1}}{EJ} ds$$

$$\delta_{11} = \frac{1}{EJ} \left[\frac{1}{2} \cdot 1 \cdot 6.5 \cdot \frac{2}{3} \cdot 1 \right] = \frac{2,1667}{EJ}$$



$$\Delta_{1P} = \int \frac{M_P \cdot M_{x1}}{EJ} ds$$

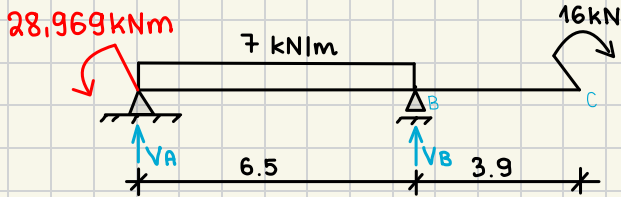


$$\Delta_{1P} = \frac{1}{EJ} \left[\frac{1}{2} \cdot 16 \cdot 6.5 \cdot \frac{1}{3} \cdot 1 - \frac{2}{3} \cdot \frac{7 \cdot 6.5^2}{8} \cdot 6.5 \cdot \frac{1}{2} \cdot 1 \right] = -\frac{62,7656}{EJ}$$

$$\delta_{11}x_1 + \Delta_{1p} = 0$$

$$\frac{2,1667}{EJ} \cdot x_1 + \left(-\frac{62,7656}{EJ} \right) = 0$$

$$x = -28,969 \text{ kNm}$$



$$\sum M_A = 7 \cdot 6,5 \cdot \frac{6,5}{2} - 6,5 V_B + 16 - 28,969 = 0$$

$$147,875 - 6,5 V_B + 16 - 28,969$$

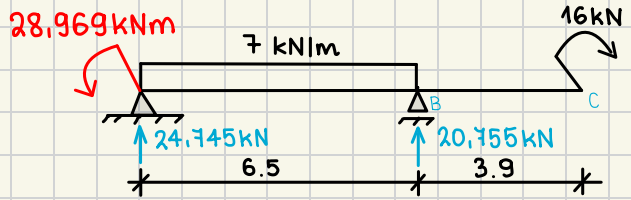
$$- 6,5 V_B + 134,906 = 0$$

$$V_B = 20,755 \text{ kN}$$

$$\sum Y = 0 \Rightarrow V_A - 7 \cdot 6,5 + 20,755 = 0$$

$$V_A - 45,5 + 20,755 = 0$$

$$V_A = 24,745 \text{ kN}$$



$$M_{x1} = -28,969 + 24,745x - 3,5x^2$$

$$M(0) = -28,969 \text{ kNm}$$

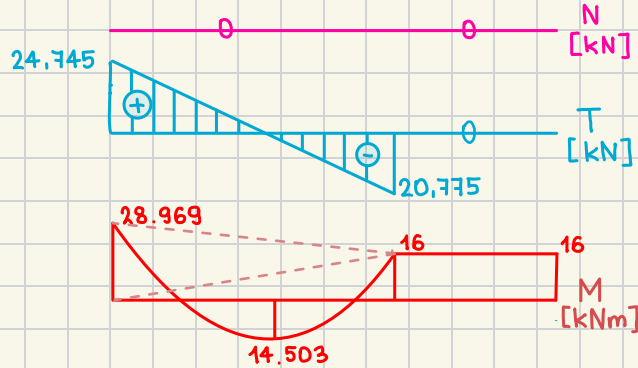
$$M(6,5) = -16 \text{ kNm}$$

$$24,745 - 6,5x = 0$$

$$6,5x = 24,745 \quad x = 3,81 \text{ m}$$

$$M(3,81) = 14,503 \text{ kNm}$$

$$M_{x2} = -16 \text{ kNm}$$



Zaprojektowanie przekroju

$$E = 210 \text{ GPa}$$

$$\sigma_{\text{dop}} = 235 \text{ MPa}$$

$$M_{\text{max}} = 28,969 \text{ kNm}$$

$$\sigma_{\text{max}} \leq \sigma_{\text{dop}} = 235 \text{ MPa}$$

$$\frac{M_{\text{max}}}{W} \leq 235 \text{ MPa}$$

$$W \geq \frac{2896,9}{23,5} = 123,27 \text{ cm}^3$$

Przyjęto IPE 180

