POLITECHNIKA POZNAŃSKA
WYDZIAŁ INŻYNIERII LĄDOWEJ I TRANSPORTU
INSTYTUT ANALIZY KONSTRUKCJI
ZAKŁAD MECHANIKI BUDOWLI

Nazwisko i imię studenta: **Fiuta Julia**
Rok akademicki: **2021/22**
Grupa **B4**
Grupa projektowa: **gr_proj_B3+B4**

ĆWICZENIE NR 1

METODA SIŁ

Termin oddania: 26.01.2022

Data	Uwagi sprawdzającego	Podpis

Dla zadanej **RAMY** (schemat nr **3**) należy:

1. Przyjąć wstępnie przekroje I_1 i I_2 z profili dwuteowych (IN, IPE, HEB, HEA).
2. Korzystając z metody sił obliczyć siły przekrojowe (M, N, T) od zadanego obciążenia i wykonać kontrolę kinematyczną. Obliczyć przemieszczenie poziome p. „F" korzystając z równania pracy wirtualnej i twierdzenia redukcyjnego. Sprawdzić naprężenia w obu grupach przekrojów I_1 i I_2 oraz sformułować wnioski (w przypadku niespełnienia warunku nośności, obliczeń nie trzeba powtarzać).
3. Korzystając z metody sił obliczyć siły przekrojowe (M, N, T) od zadanego wpływu temperatury i osiadań podpór, wykonać kontrolę kinematyczną. Obliczyć obrót przekroju w p. „S" korzystając z równania pracy wirtualnej i twierdzenia redukcyjnego.

Dla zadanej **BELKI** (schemat nr **7**) należy:

1. Przyjąć wstępnie przekroje I_1 i I_2 z profili dwuteowych (IN, IPE, HEB, HEA).
2. Korzystając z metody sił obliczyć siły przekrojowe (M, N, T) od zadanego obciążenia i wykonać kontrolę kinematyczną. Obliczyć obrót przekroju w p. „D" korzystając z równania pracy wirtualnej i twierdzenia redukcyjnego. Sprawdzić naprężenia w obu grupach przekrojów I_1 i I_2 oraz sformułować wnioski (w przypadku niespełnienia warunku nośności, obliczeń nie trzeba powtarzać).

We wszystkich obliczeniach przyjąć: E = 210 GPa, f_y = 215 MPa.

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DANE DLA RAMY:

Nr schematu: **3**

Dane:

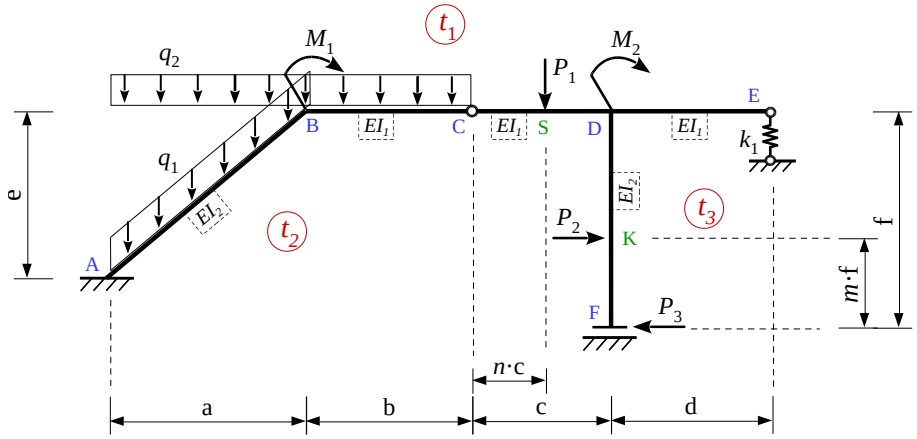
a [m]	b [m]	c [m]	d [m]	e [m]	f [m]	k_1 [kN/m]	k_2 [kN/m]
3,5	4,4	5,6	4,4	3,5	6,5	∞	-

Obciążenie:

q_1 [kN/m]	q_2 [kN/m]	P_1 [kN]	P_2 [kN]	P_3 [kN]	M_1 [kNm]	M_2 [kNm]	*n*	*m*
-	9,1	-	16,0	18,0	-	20,0	0,7	0,5

Osiadanie podpór:		Rozkład temperatur:	
osiadanie kątowe[2] węzła nr F [º]	2,0	temperatura montażu t_m [ºC]	20,0
osiadanie pionowe[1] węzła nr F [cm]	3,9	t_1 [ºC]	-5,0
osiadanie poziome[1] węzła nr A [cm]	-1,7	t_2 [ºC]	20,0
		t_3 [ºC]	35,0

Schemat 3:



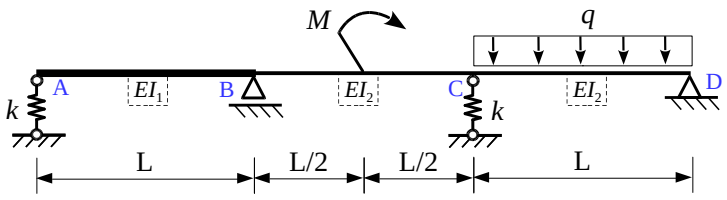
DANE DLA BELKI:

Nr schematu: **7**

Dane:

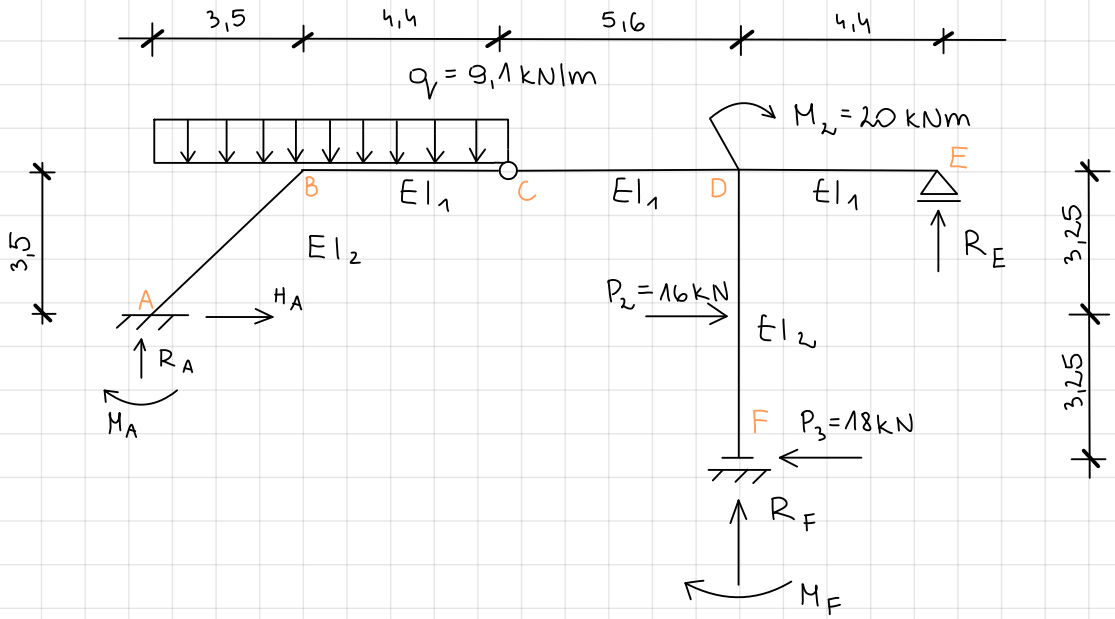
L [m]	k [kN/m]	q [kN/m]	M [kNm]
5,0	4900	11,3	-42,0

Schemat 7:



[1] wartość dodatnia oznacza zwrot w dół lub w prawo, ujemna – w górę lub w lewo
[2] wartość dodatnia oznacza zwrot w prawo, ujemna – w lewo

RAMA



analiza kinematyczna

$t = 2$ $\quad p = 3 + 2 + 2 + 1$

$3t = p$

$6 \neq 8 \longrightarrow$ nie może być statycznie wyznaczalny

$n = p - 3t$

$n = 2$

↓

układ statycznie niewyznaczalny, geometrycznie niezmienny,

$ssn = 2$

wstępne przyjęcie przekrojów

$E = 210$ GPa

I140 $I_1 = 573,0$ cm^4

$EI_1 = 210 \cdot 10^6 \cdot 573 \cdot 10^{-8} = 1203,3$ kNm^2

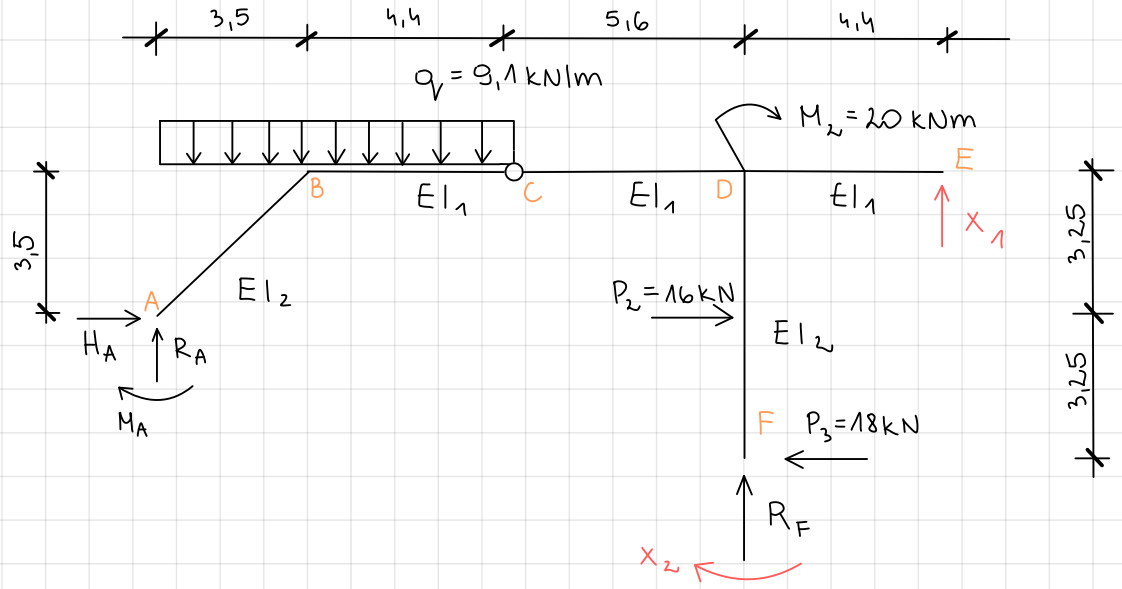
I200 $I_2 = 2140,0$ cm^4

$EI_2 = 210 \cdot 2140 \cdot 10^6 \cdot 10^{-8} = 4494$ kNm^2

$\frac{EI_1}{EI_2} = \frac{1203,3}{4494} = 0,2678 \quad | \cdot EI_2$

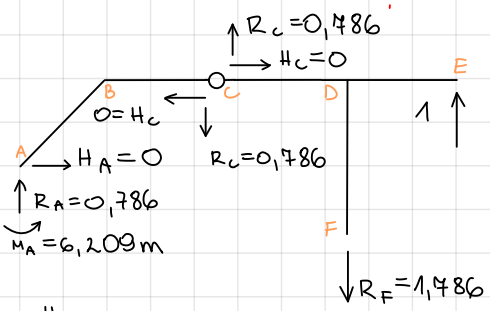
$EI_1 = 0,2678 \cdot EI_2$

układ podstawowy



$$\begin{cases} \delta_{11} X_1 + \delta_{12} X_2 + \delta_{1P} = 0 \\ \delta_{21} X_1 + \delta_{22} X_2 + \delta_{2P} = 0 \end{cases}$$

stan $x_1 = 1$



$\Sigma M_C'' = 0$

$-1 \cdot 10 + R_F \cdot 5,6 = 0 \quad R_F = 1,786$

$\Sigma Y'' = 0$

$R_C - 1,786 + 1 = 0 \quad R_C = 0,1786$

$\Sigma X'' = 0 \quad H_C = 0$

$\Sigma X' = 0 \quad H_A = 0$

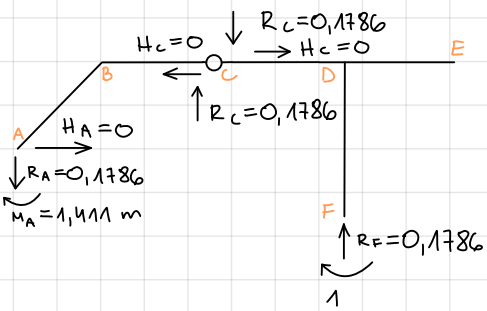
$\Sigma Y' = 0$

$R_A - 0,1786 = 0 \quad R_A = 0,1786$

$\Sigma M_A' = 0$

$0,1786 \cdot 7,9 - M_A = 0 \quad M_A = 6,209$ m

stan $x_2 = 1$



$\Sigma M_C'' = 0$

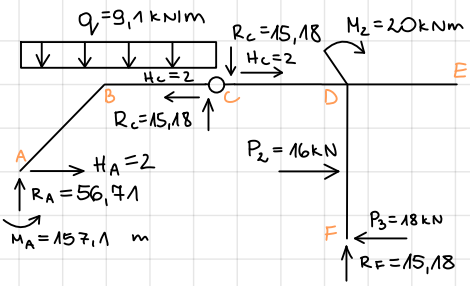
$1 - R_F \cdot 5,6 = 0 \quad R_F = 0,1786$

$\Sigma M_A' = 0$

$M_A - 0,1786 \cdot 7,9 = 0$

$M_A = 1,411$ m

stan "P"



$\Sigma M_C'' = 0$

$20 - 16 \cdot 3,25 + 18 \cdot 6,5 - R_F \cdot 5,6 =$

$= 0 \quad R_F = 15,18$

$\Sigma X'' = 0 \quad H_C + 16 - 18 = 0$

$H_C = 2$

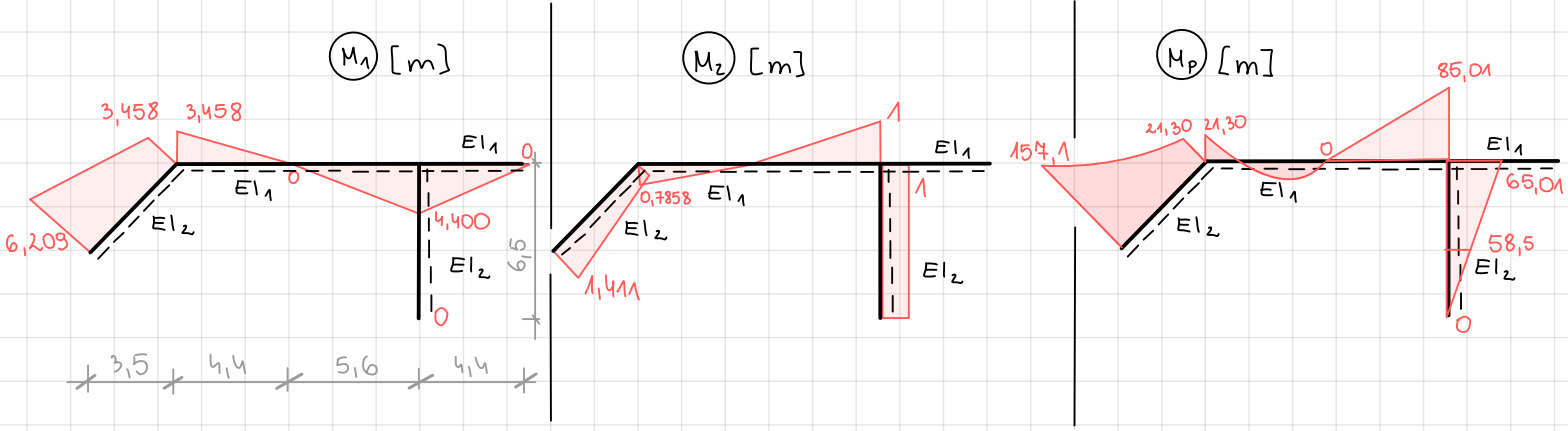
$\Sigma Y' = 0 \quad R_A - 9,1 \cdot 7,9 + 15,18 = 0$

$R_A = 56,71$

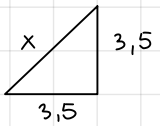
$\Sigma M_A = 0$

$-M_A + 9,1 \cdot 7,9 \cdot 3,95 - 2 \cdot 3,5 - 15,18 \cdot 7,9 = 0$

$M_A = 157,1$ m



$$\begin{cases} \delta_{11}X_1 + \delta_{12}X_2 + \delta_{1P} = 0 \\ \delta_{21}X_1 + \delta_{22}X_2 + \delta_{2P} = 0 \end{cases}$$