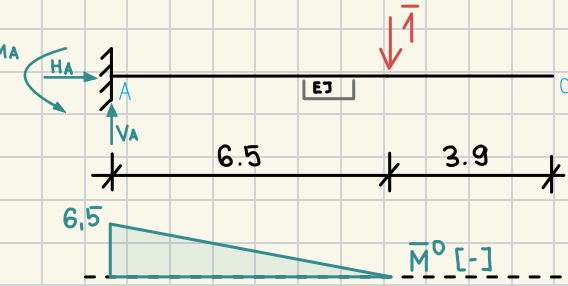
$$W = 146,00 \text{ cm}^3$$

$$J = 1320,00 \text{ cm}^4$$

$$A = 23,90 \text{ cm}^2$$

$$EJ = 210 \cdot 10^6 \cdot 1320 \cdot 10^{-8} = 2772 \text{ kNm}^2$$

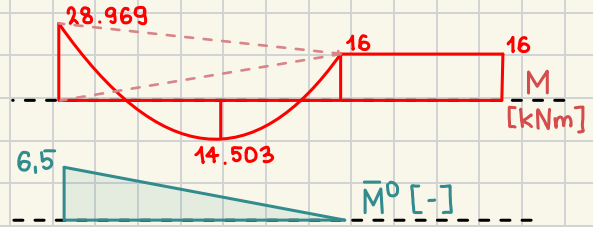
Kontrola kinematyczna



$$\sum Y = 0 \Rightarrow V_A - 1 = 0 \Rightarrow V_A = 2.1 \text{ kN}$$

$$\sum M_A = 0 \Rightarrow$$

$$\delta_0^V = \sum \int \frac{M \cdot \bar{M}^0}{EJ} dx$$

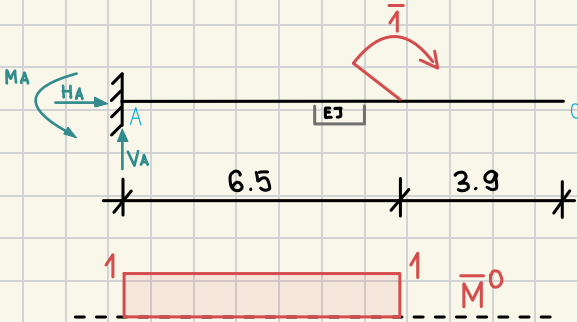


$$\delta_0^V = \frac{1}{EJ} \left[\frac{1}{2} \cdot 28.969 \cdot 6.5 \cdot \frac{2}{3} \cdot 6.5 + \frac{1}{2} \cdot 16 \cdot 6.5 \cdot \frac{1}{3} \cdot 6.5 - \frac{2}{3} \cdot \frac{7 \cdot 6.5^2}{8} \cdot 6.5 \cdot \frac{1}{2} \cdot 6.5 \right] =$$

$$= \frac{0.003520}{EJ} \approx 0$$

Warunek spełniony

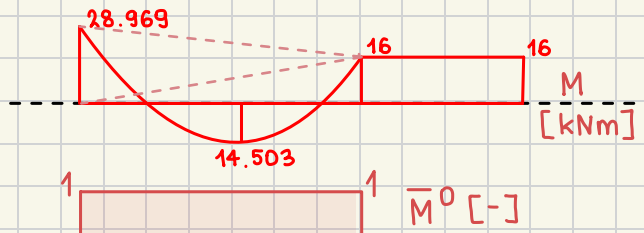
Obrót przekroju w punkcie B



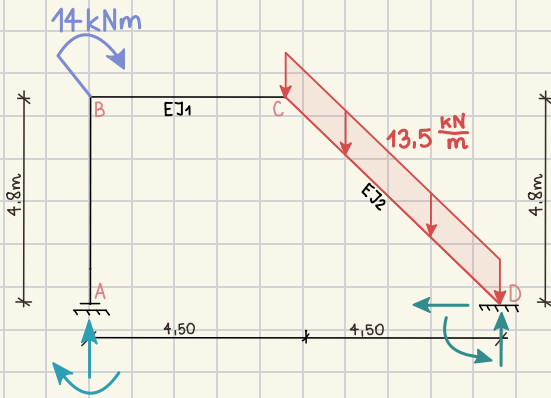
$$\varphi_B = \sum \int \frac{M \cdot \bar{M}^0}{EJ} dx$$

$$\varphi_B = \frac{1}{EJ} \left[\frac{1}{2} \cdot 28.969 \cdot 6.5 \cdot 1 + \frac{1}{2} \cdot 16 \cdot 6.5 \cdot 1 - \frac{2}{3} \cdot \frac{7 \cdot 6.5^2}{8} \cdot 6.5 \cdot 1 \right] =$$

$$= -0.005068 \text{ rad} = -26.49^\circ$$



Rama, schemat 3



$$SSN = LR - LRR - LPW$$

$$5 - 3 - 0$$

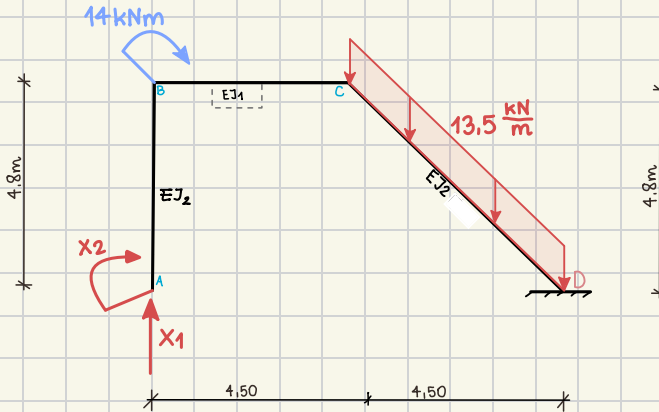
$$SSN = 2$$

LR - liczba reakcji podporowych

LRR - liczba równań równowagi

LPW - liczba przegubów wewnętrznych

układ zastępczy



1 Wstępne przyjęcie przekrojów

EJ₁: I 200
 $J = 2140 \text{ cm}^4$
 $A = 33,5 \text{ cm}^2$
 $W = 214 \text{ cm}^3$

EJ₂: I 300
 $J = 9800 \text{ cm}^4$
 $A = 69,1 \text{ cm}^2$
 $W = 653 \text{ cm}^3$

$$J_2 = J_0$$

$$EJ_1 = 10^6 \cdot 210 \cdot 10^{-8} \cdot 2140 = 4494 \text{ kNm}^2$$

$$EJ_2 = 10^6 \cdot 210 \cdot 10^{-8} \cdot 9800 = 20580 \text{ kNm}^2$$

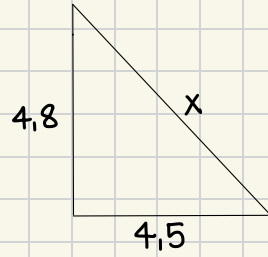
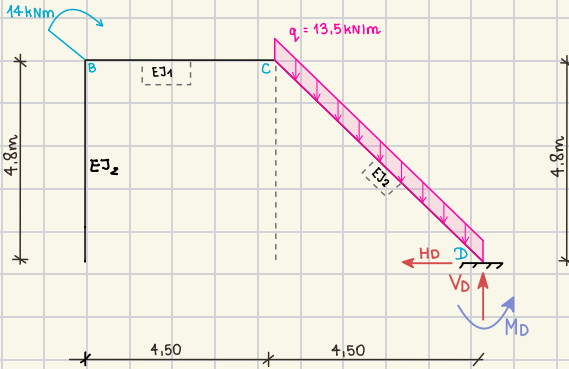
$$\frac{J_1}{J_2} = \frac{2140}{9800}$$

$$J_1 \cdot 9800 = 2140 J_0 \quad | : 9800$$

$$\frac{J_1}{J_0} = \frac{2140}{9800}$$

$$J_1 = 0,2184 J_0$$

Stan P, $x_1 = 0, x_2 = 0$



$$x = \sqrt{(4,8)^2 + (4,5)^2} = 6,5795$$

$$\sin \alpha = \frac{4,8}{6,5795} = 0,7295$$

$$\cos \alpha = \frac{4,5}{6,5795} = 0,6839$$

$$\sum Y = 0 \Rightarrow V_D - 13,5 \cdot 6,5795 = 0$$

$$V_D = 88,823 \text{ kN}$$

$$\sum M_D = 0 \Rightarrow -13,5 \cdot 6,5795 \cdot 2,25 + 14 - M_D = 0$$

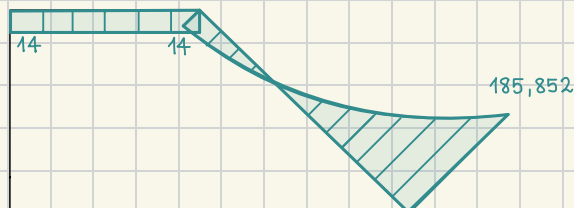
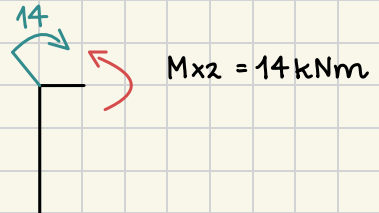
$$M_D = -185,853 \text{ kNm}$$

$$M_{x1} = 13,5 \cdot \cos \alpha \cdot x \cdot \frac{x}{2} - 185,853 + 88,823 \cdot x \cos \alpha$$

$$M_{x1} = 4,616 x^2 - 185,852 + 60,75 x$$

$$M(0) = -185,852 \text{ kNm}$$

$$M(6,5795) = 14 \text{ kNm}$$



Stan $x_1 = 1$

$$\sum Y = 0 \Rightarrow 1 + V_D = 0 \Rightarrow V_D = -1 \text{ kN}$$

$$\sum X = 0 \Rightarrow H_D = 0 \text{ kN}$$

$$\sum M_D = 0 \Rightarrow 9 - M_D = 0 \Rightarrow M_D = 9 \text{ kNm}$$

$$M_{x1} = 0 \text{ kNm}$$

$$M_{x2} = 1x$$

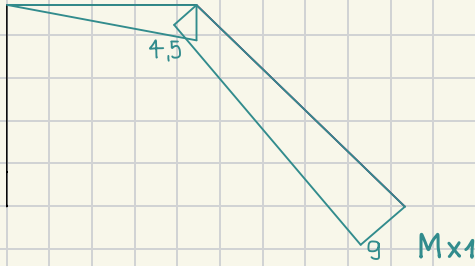
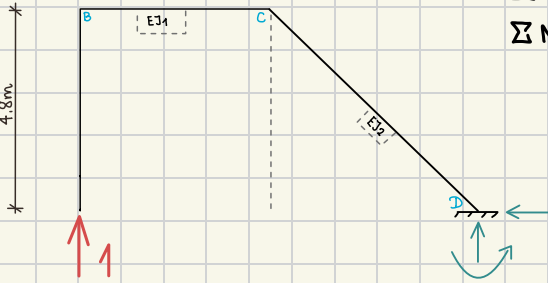
$$M_{x3} = 1 \cos x + 9$$

$$M(0) = 0 \text{ kNm}$$

$$M(4,5) = 4,5 \text{ kNm}$$

$$M(0) = 9 \text{ kNm}$$

$$M(6,5795) = 4,5 \text{ kNm}$$



Stan $x_2 = 1$

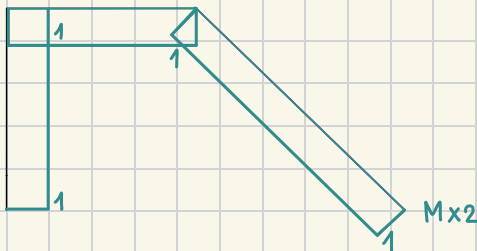
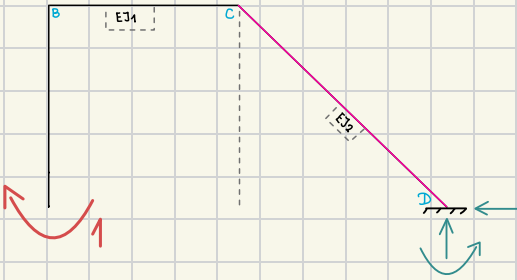
$$\sum Y = 0 \Rightarrow V_D = 0 \text{ kN}$$