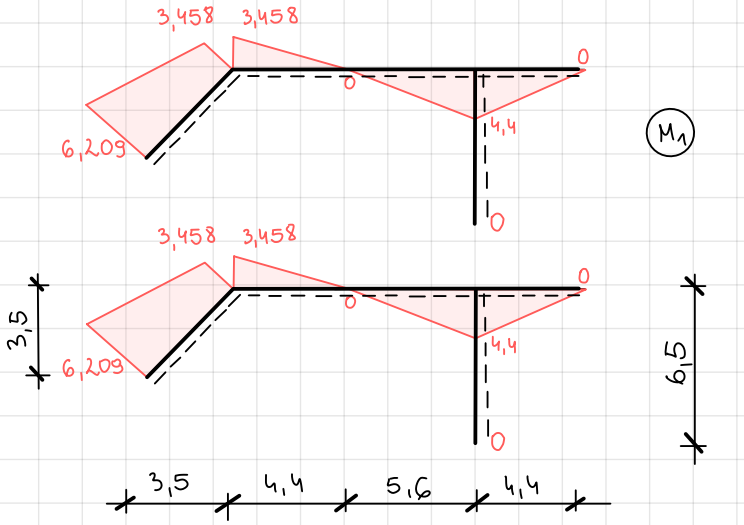


$3,5^2 + 3,5^2 = x^2$

$x = 4,95\ m$

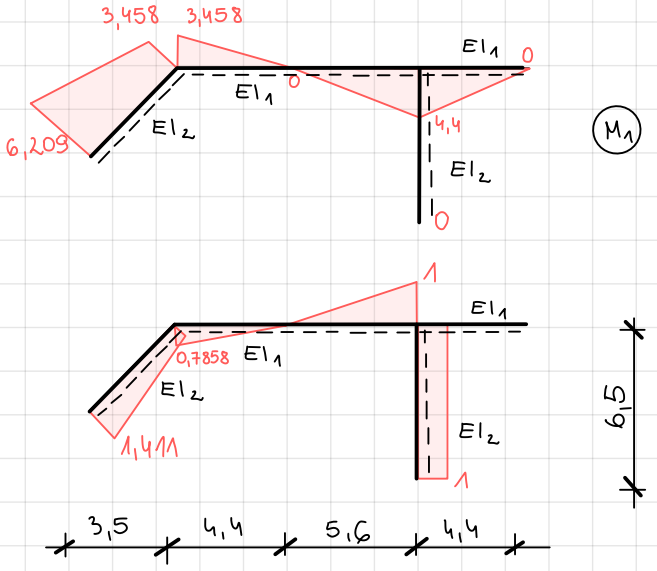
$$\delta_{11} = \sum \int_S \frac{M_1 M_1}{EI} ds = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 3,458\cdot 4,4\cdot\frac{2}{3}\cdot 3,458 + \frac{1}{2}\cdot 4,4\cdot 5,6\cdot\frac{2}{3}\cdot 4,4 + \frac{1}{2}\cdot 4,4\cdot 4,4\cdot\frac{2}{3}\cdot 4,4\right] +$$

$$\frac{1}{EI_2}\left[\frac{1}{2}\cdot 6,209\cdot 4,95\cdot\left(\frac{2}{3}\cdot 6,209 + \frac{1}{3}\cdot 3,458\right) + \frac{1}{2}\cdot 3,458\cdot 4,95\cdot\left(\frac{1}{3}\cdot 6,209 + \frac{2}{3}\cdot 3,458\right)\right] = \frac{82,07}{0,2678EI_2} + \frac{118,8}{EI_1} = \frac{425,2}{EI_2}$$



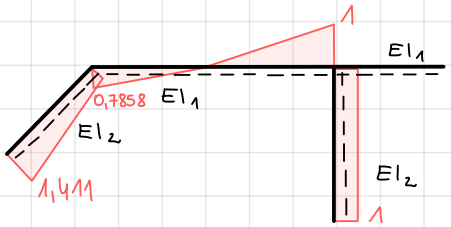
$$\delta_{12} = \delta_{21} = \frac{1}{EI_1}\left[-\frac{1}{2}\cdot 3,458\cdot 4,4\cdot\frac{2}{3}\cdot 0,7858 - \frac{1}{2}\cdot 5,6\cdot 4,4\cdot\frac{2}{3}\cdot 1\right] + \frac{1}{EI_2}\left[-\frac{1}{2}\cdot 6,209\cdot 4,95\left(\frac{2}{3}\cdot 1,4111 + \frac{1}{3}\cdot 0,7858\right) -\right.$$

$$\left. + \frac{1}{2}\cdot 3,458\cdot 4,95\left(\frac{2}{3}\cdot 0,7858 + \frac{1}{3}\cdot 1,4111\right)\right] = \frac{-12,2}{0,2678EI_2} - \frac{26,99}{EI_2} = \frac{-72,55}{EI_2}$$

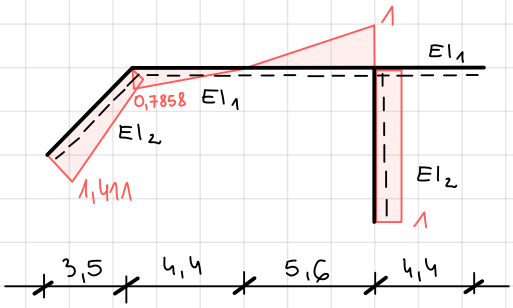


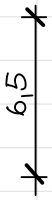
$$\delta_{22} = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 0{,}7858\cdot 4{,}4\cdot\frac{2}{3}\cdot 0{,}7858 + \frac{1}{2}\cdot 1\cdot 5{,}6\cdot\frac{2}{3}\cdot 1\right] + \frac{1}{EI_2}\left[\frac{1}{2}\cdot 4{,}95\cdot 1{,}411\cdot\left(\frac{2}{3}\cdot 1{,}411 + \frac{1}{3}\cdot 0{,}7858\right) +\right.$$

$$\left. + \frac{1}{2}\cdot 4{,}95\cdot 0{,}7858\left(\frac{2}{3}\cdot 0{,}7858 + \frac{1}{3}\cdot 1{,}411\right) + 1\cdot 6{,}5\cdot 1\right] = \frac{2{,}773}{EI_1} + \frac{12{,}63}{EI_2} = \frac{2{,}766}{0{,}2678EI_2} + \frac{12{,}63}{EI_2} = \frac{22{,}99}{EI_2}$$



M_2



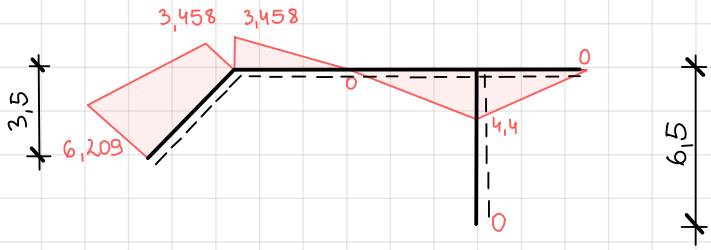


M_2

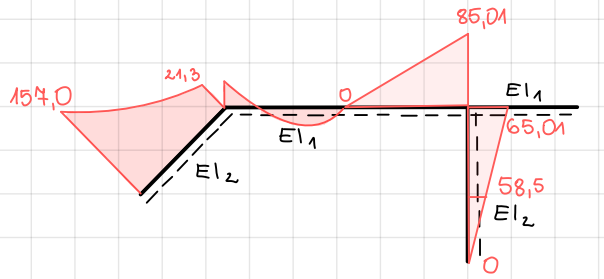
$$\delta_{1P} = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 21{,}3\cdot 4{,}4\cdot\frac{2}{3}\cdot 3{,}458 - \frac{2}{3}\cdot\frac{9{,}1\cdot(4{,}4)^2}{8}\cdot 4{,}4\cdot\frac{1}{2}\cdot 3{,}458 - 4{,}4\cdot\frac{1}{2}\cdot 5{,}6\cdot\frac{2}{3}\cdot 85{,}01\right] +$$

$$+ \frac{1}{EI_2}\left[\frac{1}{2}\cdot 157\cdot 4{,}95\left(\frac{2}{3}\cdot 6{,}209 + \frac{1}{3}\cdot 3{,}458\right) + \frac{1}{2}\cdot 21{,}3\cdot 4{,}95\left(\frac{2}{3}\cdot 3{,}458 + \frac{1}{3}\cdot 6{,}209\right) - \frac{2}{3}\cdot\frac{9{,}1\cdot(3{,}5)^2}{8}\cdot 4{,}95\cdot\frac{1}{2}\right.$$

$$\left.(6{,}209 + 3{,}458)\right] = \frac{-701{,}9}{EI_1} + \frac{2064}{EI_2} = \frac{-557{,}0}{EI_2}$$



M_1



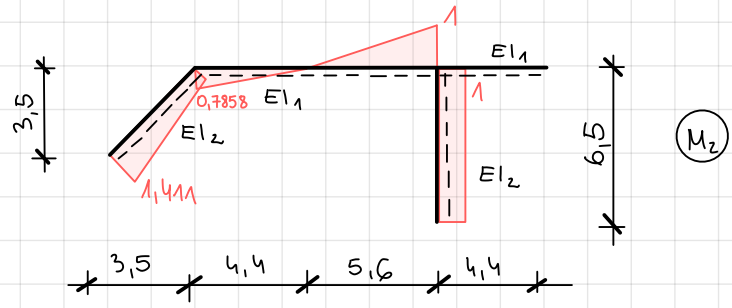
M_P

$$\delta_{2P} = \frac{1}{EI_1}\left[-\frac{1}{2}\cdot 21{,}3\cdot 4{,}4\cdot\frac{2}{3}\cdot 0{,}7858+\frac{2}{3}\cdot\frac{9{,}1\cdot(4{,}4)^2}{8}\cdot 4{,}4\cdot\frac{1}{2}\cdot 0{,}7858+\frac{1}{2}\cdot 1\cdot 5{,}6\cdot\frac{2}{3}\cdot 85{,}01\right]+$$

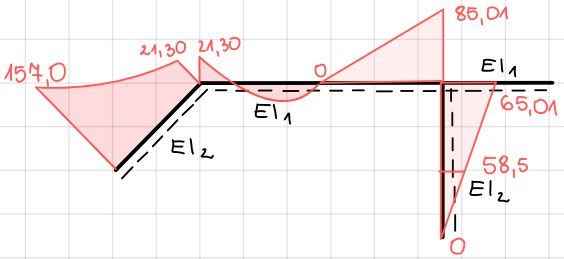
$$+\frac{1}{EI_2}\left[-\frac{1}{2}\cdot 157\cdot 4{,}95\left(\frac{2}{3}\cdot 1{,}4111+\frac{1}{3}\cdot 0{,}7858\right)-\frac{1}{2}\cdot 21{,}3\cdot 4{,}95\left(\frac{1}{3}\cdot 1{,}4111+\frac{2}{3}\cdot 0{,}7858\right)+\frac{2}{3}\cdot\frac{9{,}1\cdot(3{,}5)^2}{8}\cdot 4{,}95\cdot\frac{1}{2}\cdot\right.$$

$$\left.(1{,}4111+0{,}7858)+1\cdot 3{,}25\cdot\frac{1}{2}(65{,}01+58{,}5)+\frac{1}{2}\,3{,}25\cdot 58{,}5\cdot 1\right]=\frac{159{,}5}{EI_1}+\frac{-173{,}4}{EI_2}=\frac{159{,}5}{0{,}2678\,EI_2}-\frac{173{,}4}{EI_2}=$$

$$=\frac{422{,}2}{EI_2}$$



M_z



M_p

$$\delta_{11}=\frac{425{,}2}{EI_2} \qquad \delta_{1P}=\frac{-557{,}0}{EI_2}$$

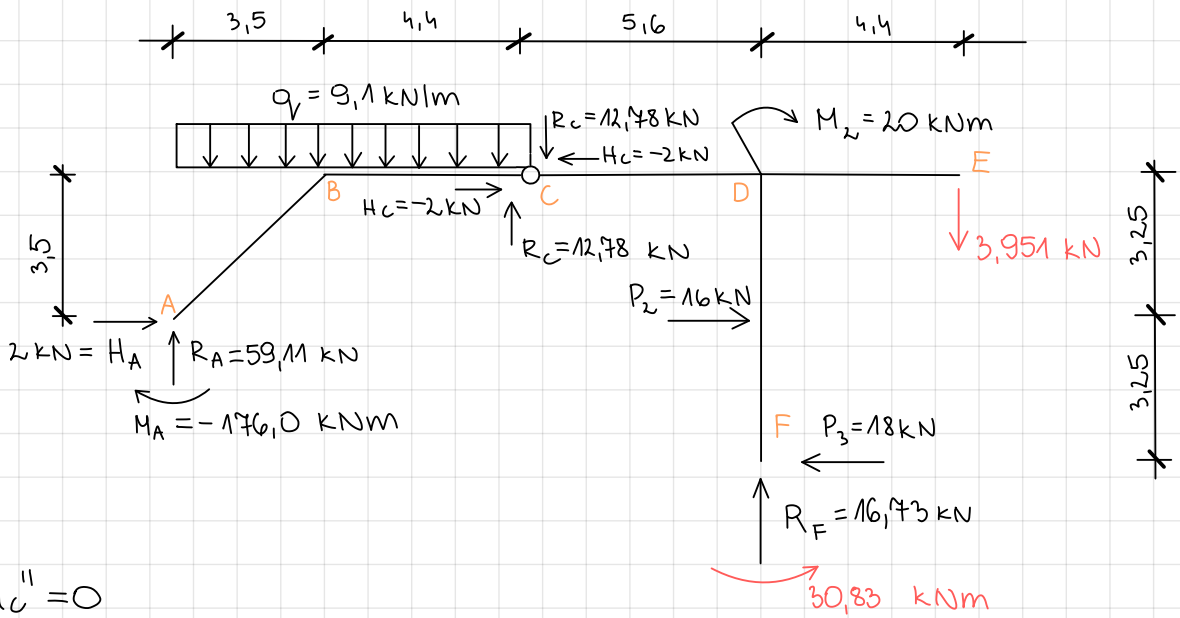
$$\delta_{22}=\frac{22{,}99}{EI_2} \qquad \delta_{2P}=\frac{422{,}2}{EI_2}$$

$$\delta_{12}=\delta_{21}=\frac{-72{,}55}{EI_2}$$

$$\begin{cases}\delta_{11}X_1+\delta_{12}X_2+\delta_{1P}=0\\ \delta_{21}X_1+\delta_{22}X_2+\delta_{2P}=0\end{cases}$$

$$\begin{cases}\frac{425{,}2}{EI_2}X_1+\frac{-72{,}55}{EI_2}X_2+\frac{-557}{EI_2}=0\ /\cdot EI_2\\ \frac{-72{,}55}{EI_2}X_1+\frac{22{,}99}{EI_2}X_2+\frac{422{,}2}{EI_2}=0\ /\cdot EI_2\end{cases}$$

$$\begin{cases}425{,}2\,X_1-72{,}55\,X_2-557=0\\ -72{,}55\,X_1+22{,}99\,X_2+422{,}2=0\end{cases}\longrightarrow\begin{aligned}X_1&=-3{,}951\ \text{kN}\\ X_2&=-30{,}83\ \text{kNm}\end{aligned}$$



$\Sigma M_C^{II} = 0$

$20 - 16 \cdot 3,25 + 18 \cdot 6,5 - R_F \cdot 5,6 - 30,83 + 3,951 \cdot 10 = 0$

$R_F = 16,73$ kN

$\Sigma Y^{II} = 0$

$-R_C + 16,73 - 3,951 = 0$

$R_C = 12,78$ kN

$\Sigma Y^{I} = 0$

$R_A - 9,1 \cdot 7,9 + 12,78 = 0$

$R_A = 59,11$ kN

$\Sigma X^{II} = 0$

$-H_C + 16 - 18 = 0$

$H_C = -2$ kN

$\Sigma X^{I} = 0$

$-2 + H_A = 0$

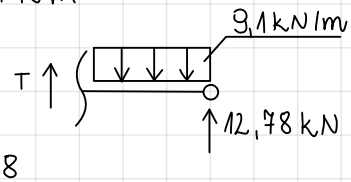
$H_A = 2$ kN

$\Sigma M_A^{I} = 0$

$M_A + 9,1 \cdot 7,9 \cdot 3,95 + (-2) \cdot 3,5 - 13,14 \cdot 7,9 = 0$

$M_A = -176,0$ kNm

Mekt.:



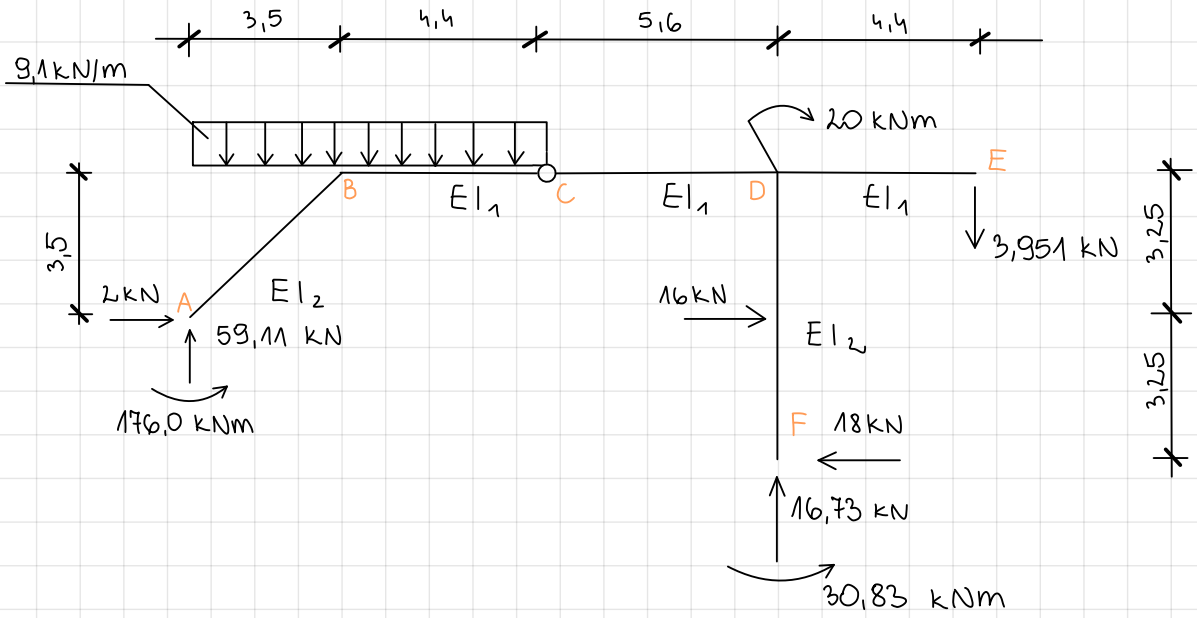
$T = 9,1x - 12,78$

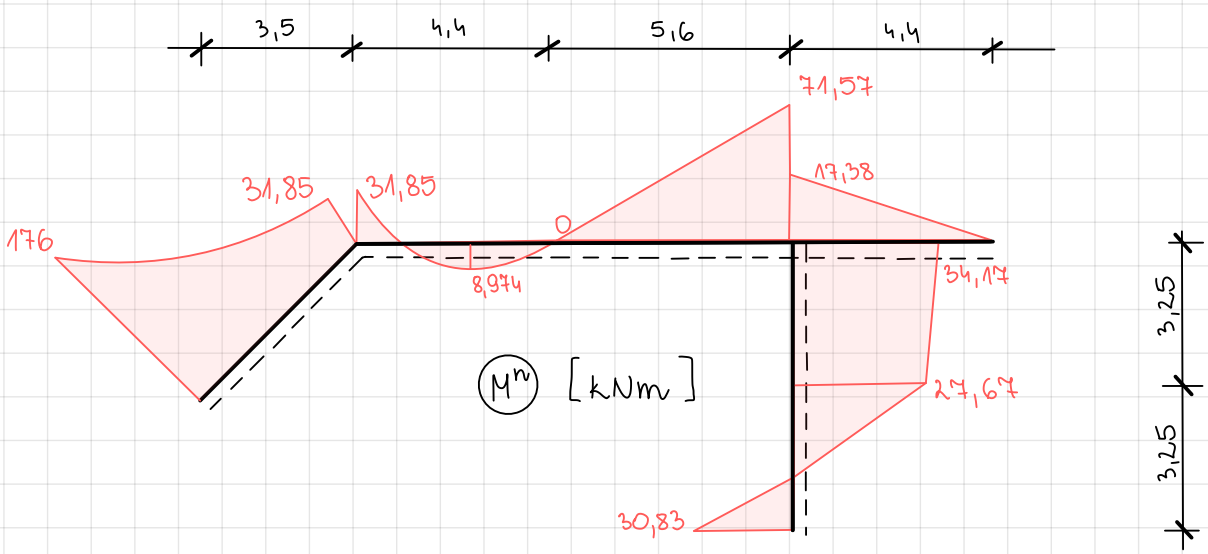
$0 = 9,1x - 12,78$

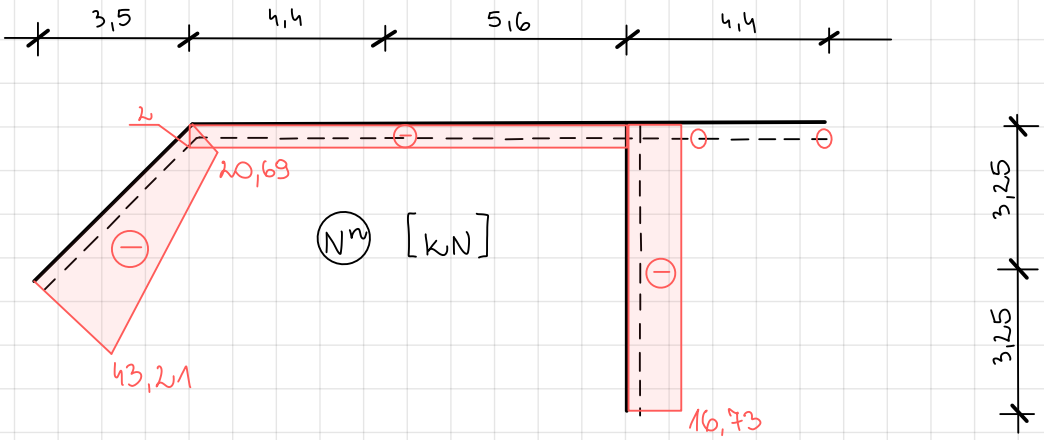
$x = 1,4$ m

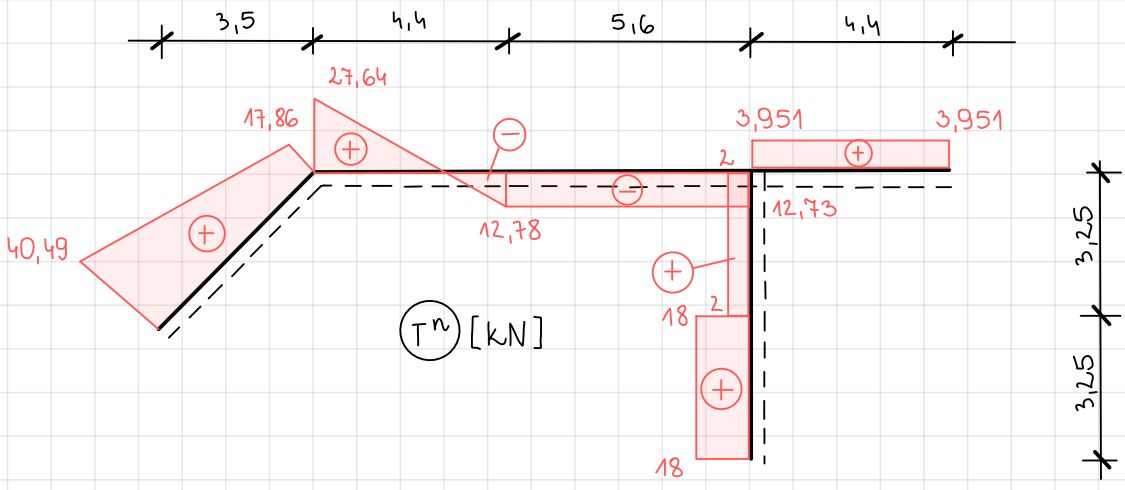
$M(1,4) = 12,78 \cdot 1,4 - 9,1 \cdot 1,4 \cdot 0,7 =$

$= 8,974$ kNm

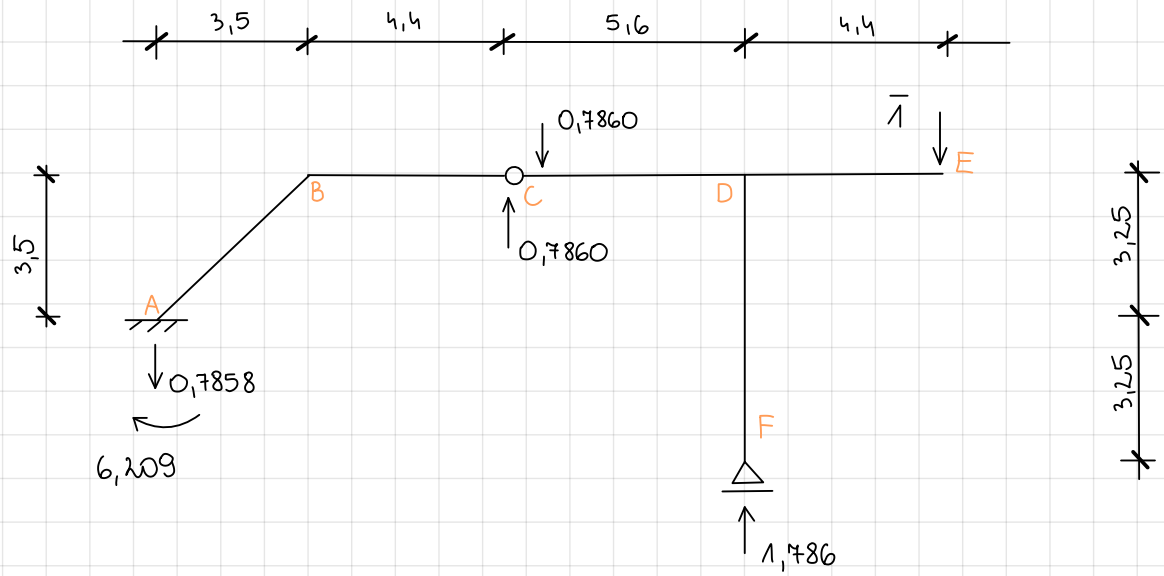


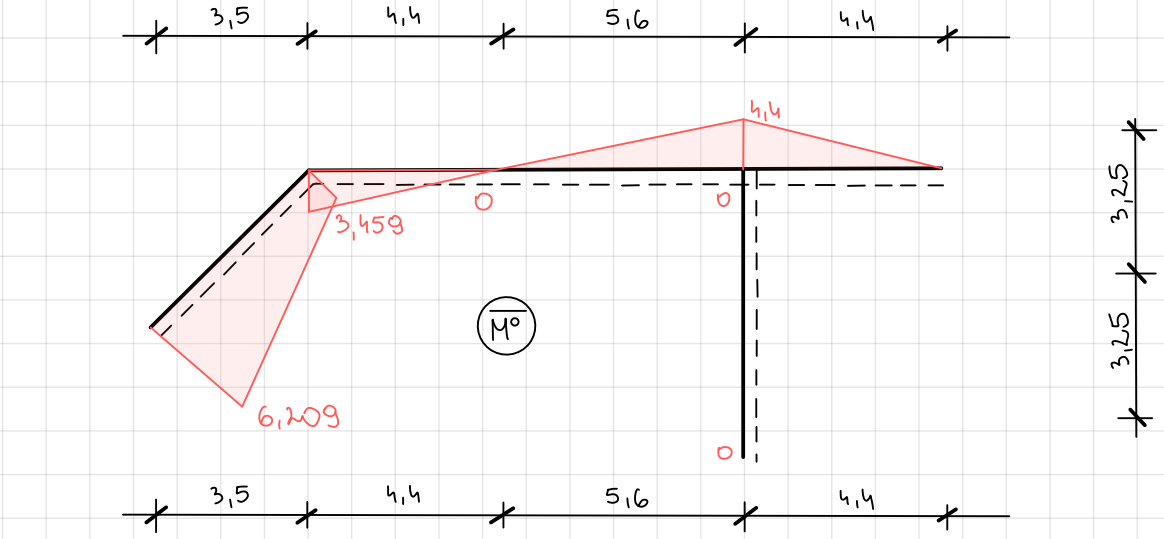

3,5
4,4
5,6
4,4
71,57
17,38
31,85
31,85
176
0
8,974
34,17
27,67
30,83
M^n [kNm]
3,25
3,25

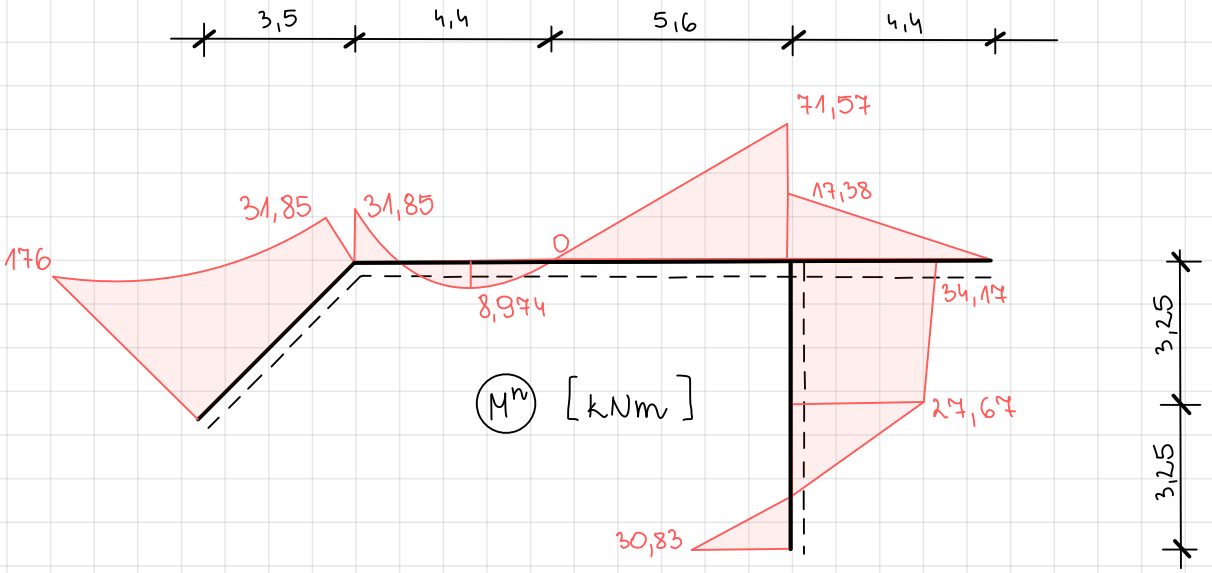

3,5
4,4
5,6
4,4
20,69
43,21
16,73
N^n [kN]
3,25
3,25


3,5
4,4
5,6
4,4
27,64
17,86
3,951
3,951
12,78
12,73
40,49
18
18
T^n [kN]
3,25
3,25

kontrola kinematyczna







$$V_E = \frac{1}{EI_1}\left[-\frac{1}{2}\cdot 31{,}85\cdot 4{,}4\cdot\frac{2}{3}\cdot 3{,}459+\frac{2}{3}\cdot\frac{9{,}1\cdot 4{,}4^2}{8}\cdot 4{,}4\cdot\frac{1}{2}\cdot 3{,}459+\frac{1}{2}\cdot 4{,}4\cdot 5{,}6\cdot\frac{2}{3}\cdot 71{,}57+\frac{1}{2}\cdot 4{,}4\cdot 4{,}4\cdot\frac{2}{3}\cdot 17{,}38\right]+\frac{1}{EI_2}\left[-\frac{1}{2}\cdot 176\cdot 4{,}95\left(\frac{2}{3}\cdot 6{,}209+\frac{1}{3}\cdot 3{,}459\right)-\frac{1}{2}\cdot 31{,}85\cdot 4{,}95\cdot\left(\frac{1}{3}\cdot 6{,}209+\frac{2}{3}\cdot 3{,}459\right)+\frac{2}{3}\cdot\frac{9{,}1\cdot 3{,}5^2}{8}\cdot 4{,}95\cdot\frac{1}{2}(6{,}209+3{,}459)\right]=\frac{650{,}1}{EI_1}+\frac{-2428}{EI_2}=\frac{650{,}1}{0{,}2678EI_2}+\frac{-2428}{4494}=\frac{-0{,}44}{EI_2}$$

$$|V_E| < \frac{1}{EI_2} \quad\Rightarrow\quad V_E \cong 0$$

sprawdzenie doboru przekrojów

przekrój J_1 $\quad$ I 140

$h = 140\,mm$

$\sigma_{dop} = 215\ MPa$

$$\sigma = \frac{|M|}{W} + \frac{|N|}{A} = \frac{7157}{81,9} + \frac{2}{18,3} = 87,50\ kN/cm^2 = 875,0\ MPa > \sigma_{dop}$$