$$\sum X = 0 \Rightarrow H_D = 0 \text{ kN}$$

$$\sum M_D = 0 \Rightarrow 1 - M_D = 0 \Rightarrow M_D = 1 \text{ kNm}$$

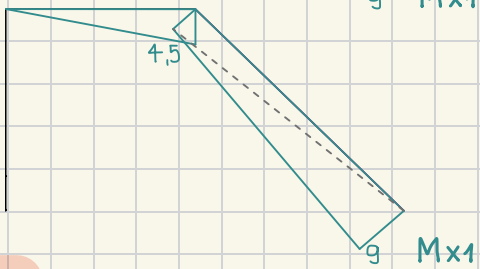
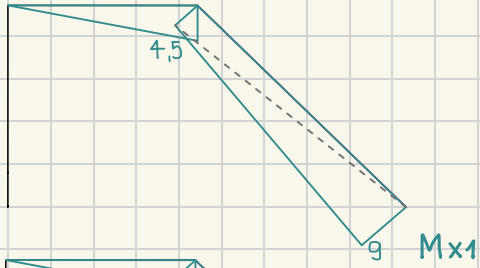
$$M_{x1} = 1 \text{ kNm}$$

$$M_{x1} = 1 \text{ kNm}$$



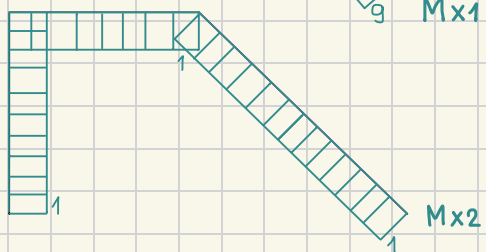
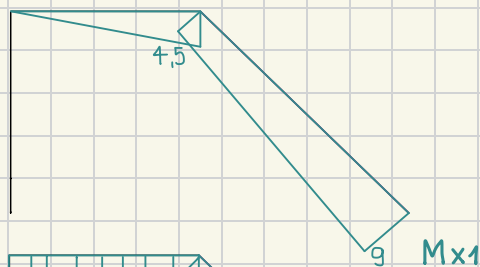
$$\delta_{11} = \sum \int \frac{\bar{M}_{x1} \cdot \bar{M}_{x1}}{EJ} dx$$

$$\delta_{11} = \frac{1}{EJ_1} \left[\frac{1}{2} \cdot 4,5 \cdot 4,5 \cdot \frac{2}{3} \cdot 4,5 \right] + \frac{1}{EJ_2} \left(\frac{1}{2} \cdot 4,5 \cdot 6,5795 \cdot \left(\frac{2}{3} \cdot 4,5 + \frac{1}{3} \cdot 9 \right) + \frac{1}{2} \cdot 9 \cdot 6,5795 \cdot \left(\frac{1}{3} \cdot 4,5 + \frac{2}{3} \cdot 9 \right) \right) = \frac{1}{0,2184 EJ_0} \cdot 30,375 + \frac{1}{EJ_0} \cdot 310,881 = \frac{449,98}{EJ_0}$$



$$\delta_{12} = \sum \int \frac{\bar{M}_{x1} \cdot \bar{M}_{x2}}{EJ} dx$$

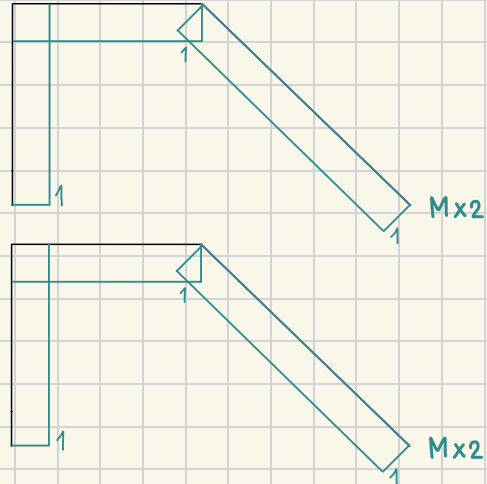
$$\delta_{12} = \frac{1}{EJ_1} \left[\frac{1}{2} \cdot 4,5 \cdot 4,5 \cdot 1 \right] + \frac{1}{EJ_2} \left[\frac{1}{2} \cdot 4,5 \cdot 6,5795 \cdot 1 + \frac{1}{2} \cdot 9 \cdot 6,5795 \cdot 1 \right] = \frac{1}{0,2184 EJ_0} \cdot 10,125 + \frac{1}{EJ_0} \cdot 44,4116 = \frac{90,778}{EJ_0}$$



$$\delta_{21} = \delta_{12}$$

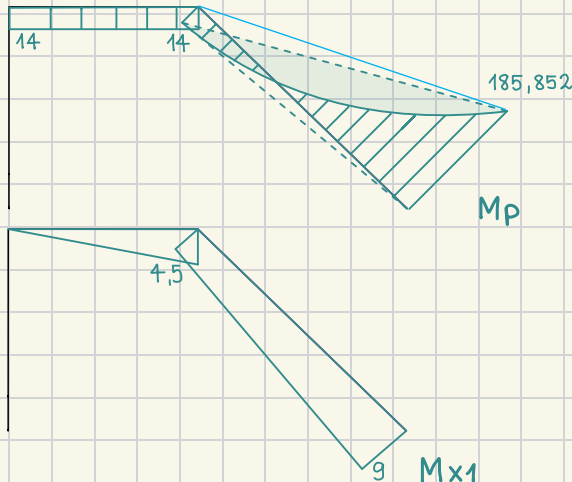
$$\delta_{22} = \sum \int \frac{\bar{M}_{x2} \cdot \bar{M}_{x2}}{EJ}$$

$$\delta_{22} = \frac{1}{EJ_1} [1 \cdot 4,5 \cdot 1] + \frac{1}{EJ_2} [1 \cdot 4,5 \cdot 1 + 1 \cdot 6,5795 \cdot 1] = \frac{1}{0,2184 EJ_0} \cdot 4,5 + \frac{1}{EJ_0} \cdot 11,3795 = \frac{31,99}{EJ_0}$$