przekrój J_2 $\quad$ I 200

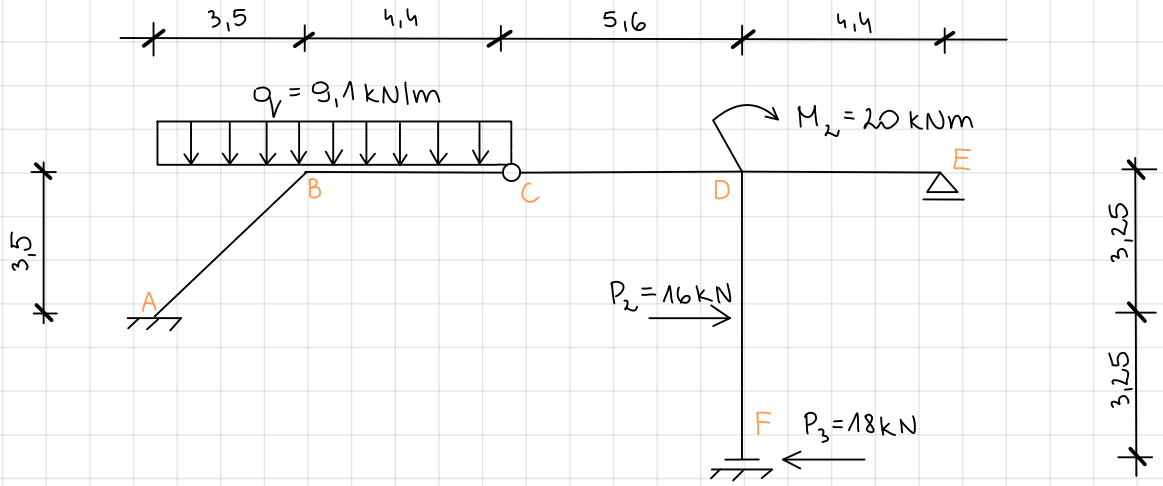
$h = 200\ mm$

$\sigma_{dop} = 215\ MPa$

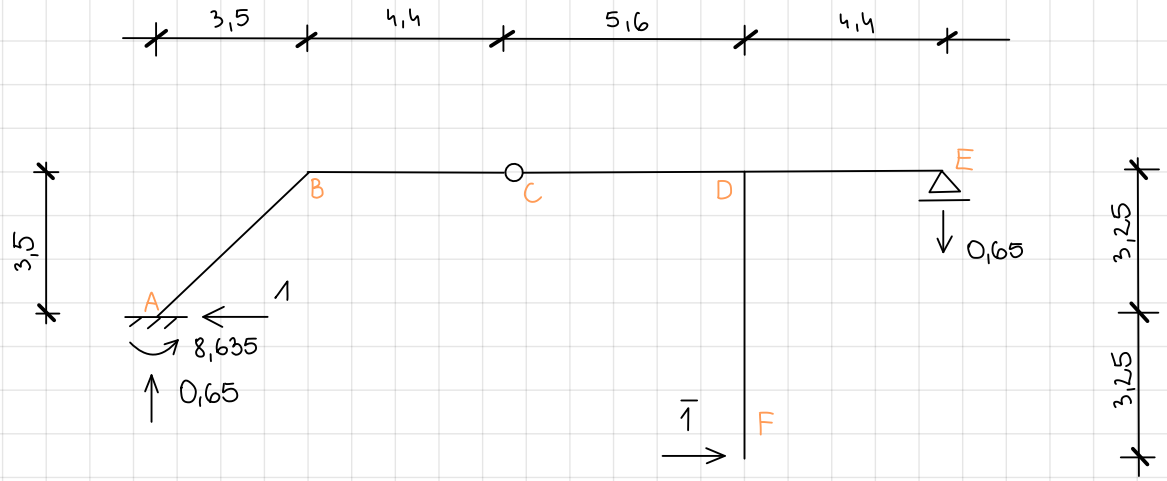
$$\sigma = \frac{M_y}{I_y} \cdot z = \frac{17600}{214} + \frac{43,21}{33,5} = 83,53\ kN/cm^2 = 835,3\ MPa > \sigma_{dop}$$

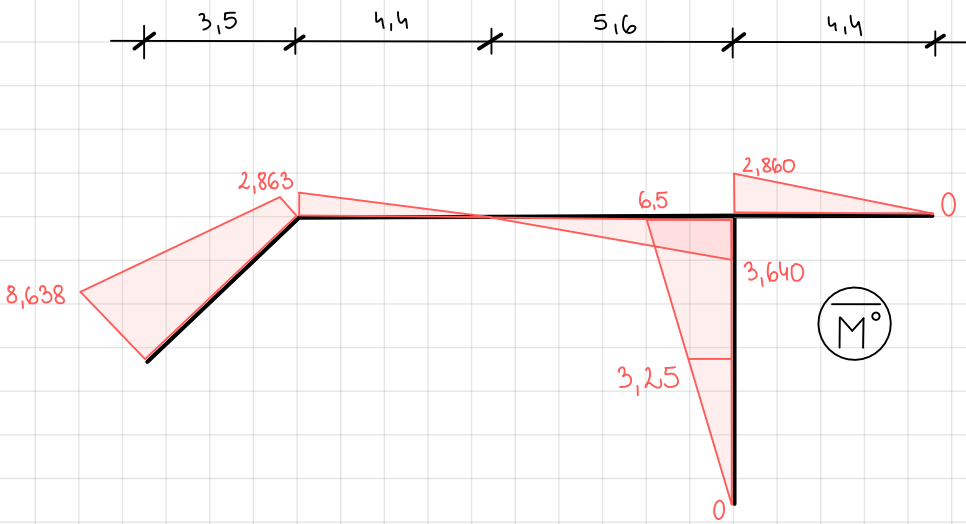
<u>wnioski</u>: Naprężenia w obydwu przekrojach są większe niż dopuszczalne. Należy przyjąć większe. Zmiana mom. bezwładności J_1 i J_2 spowoduje zmianę współczynnika $n = \frac{J_1}{J_2}$, a więc całe zadanie trzeba przeliczyć dla nowego "n" i ponownie sprawdzić naprężenia.

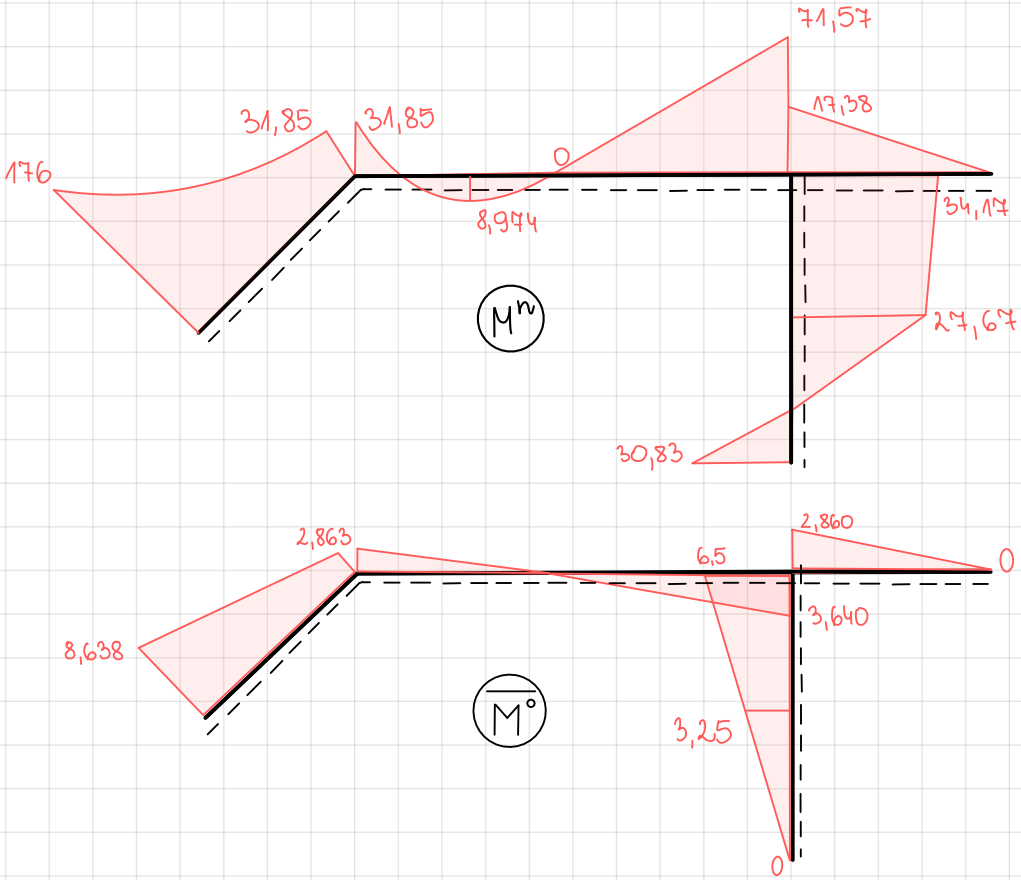
przemieszczenie poziome p. „F”



$$V_F \cdot \overline{1} = \sum \int \frac{M_p^n \overline{M^o}}{EI} dx$$







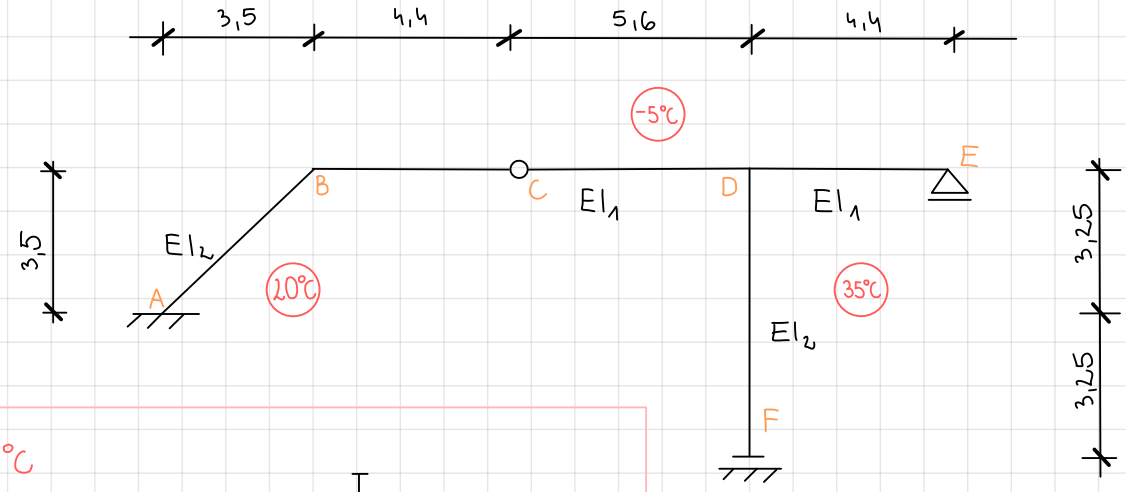
$$H_F = \frac{1}{EI_2}\Big[\frac{1}{2}\cdot 176\cdot 4{,}95\left(\frac{2}{3}\cdot 8{,}638+\frac{1}{3}\cdot 2{,}863\right)-\frac{2}{3}\,\frac{9{,}1\cdot(3{,}5)^2}{8}\cdot 4{,}95\cdot\frac{1}{2}\cdot(8{,}638+2{,}863)+\frac{1}{2}\cdot 31{,}85\cdot 4{,}95\cdot$$

$$\left(\frac{2}{3}\cdot 2{,}863+\frac{1}{3}\cdot 8{,}638\right)-\frac{1}{2}\cdot 34{,}17\cdot 3{,}25\cdot\left(\frac{2}{3}\cdot 6{,}5+\frac{1}{3}\cdot 3{,}25\right)-\frac{1}{2}\cdot 3{,}25\cdot 27{,}67\cdot\left(\frac{2}{3}\cdot 3{,}25+\frac{1}{3}\cdot 6{,}5\right)$$

$$+\frac{1}{2}\cdot 3{,}25\cdot 3{,}25\left(\frac{1}{3}\cdot 30{,}83-\frac{2}{3}\cdot 27{,}67\right)+\frac{1}{EI_1}\Big[\frac{1}{2}\cdot 4{,}4\cdot 31{,}85\cdot\frac{2}{3}\cdot 2{,}863-\frac{2}{3}\cdot\frac{9{,}1\cdot 4{,}4^2}{8}\cdot 4{,}4\cdot\frac{1}{2}\cdot 2{,}863-\frac{1}{2}\cdot 5{,}6\cdot 3{,}640\cdot$$

$$\cdot\frac{2}{3}\cdot 71{,}57+\frac{1}{2}\cdot 4{,}4\cdot 2{,}86\cdot\frac{2}{3}\cdot 17{,}38\Big]=\frac{2498}{EI_2}+\frac{-372{,}2}{EI_1}=\frac{2498}{4494}+\frac{-372{,}2}{1203{,}3}=0{,}2465\ m$$

obliczanie sił przekrojowych od zadanych wypływów temperatury



$t_m = 20°C$

$\Delta t = |t_d - t_g|$

$t_o = \frac{t_d + t_g}{2} - t_m$

EI_1 : I140 $h = 0{,}14\,m$

EI_2 : I200 $h = 0{,}2\,m$

$t_{AB} = t_{BC} = t_{CD} = \frac{20+(-5)}{2} - 20 = -12{,}5°C$

$t_{DE} = \frac{35+(-5)}{2} - 20 = -5°C$

$t_{DF} = \frac{35+20}{2} - 20 = 7{,}5°C$

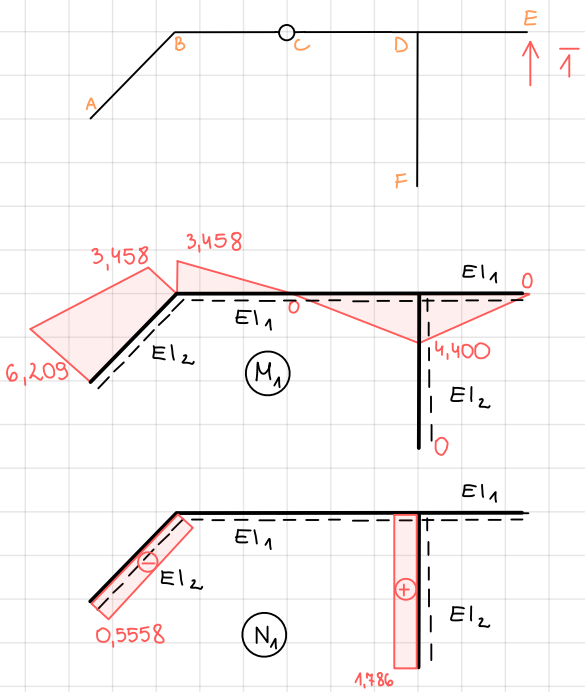
$\Delta t_{AB} = \Delta t_{BC} = \Delta t_{CD} = |20-(-5)| = 25°C$