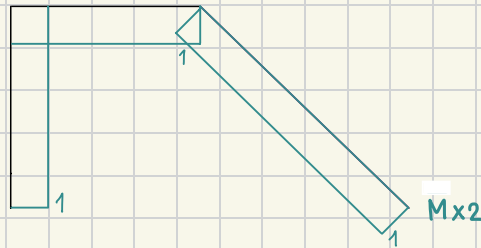
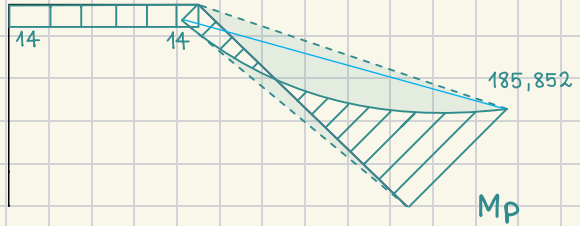
$$\sigma_{1p} = \sum \int \frac{M_p \cdot \bar{M}_{x1}}{EJ} dx \quad q_{\perp} = 13,5 \cdot 0,6839 = 9,2332$$

$$\delta_{1p} = \frac{1}{EJ_1} [14 \cdot 4,5 \cdot \frac{1}{2} \cdot 4,5] + \frac{1}{EJ_2} \left(-\frac{1}{2} \cdot 185,852 \cdot 6,5795 \cdot \left(\frac{2}{3} \cdot 9 + \frac{1}{3} \cdot 4,5 \right) + \frac{1}{2} \cdot 14 \cdot 6,5795 \cdot \left(\frac{1}{3} \cdot 9 + \frac{2}{3} \cdot 4,5 \right) + \frac{2}{3} \cdot \frac{9,2332 \cdot 6,5795^2}{8} \cdot 6,5795 \cdot \left(\frac{1}{2} \cdot 9 + \frac{1}{2} \cdot 4,5 \right) \right) = \frac{1}{0,2184 EJ_0} \cdot 141,75 - \frac{1}{EJ_0} \cdot 2830 = -\frac{2180,8}{EJ_0}$$



$$\sigma_{2p} = \sum \int \frac{M_p \cdot \bar{M}_{x2}}{EJ}$$

$$\sigma_{2p} = \frac{1}{EJ_1} [14 \cdot 4,5 \cdot 1] + \frac{1}{EJ_2} \left(-\frac{1}{2} \cdot 185,852 \cdot 6,5795 \cdot 1 + \frac{1}{2} \cdot 14 \cdot 6,5795 \cdot 1 + \frac{2}{3} \cdot \frac{9,2332 \cdot 6,5795^2}{8} \cdot 6,5795 \cdot 1 \right) = \frac{1}{0,2184 EJ_0} \cdot 63 - \frac{1}{EJ_0} \cdot 346,21 = -\frac{57,7}{EJ_0}$$



$$\begin{cases} \delta_{11} x_1 + \delta_{12} x_2 + \delta_{1p} = 0 \\ \delta_{21} x_1 + \delta_{22} x_2 + \delta_{2p} = 0 \end{cases}$$

$$\delta_{1p} = -\frac{2180,8}{EJ_0}$$

$$\delta_{2p} = -\frac{57,7}{EJ_0}$$

$$\delta_{11} = \frac{449,98}{EJ_0}$$

$$\delta_{12} = \frac{90,78}{EJ_0}$$

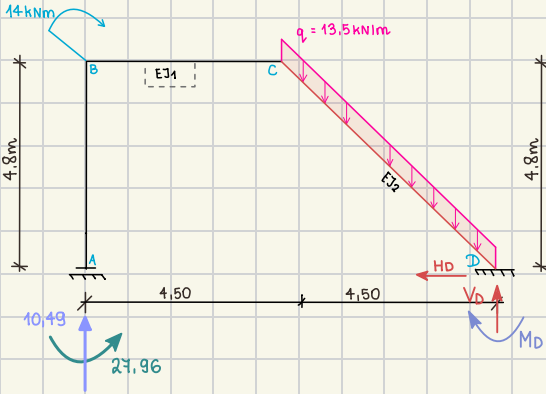
$$\delta_{21} = \frac{90,78}{EJ_0}$$

$$\delta_{22} = \frac{31,99}{EJ_0}$$

$$\begin{cases} \frac{449,98}{EJ_0} x_1 + \frac{90,78}{EJ_0} x_2 - \frac{2180,8}{EJ_0} = 0 \\ \frac{90,78}{EJ_0} x_1 + \frac{31,99}{EJ_0} x_2 - \frac{57,7}{EJ_0} = 0 \end{cases}$$

$$x_1 = 10,49 \text{ kN}$$

$$x_2 = -27,96 \text{ kNm}$$



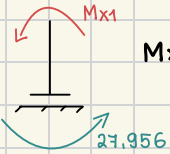
$$\sum Y = 0 \Rightarrow 10,49 - 13,5 \cdot 6,5795 + v_D = 0$$

$$v_D = 78,34 \text{ kN}$$

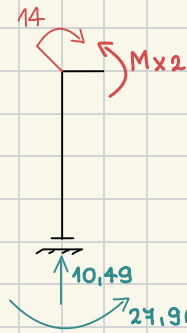
$$\sum X = 0 \Rightarrow H_D = 0 \text{ kN}$$

$$\sum M_D = 0 \Rightarrow -27,96 + 14 + 10,49 \cdot 9 - 13,5 \cdot 6,5795 \cdot 2,25 + M_D = 0$$

$$M_D = 119,43 \text{ kNm}$$



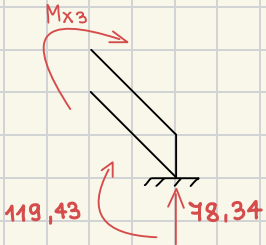
$$M_{x1} = -27,96 \text{ kNm}$$



$$M_{x2} = 10,49 \cdot x - 27,96 + 14 = 0$$

$$M(0) = -13,96 \text{ kNm}$$

$$M(4,5) = 33,23 \text{ kNm}$$



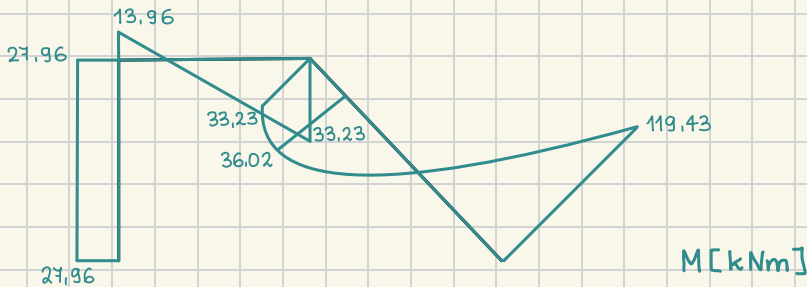
$$M_{x3} = -119,43 + 78,34 \cdot \cos \cdot x - 13,5 \cdot \cos \cdot x \cdot \frac{x}{2}$$

$$M_{x3} = -119,43 + 53,575x - 4,616x^2$$

$$M(0) = -119,43 \text{ kNm}$$

$$M(6,5795) = 33,23 \text{ kNm}$$

$$M(5,80) = 36,02 \text{ kNm}$$



Kontrola kinematyczna - nowy układ podstawowy

$$\sum M_A = 0 \Rightarrow 1 - V_D \cdot 9 = 0$$

$$9V_D = 1$$

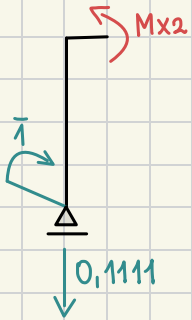
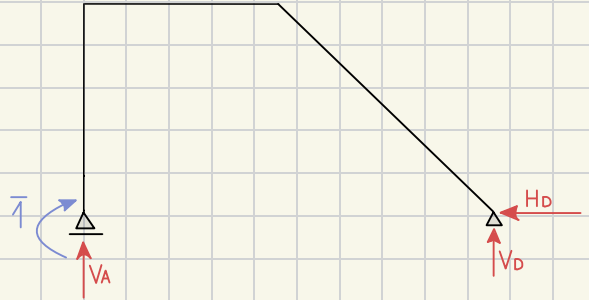
$$V_D = 0,1111$$

$$\sum M_D = 0 \Rightarrow 1 + V_A \cdot 9 = 0$$

$$9V_A = -1$$

$$V_A = -0,1111$$

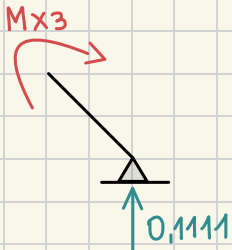
$$\text{Spr. } \sum Y = 0 \Rightarrow -0,1111 + 0,1111 = 0 \Rightarrow 0 = 0 \square$$



$$M_{x2} = -0,1111x + 1$$

$$M(0) = 1$$

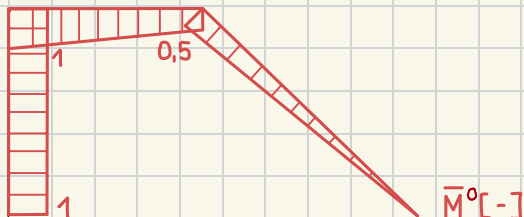
$$M(4,5) = 0,5$$



$$M_{x3} = 0,1111 \cos \cdot x$$

$$M(0) = 0$$

$$M(6,5795) = 0,5$$



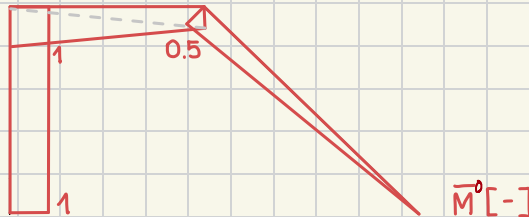
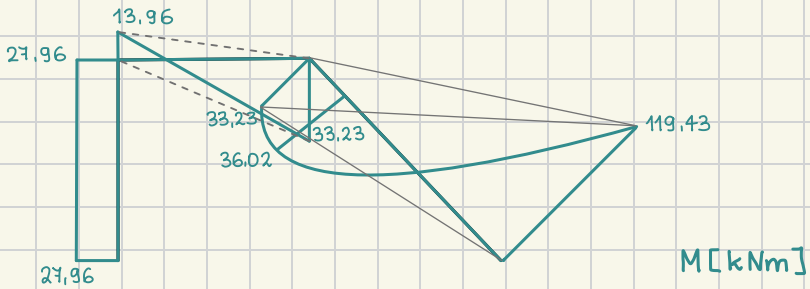
$$\bar{1} \cdot H = \Sigma \int \frac{M \cdot \bar{M}^0}{EJ} dx = 0$$