$\Delta t_{DE} = |35-(-5)| = 40°C$

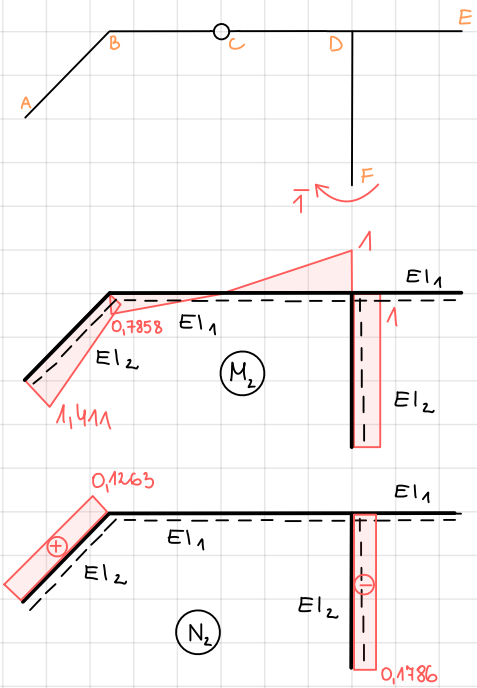
$\Delta t_{DF} = |35-20| = 15°C$

$$\delta_{ik} = \sum \int M_i \alpha_t \frac{\Delta t}{h} dx + \sum \int N_i \alpha_t t_o \, dx$$

stan $x_1 = 1$



stan $x_2 = 1$



$$\delta_{11} = \frac{425{,}2}{EI_2}$$

$$\delta_{22} = \frac{22{,}99}{EI_2}$$

$$\delta_{12} = \frac{-72{,}55}{EI_2}$$

$$\delta_{1T} = \frac{1{,}2\cdot 10^{-5}}{0{,}14}\left[\left(-\frac{1}{2}\cdot 3{,}458\cdot 4{,}4 + \frac{1}{2}\cdot 4{,}4\cdot 5{,}6\right)\cdot 25 + \frac{1}{2}\ 4{,}4\cdot 4{,}4\cdot 40\right] + \frac{1{,}2\cdot 10^{-5}}{0{,}2}\left[\left(-\frac{1}{2}\cdot 3{,}458\cdot 4{,}95 + -\frac{1}{2}\cdot 6{,}209\cdot 4{,}95\right)\cdot 25\right] + 1{,}2\cdot 10^{-5}\left[-0{,}5558\cdot 4{,}95\cdot(-12{,}5) + 1{,}786\cdot 6{,}5\cdot 7{,}5\right] = 0{,}008857$$

$$\delta_{2T} = \frac{1{,}2\cdot 10^{-5}}{0{,}14}\left[\left(\frac{1}{2}\cdot 0{,}7858\cdot 4{,}4 - \frac{1}{2}\cdot 1\cdot 5{,}6\right)\cdot 25\right] + \frac{1{,}2\cdot 10^{-5}}{0{,}2}\left[1\cdot 6{,}5\cdot 15 + \left(\frac{1}{2}\cdot 0{,}7858\cdot 4{,}95 + \frac{1}{2}\cdot 1{,}411\cdot 4{,}95\right)\cdot 25\right] + 1{,}2\cdot 10^{-5}\left[0{,}1263\cdot 4{,}95\cdot(-12{,}5) - 0{,}1786\cdot 6{,}5\cdot 7{,}5\right] = 0{,}01151$$

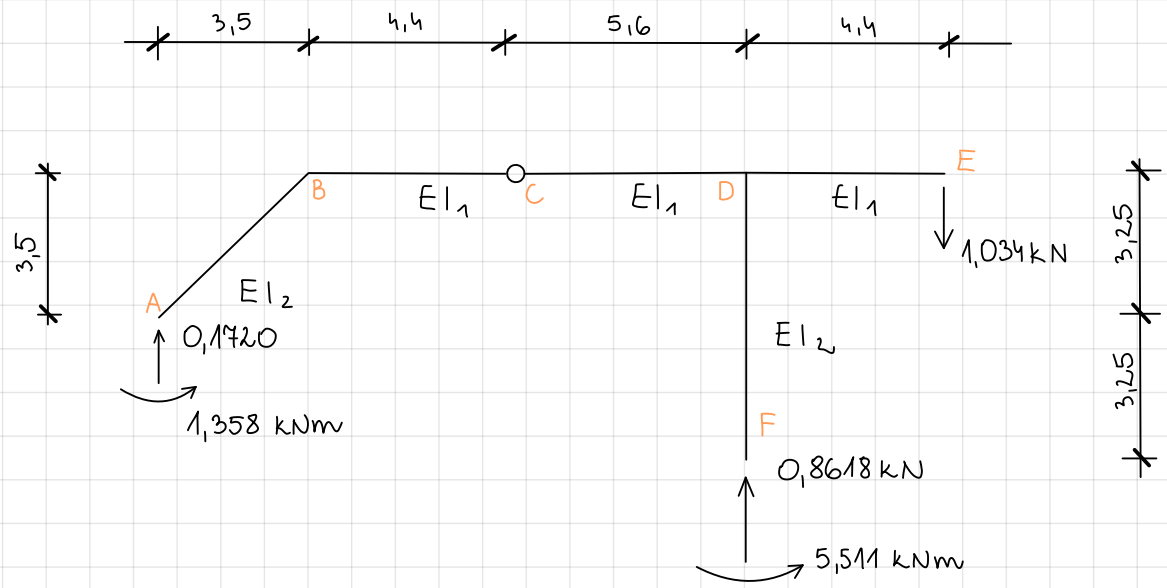
$$\delta_{11} = \frac{425{,}2}{EI_2} \qquad \delta_{1T} = 0{,}008857$$

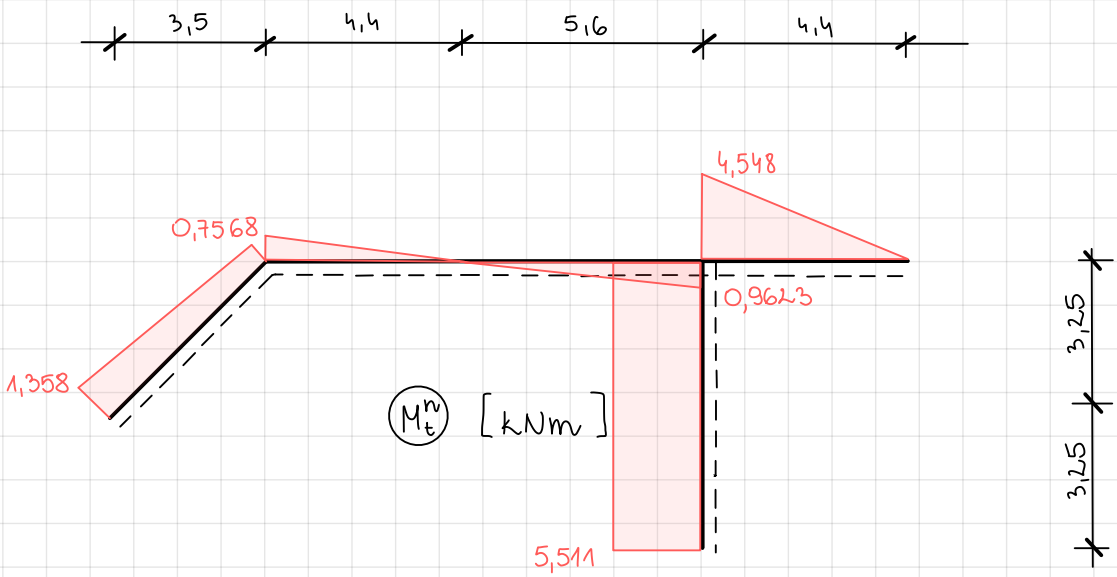
$$\delta_{22} = \frac{22{,}99}{EI_2} \qquad \delta_{2T} = 0{,}01151$$

$$\delta_{12} = \delta_{21} = \frac{-72{,}55}{EI_2}$$

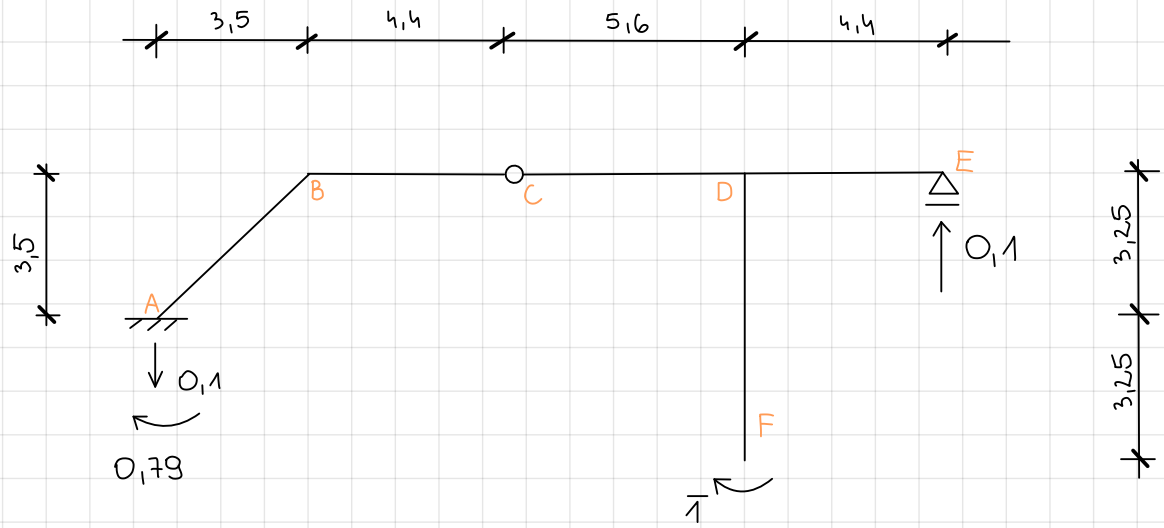
$$\begin{cases} \delta_{11} X_1 + \delta_{12} X_2 + \delta_{1T} = 0 \\ \delta_{21} X_1 + \delta_{22} X_2 + \delta_{2T} = 0 \end{cases}$$

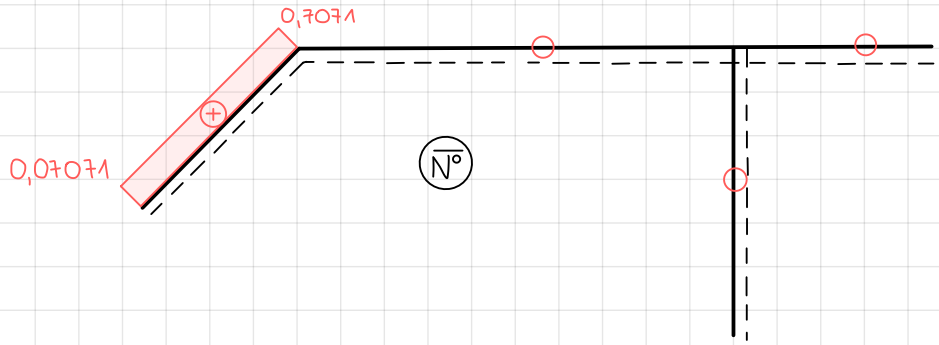
$$\begin{cases} \frac{425{,}2}{4494} X_1 + \frac{-72{,}55}{4494} X_2 + 0{,}008855 = 0 \\ \frac{-72{,}55}{4494} X_1 + \frac{22{,}99}{4494} X_2 + 0{,}01151 = 0 \end{cases} \longrightarrow \begin{array}{l} X_1 = -1{,}034 \text{ kN} \\ X_2 = -5{,}511 \text{ kNm} \end{array}$$

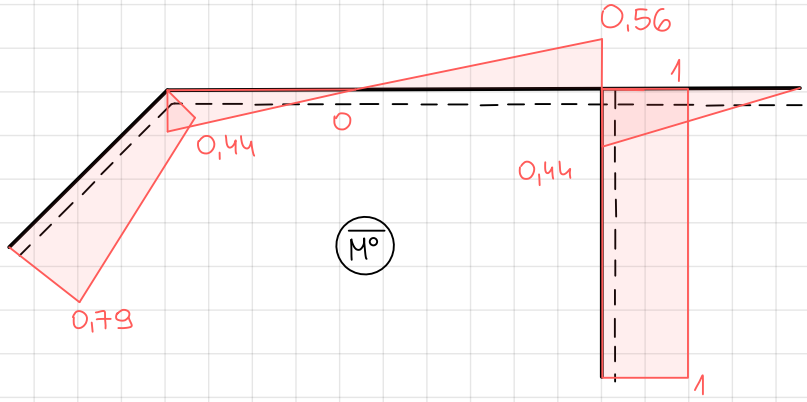


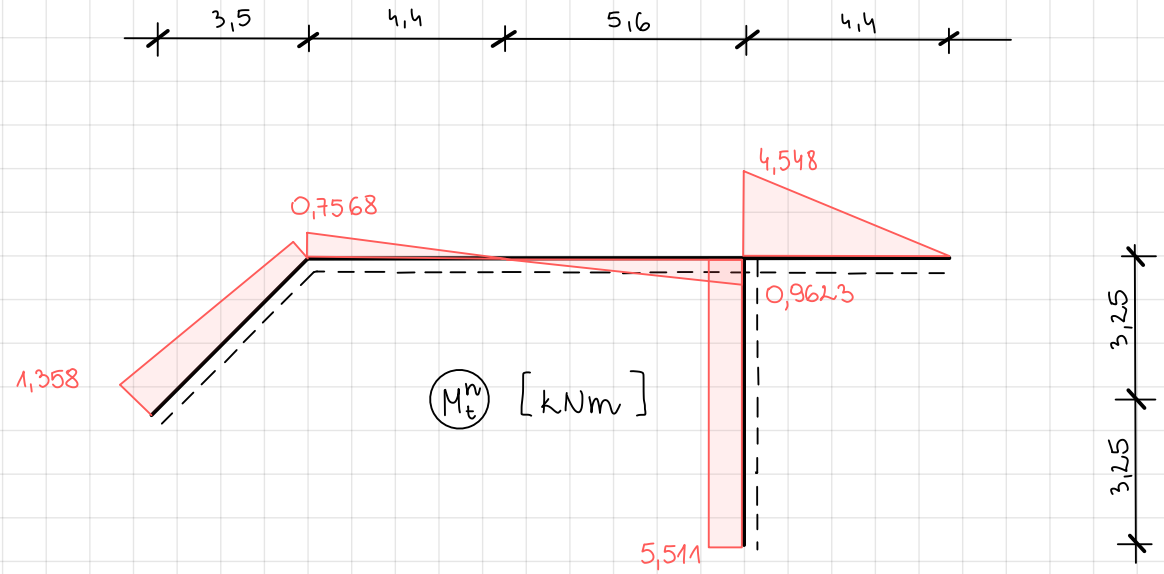

3,5
4,4
5,16
4,4
4,548
0,7568
0,9623
1,358
M_t^n [kNm]
5,511
3,25
3,25
0,8618
N_t^n [kN]
0,1216
0,8618
0,1720
0,1720
1,034
1,034
0,1216
T_t^n [kN]

kontrola kinematyczna









$$\varphi_F = \sum \int \frac{M_t^n \bar{M}^o}{EJ} dx + \sum \int \bar{M}^o \alpha_t \frac{\Delta t}{h} dx + \sum \int \bar{N}^o \alpha_t t_o \, dx$$

$$\varphi_F = \frac{1}{EI_1}\left[-\frac{1}{2}\cdot 0{,}7568 \cdot 4{,}4 \cdot \frac{2}{3}\cdot 0{,}44 - \frac{1}{2}\cdot 0{,}9623 \cdot 5{,}6 \cdot \frac{2}{3}\cdot 0{,}56 - \frac{1}{2}\cdot 4{,}548 \cdot 4{,}4 \cdot \frac{2}{3}\cdot 0{,}44\right] +$$

$$+ \frac{1}{EI_2}\left[-5{,}511 \cdot 6{,}5 \cdot 1 - \frac{1}{2}\cdot 0{,}7568 \cdot 4{,}95 \cdot \left(\frac{2}{3}\cdot 0{,}44 + \frac{1}{3}\cdot 0{,}79\right) - \frac{1}{2}\cdot 1{,}358\ 4{,}95\left(\frac{2}{3}\cdot 0{,}79 + \frac{1}{3}\cdot 0{,}44\right)\right] +$$

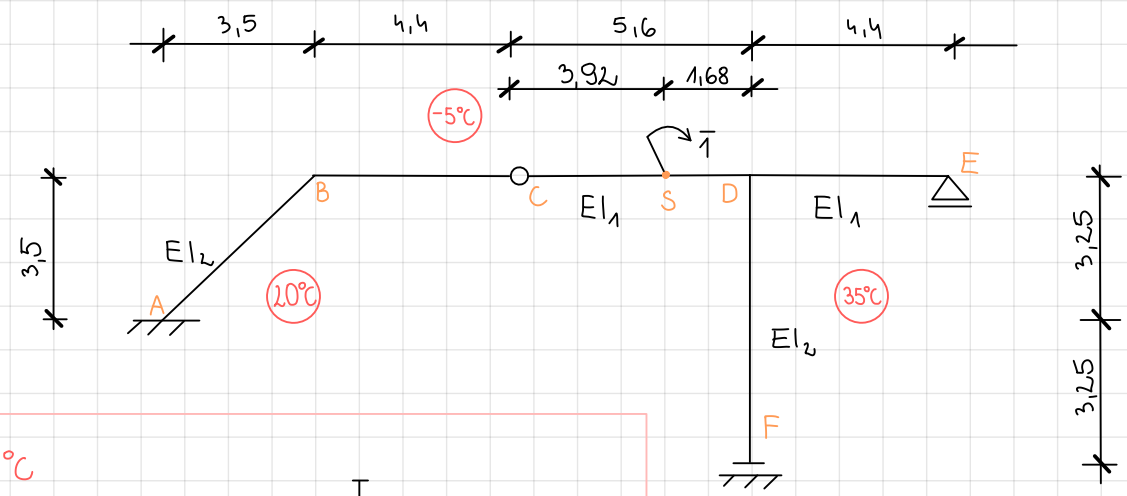
$$+ \frac{1{,}2\cdot 10^{-5}}{0{,}14}\left[\left(\frac{1}{2}\cdot 0{,}44 \cdot 4{,}4 - \frac{1}{2}\cdot 0{,}56 \cdot 5{,}6\right)\cdot 25 + \frac{1}{2}\cdot 0{,}44 \cdot 4{,}4 \cdot 40\right] + \frac{1{,}2\cdot 10^{-5}}{0{,}2}\left[1 \cdot 6{,}5 \cdot 15 +\right.$$

$$\left.\left(\frac{1}{2}\cdot 0{,}44 \cdot 4{,}95 + \frac{1}{2}\cdot 0{,}79 \cdot 4{,}95\right)\cdot 25 + 1{,}2\cdot 10^{-5}\left[0{,}07071 \cdot 4{,}95 \cdot (-12{,}5)\right] = \frac{-4{,}429}{EI_1} - \frac{39{,}13}{EI_2} + 0{,}01245 +\right.$$

$$-0{,}00005250 = \frac{-4{,}429}{0{,}2678\, EI_2} - \frac{39{,}13}{EI_2} + 0{,}01240 \cdot \frac{4494}{EI_2} = \frac{0{,}046}{EI_2}$$

$$|\varphi_F| < \frac{1}{EI_2} \Rightarrow \varphi_F \cong 0$$

obrót przekroju w p. S



$t_m = 20°C$

$\Delta t = |t_d - t_g|$

$t_o = \frac{t_d + t_g}{2} - t_m$

EI_1 : I140 $h = 0{,}14\,m$

EI_2 : I200 $h = 0{,}2\,m$

$t_{AB} = t_{BC} = t_{CD} = \frac{20 + (-5)}{2} - 20 = -12{,}5°C$

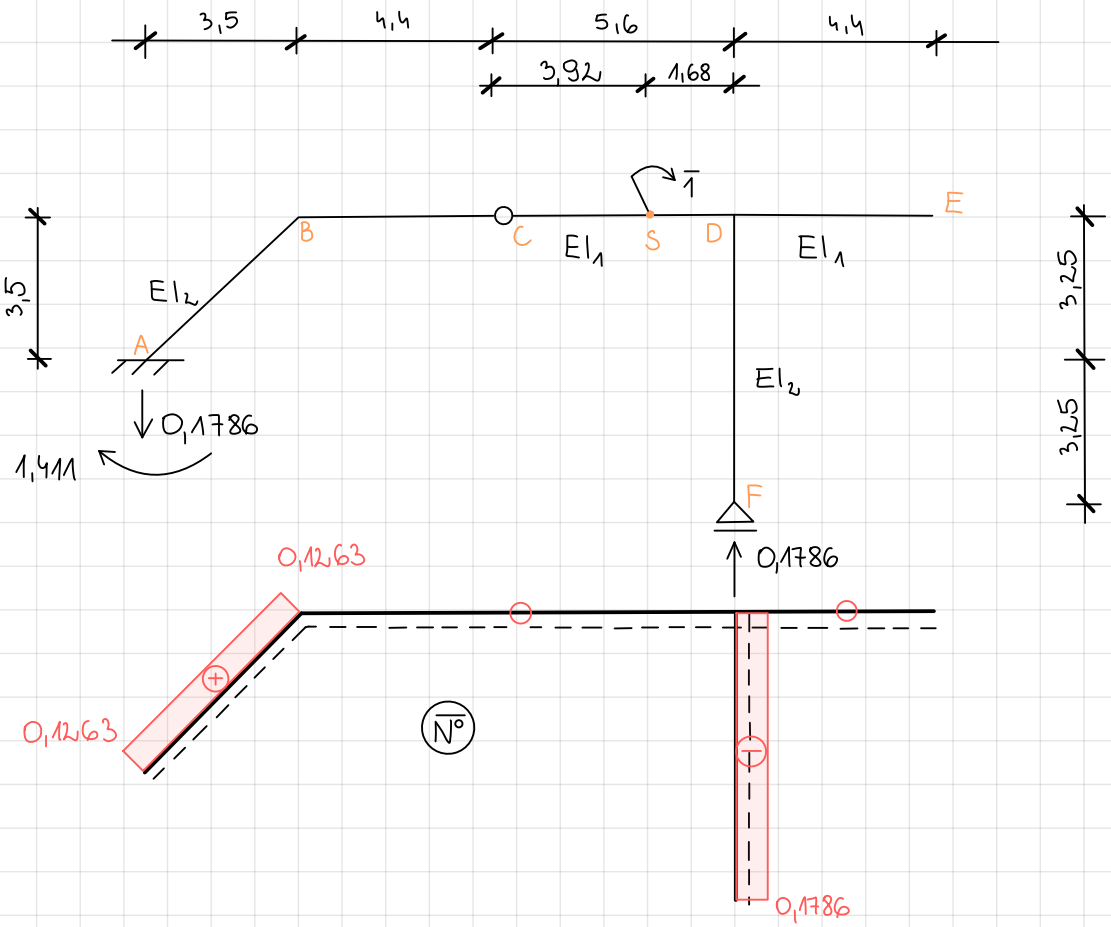
$\Delta t_{AB} = \Delta t_{BC} = \Delta t_{CD} = |20 - (-5)| = 25°C$

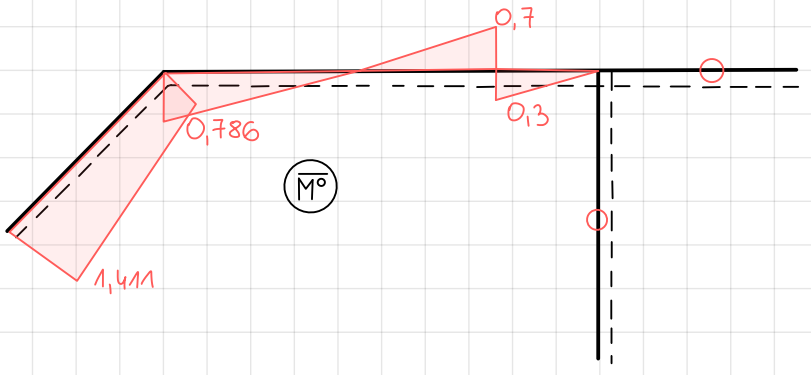
$t_{DE} = \frac{35 + (-5)}{2} - 20 = -5°C$

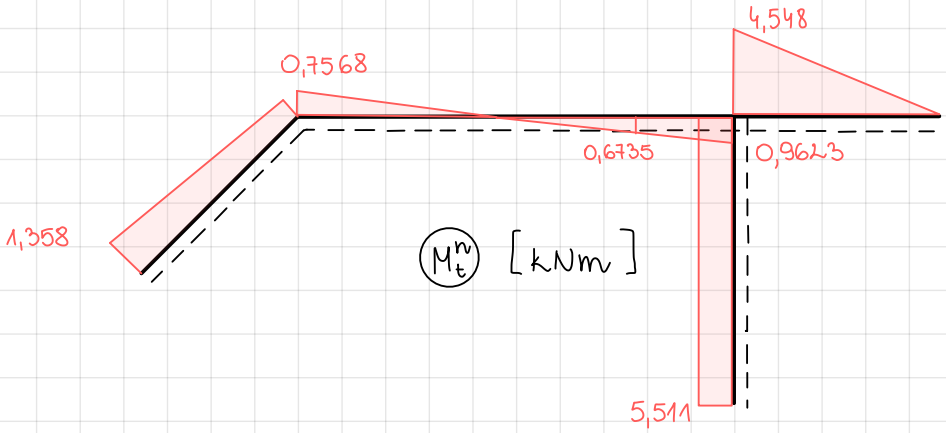
$\Delta t_{DE} = |35 - (-5)| = 40°C$

$t_{DF} = \frac{35 + 20}{2} - 20 = 7{,}5°C$

$\Delta t_{DF} = |35 - 20| = 15°C$



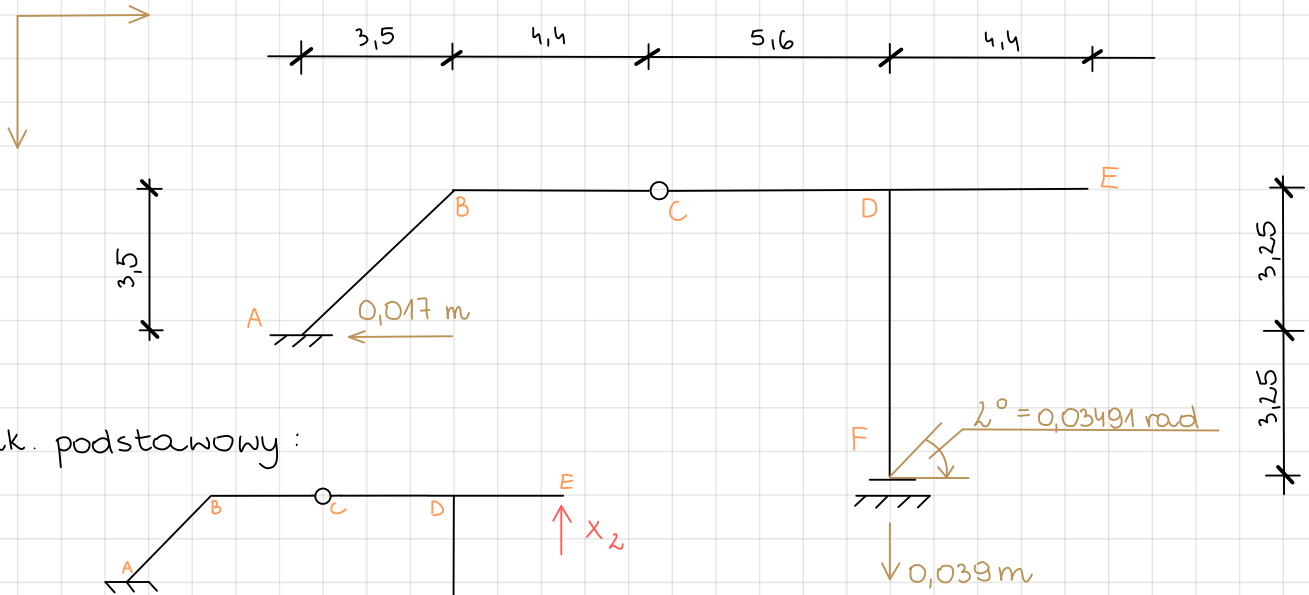




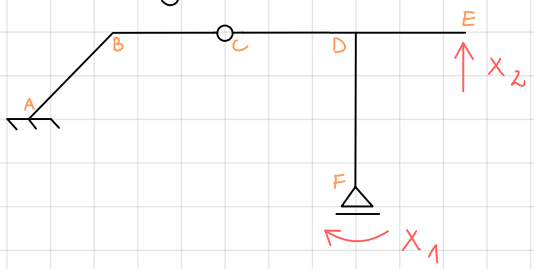
$$\varphi_s = \sum\int \frac{M_t^n \overline{M}^0}{EJ} + \sum\int \overline{M}^0 \alpha_t \frac{\Delta t}{h} dx + \sum\int \overline{N}^0 \alpha_t t_0 \, dx$$

$$\varphi_s = \frac{1}{EI_1}\left[-\frac{1}{2}\cdot 0{,}7568\cdot 4{,}4\cdot\frac{2}{3}\cdot 0{,}786 - \frac{1}{2}\cdot 0{,}6735\cdot 3{,}92\cdot\frac{2}{3}\cdot 0{,}7 + \frac{1}{2}\cdot 0{,}3\cdot 1{,}68\cdot\right.$$
$$\left.\left(\frac{2}{3}\cdot 0{,}6735 + \frac{1}{3}\cdot 0{,}9623\right)\right] + \frac{1}{EI_2}\left[-\frac{1}{2}\cdot 0{,}7568\cdot 4{,}95\left(\frac{2}{3}\cdot 0{,}786 + \frac{1}{3}\cdot 1{,}411\right) - \frac{1}{2}\cdot 1{,}3584\cdot 4{,}95\cdot\right.$$
$$\left.\left(\frac{2}{3}\cdot 1{,}411 + \frac{1}{3}\cdot 0{,}786\right)\right] + \frac{1{,}2\cdot 10^{-5}}{0{,}14}\left[\left(\frac{1}{2}\cdot 0{,}786\cdot 4{,}4 - \frac{1}{2}\cdot 0{,}7\cdot 3{,}92 + \frac{1}{2}\cdot 0{,}3\cdot 1{,}68\right)\cdot 25\right] +$$
$$+ \frac{1{,}2\cdot 10^{-5}}{0{,}2}\left[\left(\frac{1}{2}\cdot 0{,}786\cdot 4{,}95 + \frac{1}{2}\cdot 1{,}411\cdot 4{,}95\right)\cdot 25\right] + 1{,}2\cdot 10^{-5}\left[0{,}11263\cdot 4{,}95\cdot(-12{,}5) +\right.$$
$$\left. - 0{,}1786\cdot 6{,}5\cdot 7{,}5\right] = 6{,}874\cdot 10^{-3} \text{ rad}$$

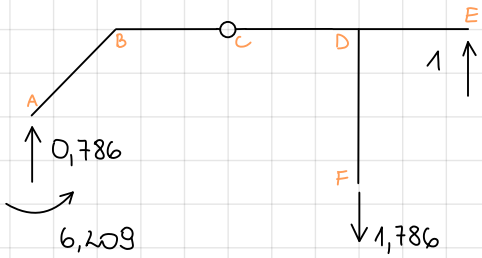
obliczenie sił przekrojowych od osiadania podpór



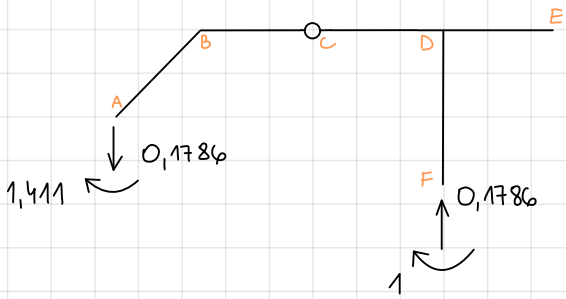
uk. podstawowy:



stan $X_1 = 1$



stan $X_2 = 1$



$$\delta_{11} = \frac{425,2}{EI_2}$$

$$\delta_{22} = \frac{22,99}{EI_2}$$

$$\delta_{12} = \delta_{21} = \frac{-72,55}{EI_2}$$

$$\delta_{1\Delta} = -\sum R_1 \cdot \Delta = -(1,786 \cdot 0,039) = -0,06965 \text{ m}$$

$$\delta_{2\Delta} = -\sum R_2 \cdot \Delta = -(1 \cdot 0,03491 - 0,1786 \cdot 0,039) = -0,02794 \text{ m}$$

$$\delta_{11} = \frac{425,2}{EI_2} \qquad \delta_{1\Delta} = -0,06965$$

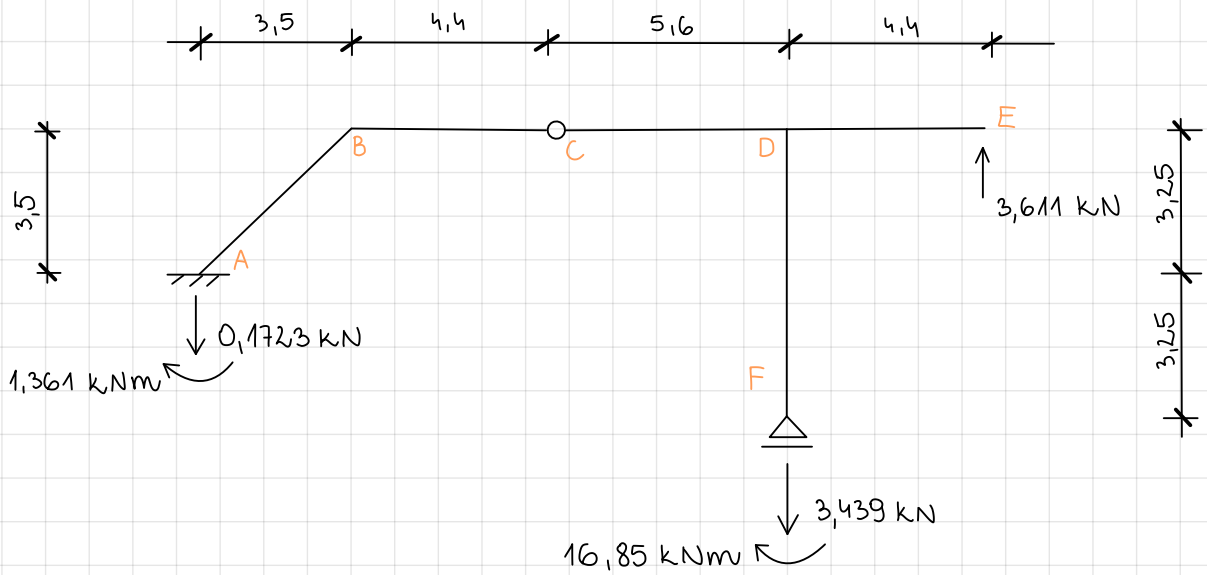
$$\delta_{22} = \frac{22,99}{EI_2} \qquad \delta_{2\Delta} = -0,02794$$