$$\begin{aligned} \delta_0^H &= \frac{1}{EJ_1} \left[-\frac{1}{2} \cdot 13,96 \cdot 4,5 \cdot \left(\frac{2}{3} \cdot 1 + \frac{1}{3} \cdot 0,5 \right) + \frac{1}{2} \cdot 33,23 \cdot 4,5 \cdot \left(\frac{1}{3} \cdot 1 + \frac{2}{3} \cdot 0,5 \right) \right. \\ &+ \frac{1}{EJ_2} \left[-27,96 \cdot 4,8 \cdot 1 - \frac{1}{2} \cdot 119,43 \cdot 6,5795 \cdot \frac{1}{3} \cdot 0,5 + \frac{1}{2} \cdot 33,23 \cdot 6,5795 \right. \\ &\cdot \frac{2}{3} \cdot 0,5 + \frac{2}{3} \cdot \frac{9,2332 \cdot 6,5795^2}{8} \cdot 6,5795 \cdot \frac{1}{2} \cdot 0,5 \left. \right] = \\ &= \frac{1}{0,2184 EJ_0} \cdot 23,68 + \frac{1}{EJ_0} \cdot (-108,443) = -\frac{0,018}{EJ_0} \end{aligned}$$

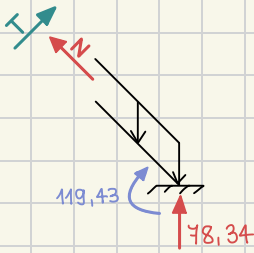
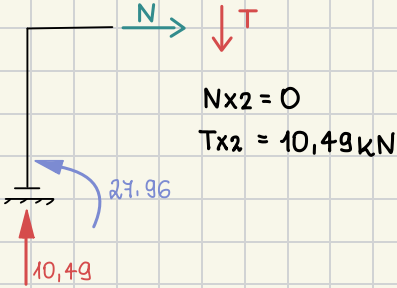
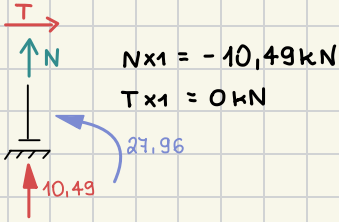
$$|\delta| < \frac{1}{EJ}$$

Warunek został spełniony

$$\frac{0,018}{EJ_0} < \frac{1}{EJ_0}$$



Siły przekrojowe (M,N,T) od zadanego obciążenia



$$T_{x3} = -78,34 \cos + 9,2332 x$$

$$T(0) = -53,58 \text{ kN}$$

$$T(6,5795) = 7,17 \text{ kN}$$

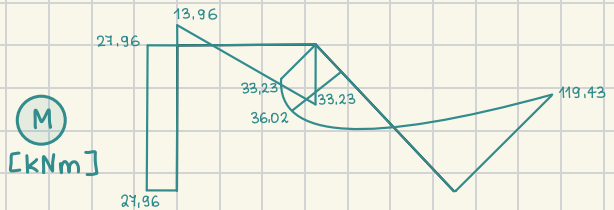
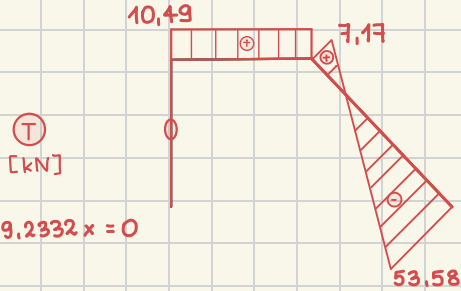
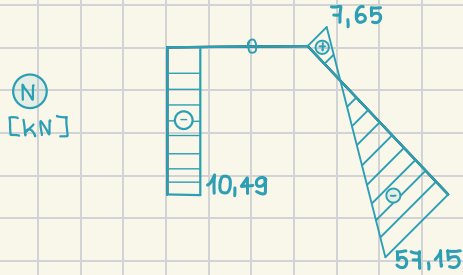
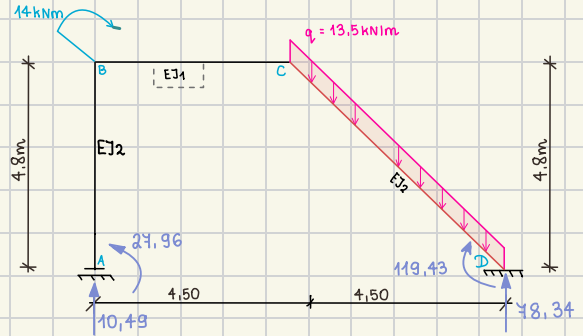
$$N_{x3} = -78,34 \sin + 9,8488 x$$

$$N(0) = -57,15 \text{ kN}$$

$$N(6,5795) = 7,65 \text{ kN}$$

$$-78,34 \sin + 9,8488 x = 0$$

$$x = 5,80 \text{ m}$$



Sprawdzenie naprężeń w obu przekrojach

$$\sigma_{dop} = 235 \text{ MPa}$$

I 200

$$M_{max} = 36,02 \text{ kNm}$$

$$\frac{M_{max}}{W} \leq 235 \text{ MPa}$$

$$\sigma_{max} = \frac{|M|}{W_y} + \frac{|N|}{A} = \frac{36,02}{214} + 0$$

$$\sigma_{max} = \frac{36,02}{214} = 16,832 \text{ kN/cm}^2 \Rightarrow 168,32 \text{ MPa}$$

$$168,32 \text{ MPa} < 235 \text{ MPa} \Rightarrow \sigma_{max} < \sigma_{dop}$$

I 300

$$M_{max} = 119,43 \text{ kNm}$$

$$\frac{M_{max}}{W} \leq 235 \text{ MPa}$$

$$\sigma_{max} = \frac{|M|}{W_y} + \frac{|N|}{A} = \frac{119,43}{653} + \frac{57,15}{69,1} = \frac{119,43}{653} + \frac{57,15}{69,1} = 19,117 \text{ kN/cm}^2$$

$$\sigma_{max} = 19,117 \text{ kN/cm}^2 \Rightarrow 191,17 \text{ MPa}$$

$$191,17 \text{ MPa} < 235 \text{ MPa} \Rightarrow \sigma_{max} < \sigma_{dop}$$

Wniosek :

W obydwu przypadkach warunków nośności został spełniony. Przekroje można by zoptymalizować i zmniejszyć, co wymagałoby ponownego obliczenia zadania dla zmienionych przekrojów.

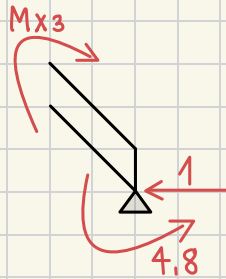
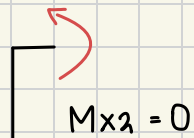
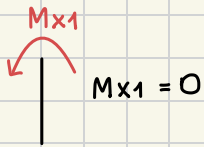
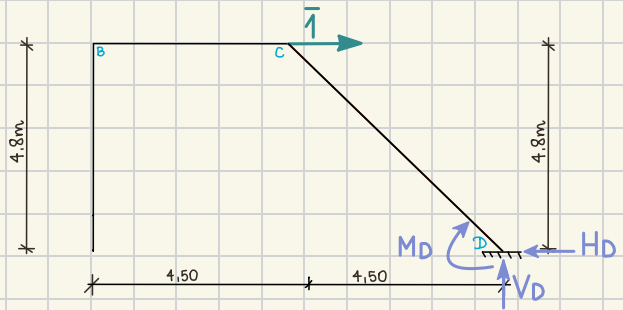
Przemieszczenie poziome w punkcie "C"

$$\sum Y = 0 \Rightarrow V_D = 0 \text{ kN}$$

$$\sum X = 0 \Rightarrow H_D = 1 \text{ kN}$$

$$\sum M_D = 0 \Rightarrow 1 \times 4,8 + M_D = 0$$

$$M_D = -4,8 \text{ kNm}$$

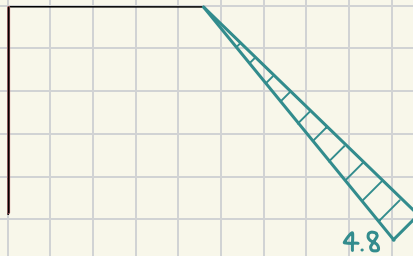


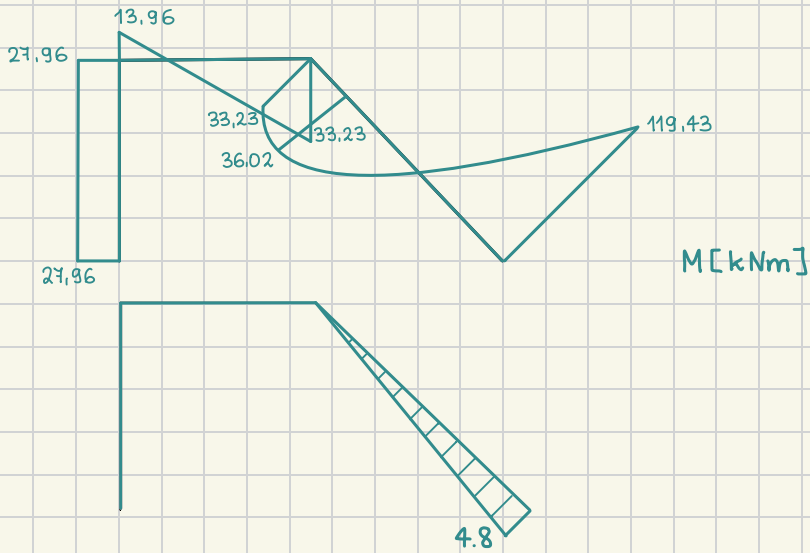
$$M_{x3} = -1 \sin \alpha \cdot x + 4,8$$

$$M_{x3} = -0,7295x + 4,8$$

$$M(0) = 4,8 \text{ kNm}$$

$$M(6,5795) = 0 \text{ kNm}$$





$$\delta_C^H = \frac{1}{EJ_2} \left[-\frac{1}{2} \cdot 119,43 \cdot 6,5795 \cdot \frac{2}{3} \cdot 4,8 + \frac{1}{2} \cdot 33,23 \cdot 6,5795 \cdot \frac{1}{3} \cdot 4,8 \right. \\ \left. + \frac{2}{3} \cdot \frac{9,2332 \cdot 6,5795^2}{8} \cdot 6,5795 \cdot \frac{1}{2} \cdot 4,8 \right] = -\frac{556,38}{EJ_2} = -0,02704 \text{ m}$$