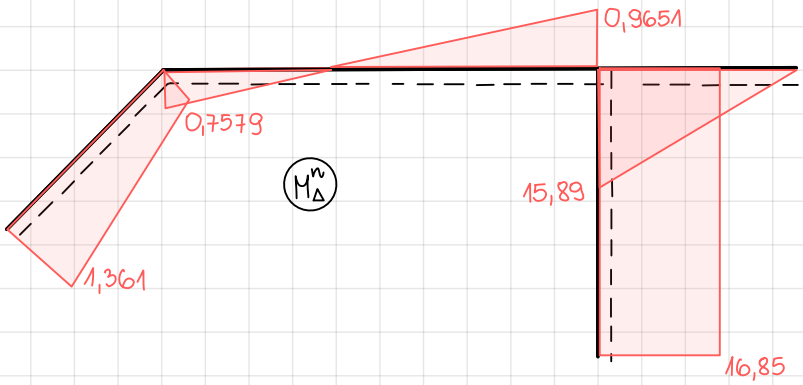
$$\delta_{12} = \delta_{21} = \frac{-72,55}{EI_2}$$

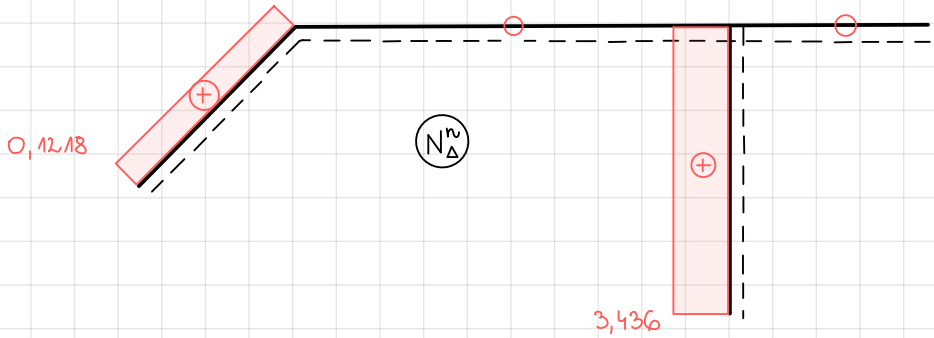
$$\begin{cases} \delta_{11} X_1 + \delta_{12} X_2 + \delta_{1\Delta} = 0 \\ \delta_{21} X_1 + \delta_{22} X_2 + \delta_{2\Delta} = 0 \end{cases}$$

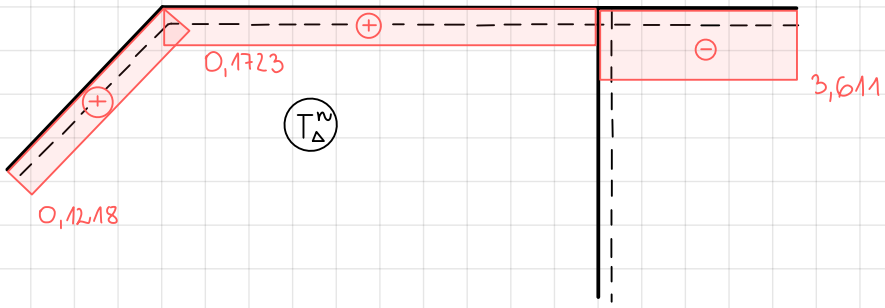
$$\begin{cases} \frac{425,2}{4494} X_1 + \frac{-72,55}{4494} X_2 - 0,06965 = 0 \\ \frac{-72,55}{4494} X_1 + \frac{22,99}{4494} X_2 - 0,02794 = 0 \end{cases}$$

$$\begin{cases} 0{,}09462\,X_1 - 0{,}01614\,X_2 - 0{,}06965 = 0 \\ -0{,}01614\,X_1 + 0{,}005116\,X_2 - 0{,}02794 = 0 \end{cases} \longrightarrow \begin{matrix} X_1 = 3{,}611\ \text{kN} \\ X_2 = 16{,}85\ \text{kNm} \end{matrix}$$

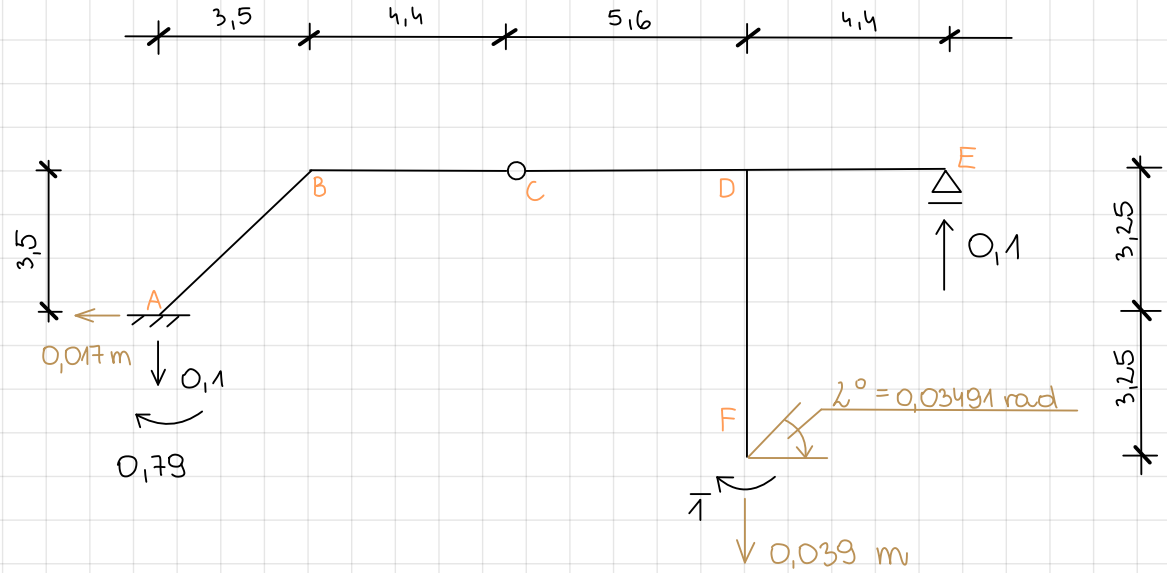


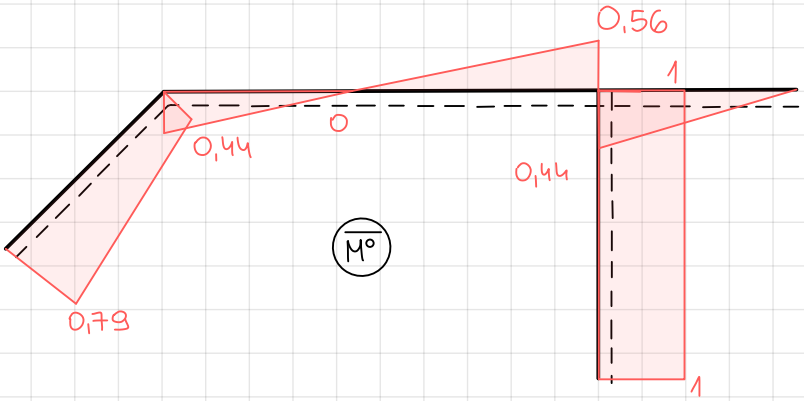


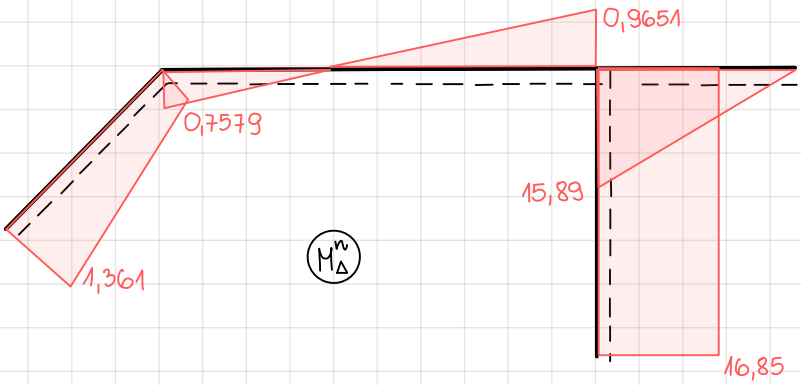




kontrola kinematyczna







$$\varphi_D \quad \overline{1} = \sum \int \frac{M_\Delta^n \overline{M}^\circ}{EJ} dx - \sum \overline{R}^\circ \Delta = \frac{1}{EI_1} \left[\frac{1}{2} \cdot 0{,}44 \cdot 4{,}4 \cdot \frac{2}{3} \cdot 0{,}75792 + \frac{1}{2} \cdot 0{,}56 \cdot 5{,}6 \cdot \frac{2}{3} \cdot 0{,}9651 + \right.$$

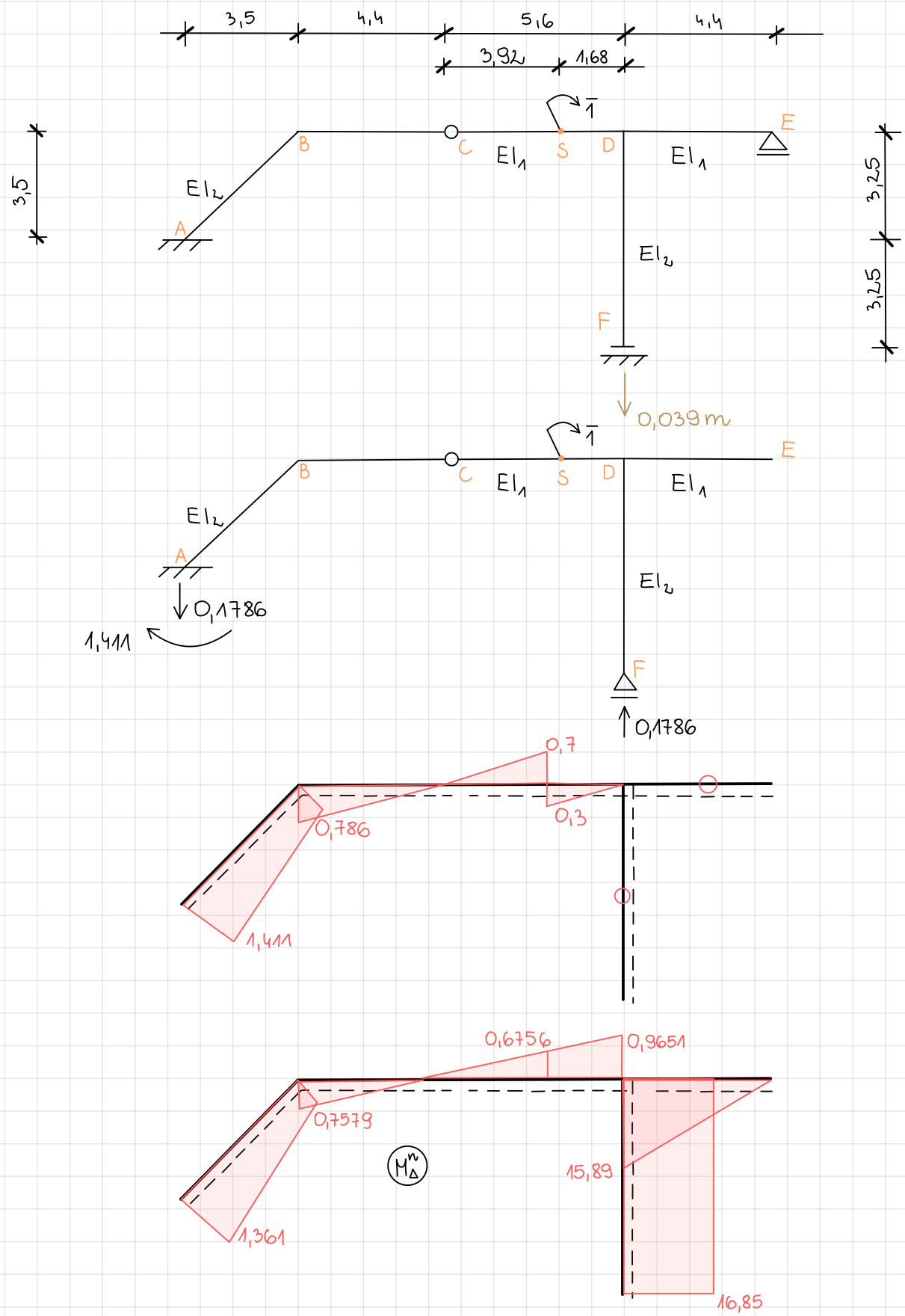
$$\left. + \frac{1}{2} \cdot 0{,}44 \cdot 4{,}4 \cdot \frac{2}{3} \cdot 15{,}89 \right] + \frac{1}{EI_2} \left[1 \cdot 6{,}5 \cdot 16{,}85 + \frac{1}{2} \cdot 0{,}44 \cdot 4{,}95 \cdot \left(\frac{2}{3} \cdot 0{,}7579 + \frac{1}{3} \cdot 1{,}361 \right) + \frac{1}{2} \cdot 0{,}79 \cdot \right.$$

$$\left. 4{,}95 \left(\frac{2}{3} \cdot 1{,}361 + \frac{1}{3} \cdot 0{,}7579 \right) \right] - \left[1 \cdot 0{,}03491 \right] \cdot \frac{EI}{EI} = \frac{11{,}175}{0{,}2678 EI_2} + \frac{112{,}8}{EI_2} - 0{,}03491 \cdot \frac{4494}{EI_2} =$$

$$= \frac{-0{,}17}{EI_2}$$

$$|\varphi_F| < \frac{1}{EI_2} \Rightarrow \varphi_F \cong 0$$

obrót przekroju w p. S

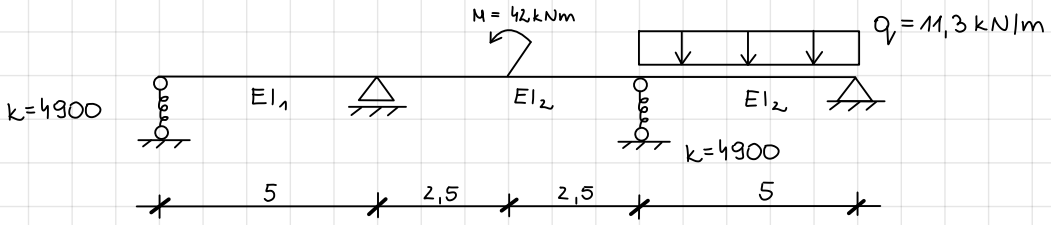


$$\varphi_S = \sum \int \frac{M_\Delta^n \overline{M}^0}{EJ} dx - \sum \overline{R}^0 \cdot \Delta = \frac{1}{EI_1} \left[\frac{1}{2} \cdot 0{,}786 \cdot 4{,}4 \cdot \frac{2}{3} \cdot 0{,}7579 + \frac{1}{2} \cdot 0{,}7 \cdot 3{,}92 \cdot \frac{2}{3} \cdot 0{,}6756 + \right.$$

$$\left. - \frac{1}{2} \cdot 0{,}3 \cdot 1{,}68 \cdot \left(\frac{2}{3} \cdot 0{,}6756 + \frac{1}{3} \cdot 0{,}9651 \right) \right] + \frac{1}{EI_2} \left[\frac{1}{2} \cdot 0{,}786 \cdot 4{,}95 \left(\frac{2}{3} \cdot 0{,}7579 + \frac{1}{3} \cdot 1{,}361 \right) + \right.$$

$$\left. + \frac{1}{2} \cdot 1{,}411 \cdot 4{,}95 \left(\frac{2}{3} \cdot 1{,}361 + \frac{1}{3} \cdot 0{,}7579 \right) \right] - \left[-0{,}1786 \cdot 0{,}039 \right] = 9{,}360 \cdot 10^{-3} \text{ rad}$$

BELKA



analiza kinematyczna

$t = 1 \qquad p = 1+1+1+2 = 5$

$3t = p$

$3 \neq 5 \longrightarrow$ nie może być statycznie wyznaczalny

$n = p - 3t$

$n = 2$

$\downarrow$

układ statycznie niewyznaczalny, geometrycznie niezmienny,

$ssn = 2$

$E = 210\ GPa$

$I140 \quad I_1 = 573{,}0\ cm^4$

$EI_1 = 210 \cdot 10^6 \cdot 573 \cdot 10^{-8} = 1203{,}3\ kNm^2$

$I200 \quad I_2 = 2140{,}0\ cm^4$

$EI_2 = 210 \cdot 2140 \cdot 10^6 \cdot 10^{-8} = 4494\ kNm^2$

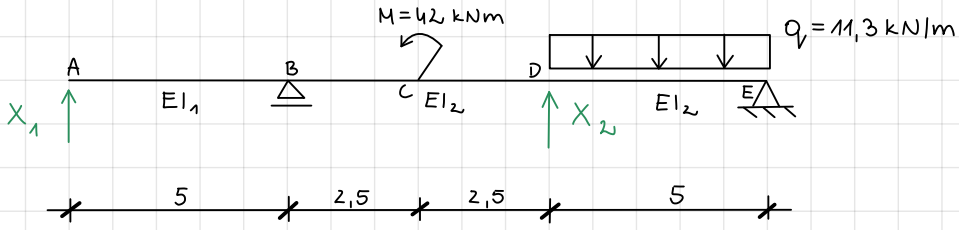
$\frac{EI_1}{EI_2} = \frac{1203{,}3}{4494} = 0{,}2678 \quad | \cdot EI_2$

$EI_1 = 0{,}2678 \cdot EI_2 = 0{,}2678\, EI_0$

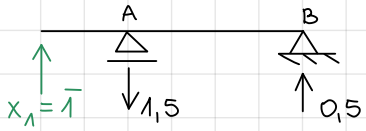
$k = 4900\ kN/m \qquad \alpha = \frac{k}{EI_2} = \frac{4900}{4494} = 1{,}090$

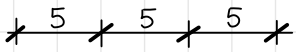
$EI_2 = 4494\ kN/m^2 \qquad k = 1{,}09\, EI_2$

układ podstawowy



stan $x_1 = 1$





$\sum M_A = 0$

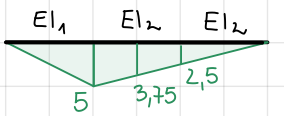
$1 \cdot 5 - V_B \cdot 10 = 0$

$V_B = 0{,}5$

$\sum M_B = 0$

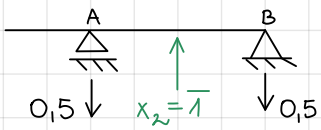
$1 \cdot 15 + V_A \cdot 10 = 0$

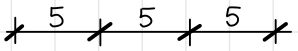
$V_A = 1{,}5$



M_1

stan $x_2 = 1$





$\sum M_A = 0$

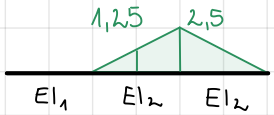
$-1 \cdot 5 - V_B \cdot 10 = 0$

$V_B = -0{,}5$

$\sum M_B = 0$

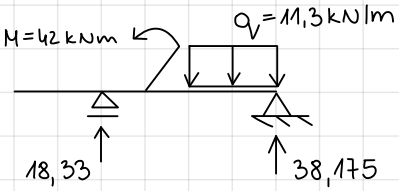
$V_A \cdot 10 + 1 \cdot 5 = 0$

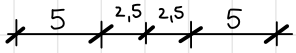
$V_A = -0{,}5$



M_2

stan "p"





$\sum M_A = 0$

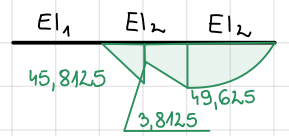
$-42 + 11{,}3 \cdot 5 \cdot 7{,}5 - V_B \cdot 10 = 0$

$V_B = 38{,}175 \text{ kN}$

$\sum M_B = 0$

$V_A \cdot 10 - 42 - 11{,}3 \cdot 5 \cdot 2{,}5 = 0$

$V_A = 18{,}325 \text{ kN}$



M_p

$$\delta_{11} = \frac{1}{EI_1}\left[\frac{1}{2}\cdot 5\cdot 5\cdot\frac{2}{3}\cdot 5\right] + \frac{1}{EI_2}\left[\frac{1}{2}\cdot 5\cdot 10\cdot\frac{2}{3}\cdot 5\right] + \frac{1\cdot 1}{1{,}09\,EI_0} = \frac{1}{0{,}2678\,EI_0}\cdot\frac{125}{3} + \frac{1}{EI_0}\frac{250}{3} + \frac{1}{1{,}09\,EI_0} =$$

$$= \frac{239{,}9}{EI_0}$$

$$\delta_{12} = \frac{1}{EI_2}\left[-\frac{1}{2}\cdot 2{,}5\cdot 5\cdot\left(\frac{2}{3}\cdot 2{,}5 + \frac{1}{3}\cdot 5\right) - \frac{1}{2}\cdot 2{,}5\cdot 5\cdot\frac{2}{3}\cdot 2{,}5\right] = \frac{1}{EI_0}\cdot(-31{,}25) = -\frac{31{,}25}{EI_0}$$

$$\delta_{22} = \frac{1}{EI_2}\left[-\frac{1}{2}\cdot 2{,}5\cdot 5\cdot\frac{2}{3}\cdot 2{,}5 + \frac{1}{2}\cdot 2{,}5\cdot 5\cdot\frac{2}{3}\cdot 2{,}5\right] + \frac{1\cdot 1}{1{,}09\,EI_0} = \frac{1}{EI_0}\cdot\frac{125}{6} + \frac{1}{1{,}09\,EI_0} =$$

$$= \frac{21{,}75}{EI_0}$$

$$\delta_{1P} = \frac{1}{EI_2}\left[\frac{1}{2}\cdot 45{,}8125\cdot 2{,}5\left(\frac{2}{3}\cdot 3{,}75 + \frac{1}{3}\cdot 5\right) + \frac{1}{2}\cdot 3{,}8125\cdot 2{,}5\left(\frac{2}{3}\cdot 3{,}75 + \frac{1}{3}\cdot 2{,}5\right) +\right.$$

$$\left. + \frac{1}{2}\cdot 49{,}625\cdot 2{,}5\left(\frac{2}{3}\cdot 2{,}5 + \frac{1}{3}\cdot 3{,}75\right) + \frac{1}{2}\cdot 49{,}625\cdot 5\cdot\frac{2}{3}\cdot 2{,}5 + \frac{2}{3}\cdot\frac{11{,}3\cdot 5^2}{8}\cdot 5\cdot\frac{1}{2}\cdot 2{,}5\right] =$$

$$= \frac{789{,}3}{EI_0}$$

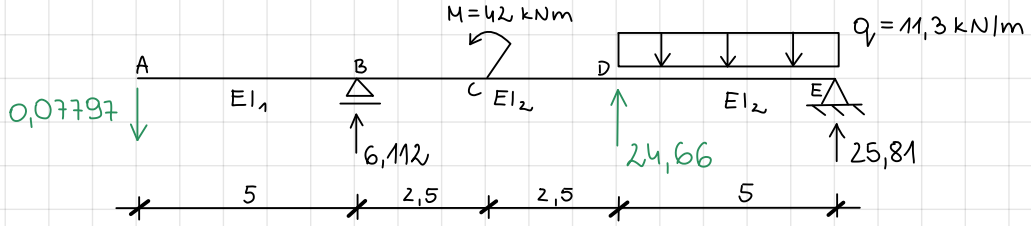
$$\delta_{2P} = \frac{1}{EI_2}\left[-\frac{1}{2}\cdot 45{,}8125\cdot 2{,}5\cdot\frac{2}{3}\cdot 1{,}25 - \frac{1}{2}\cdot 3{,}8125\cdot 2{,}5\left(\frac{2}{3}\cdot 1{,}25 + \frac{2}{3}\cdot 2{,}5\right) - \frac{1}{2}\cdot 49{,}625\cdot 2{,}5\left(\frac{2}{3}\cdot 2{,}5 +\right.\right.$$

$$\left.\left. + \frac{1}{3}\cdot 1{,}25\right) - \frac{1}{2}\cdot 49{,}625\cdot 5\cdot\frac{2}{3}\cdot 2{,}5 - \frac{2}{3}\cdot\frac{11{,}3\cdot 5^2}{8}\cdot 5\cdot\frac{1}{2}\cdot 2{,}5\right] = -\frac{538{,}8}{EI_0}$$

$$\begin{cases} \frac{239{,}9}{EI_0}X_1 - \frac{31{,}25}{EI_0}X_2 + \frac{789{,}3}{EI_0} = 0 \\ -\frac{31{,}25}{EI_0}X_1 + \frac{21{,}75}{EI_0}X_2 - \frac{538{,}8}{EI_0} = 0 \end{cases} \longrightarrow$$

$X_1 = -0{,}07797$ kN

$X_2 = 24{,}66$ kN

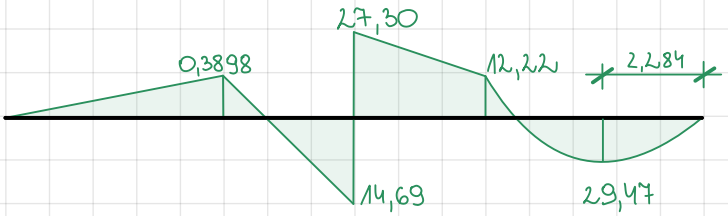


$\sum M_B = 0$

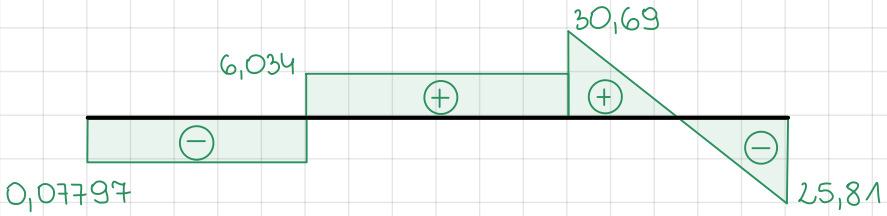
$-0{,}07797\cdot 5 - 42 + 11{,}3\cdot 5\cdot 7{,}5 - 24{,}66\cdot 5 - V_E\cdot 10 = 0$ $\qquad V_E = 25{,}81$ kN

$\sum M_E = 0$

$-0{,}07797\cdot 15 + V_B\cdot 10 - 42 + 24{,}66\cdot 5 - 11{,}3\cdot 5\cdot 2{,}5 = 0$ $\qquad V_B = 6{,}112$ kN

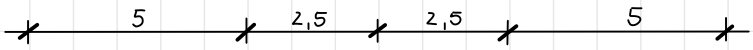


M_p^n [kNm]

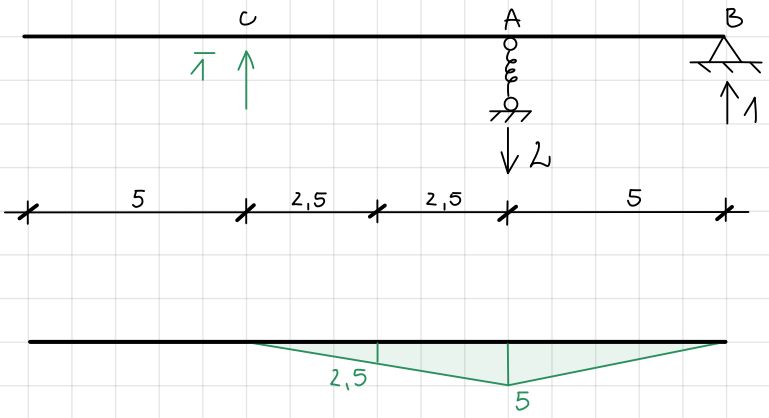


T_p^n [kN]

N_p^n [kN]



kontrola kinematyczna



$\sum M_A \quad 1 \cdot 5 - V_B \cdot 5 = 0 \quad V_B = 1$

$\sum M_B \quad V_A \cdot 5 + 1 \cdot 10 = 0 \quad V_A = -2$

$\overline{M^0}$ [m]

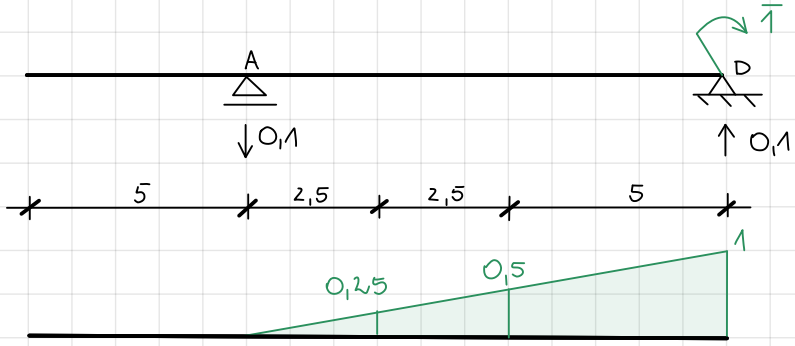
$$\overline{1} \cdot v_C = \sum \int \frac{M_P^n \cdot \overline{M}^0}{EI} dx + \sum \frac{R_P^n \cdot \overline{R}^0}{k} = \frac{1}{EI_2}\Big[\frac{1}{2} 2{,}5 \cdot 2{,}5 \left(\frac{2}{3} \cdot 14{,}69 - \frac{1}{3} \cdot 0{,}3898\right) - \frac{1}{2} \cdot 2{,}5 \cdot 2{,}5 \left(\frac{2}{3} \cdot 27{,}30 +$$

$$+ \frac{1}{3} \cdot 12{,}22\right) - \frac{1}{2} \cdot 5 \cdot 2{,}5 \left(\frac{2}{3} \cdot 12{,}22 + \frac{1}{3} \cdot 27{,}30\right) - \frac{1}{2} \cdot 5 \cdot 5 \cdot \frac{2}{3} \cdot 12{,}22 + \frac{2}{3} \cdot \frac{11{,}3 \cdot 5^2}{8} \cdot 5 \cdot \frac{1}{2} \cdot 5 \Big] +$$

$$- \frac{24{,}66 \cdot 2}{1{,}09 EI_0} = \frac{1}{EI_0} \cdot 45{,}23 - \frac{45{,}25}{EI_0} = - \frac{0{,}02}{EI_0}$$

$$|v_C| < \frac{1}{EI_0} \Rightarrow v_C \cong 0$$

obrót w punkcie D



$\sum M_A = 0 \quad 1 - V_D \cdot 10 = 0 \quad V_D = 0{,}1$

$\sum M_D = 0 \quad V_A \cdot 10 + 1 = 0 \quad V_A = -0{,}1$

$\overline{M^0}$ [–]

$$\overline{1} \cdot \varphi_D = \sum \int \frac{M_P^n \cdot \overline{M}^0}{EI} dx + \sum \frac{R_P^n \cdot \overline{R}_0}{k} = \frac{1}{EI_2}\Big[\frac{1}{2} \cdot 0{,}25 \cdot 2{,}5 \left(-\frac{2}{3} \cdot 14{,}69 + \frac{1}{3} \cdot 0{,}3898\right) + \frac{1}{2} \cdot 0{,}25 \cdot 2{,}5$$

$$\left(\frac{2}{3} \cdot 27{,}30 + \frac{1}{3} \cdot 12{,}22\right) + \frac{1}{2} \cdot 0{,}5 \cdot 2{,}5 \left(\frac{2}{3} 12{,}22 + \frac{1}{3} \cdot 27{,}30\right) + \frac{1}{2} \cdot 12{,}22 \cdot 5 \cdot \left(\frac{2}{3} \cdot 0{,}5 + \frac{1}{3} \cdot 1\right) -$$

$$+ \frac{2}{3} \frac{11{,}3 \cdot 5^2}{8} \cdot 5 \cdot \frac{1}{2} \cdot (0{,}5 + 1)\Big] = \frac{1}{EI_0} (-53{,}19) = -0{,}01184 \text{ rad} = -0{,}6784^\circ$$

sprawdzenie doboru przekrojów

przekrój y_1 I 140

$h = 140\,mm$

$\sigma_{dop} = 215\ MPa$

$$\sigma = \frac{|M|}{W} = \frac{38,98}{81,9} = 0,4760\ kN/cm^2 = 4,760\ MPa < \sigma_{dop}$$

przekrój y_2 I 200

$h = 200\ mm$

$\sigma_{dop} = 215\ MPa$

$$\sigma = \frac{|M|}{W} = \frac{2947}{214} = 13,77\ kN/cm^2 = 137,7\ MPa < \sigma_{dop}$$

wnioski: Naprężenia w obydwu przekrojach są mniejsze niż dopuszczalne z dużym zapasem nośności. Należy przyjąć mniejsze, które będą bardziej ekonomiczne.