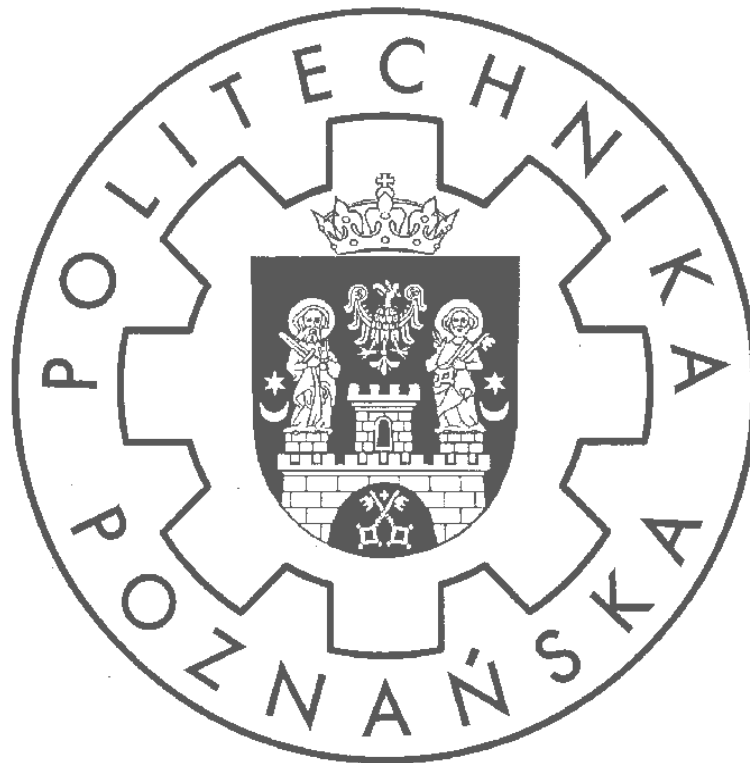


Politechnika Poznańska
Instytut Konstrukcji Budowlanych
Zakład Mechaniki Budowli



Ćwiczenie projektowe nr 1

Metoda przemieszczeń

Prowadzący:

Autor:

Nr indeksu:

Semestr:

Rok:

mgr inż. Anita Kaczor

Robert Dybionka

Jakub Stasiak

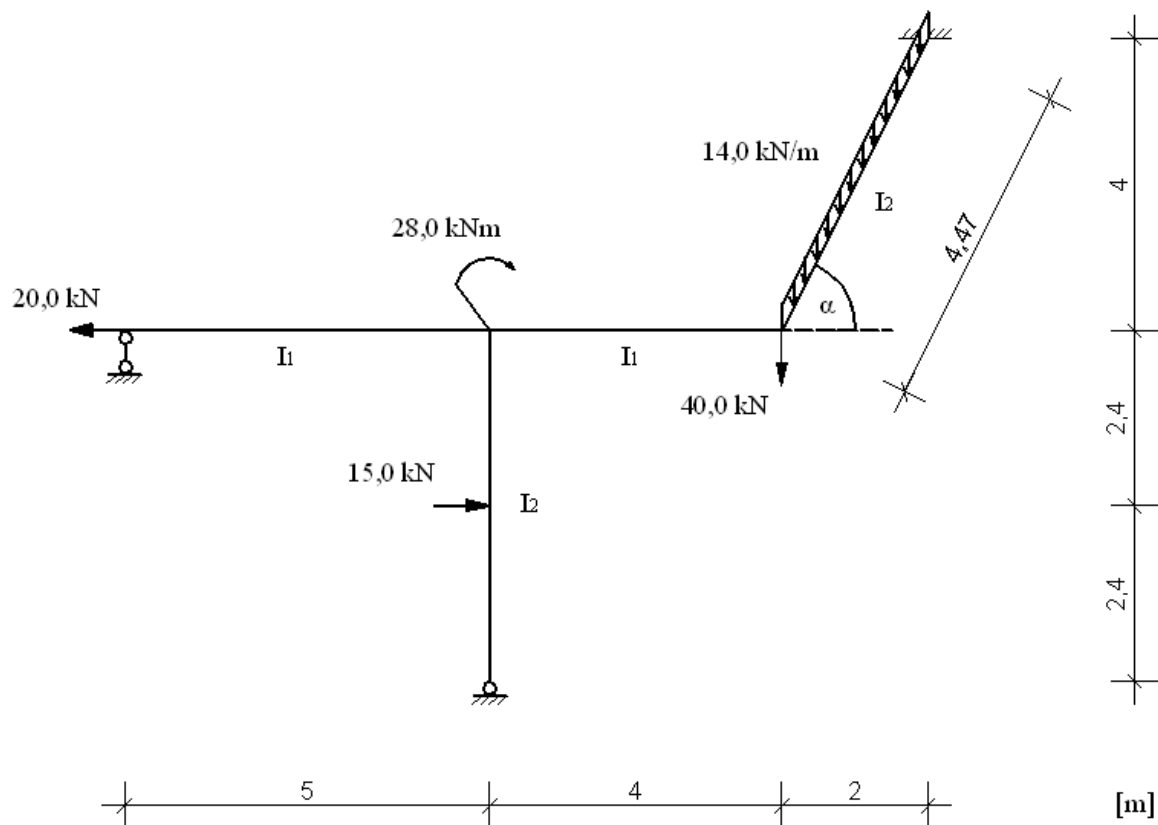
113012, 113010

IV (pierwszy stopień)

2013/2014

Ćwiczenie projektowe nr 1 – Metoda przemieszczeń

1. Schemat statyczny ramy:

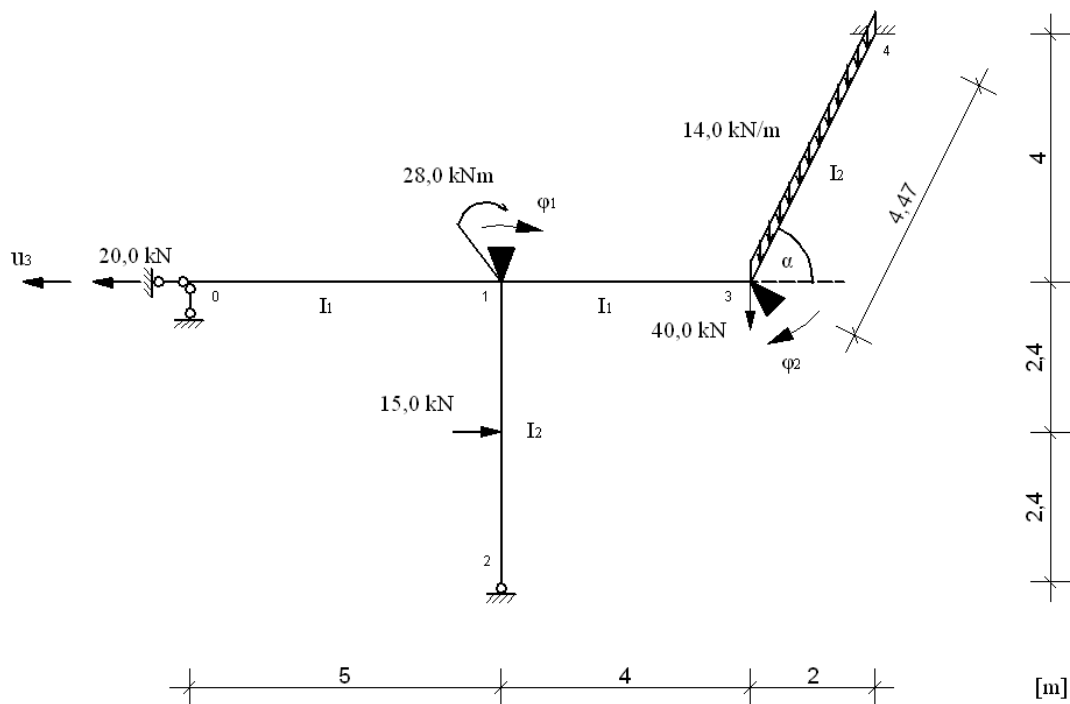


2. Stopień geometrycznej niewyznaczalności: $SGN = 3$

3. Dane wyjściowe do projektu:

- $\sin \alpha = 0,8949$
- $\cos \alpha = 0,4474$
- $\sigma_{dop} = 215 \text{ MPa}$
- $E = 205 \text{ GPa}$
- I 160 PE; $I_1 = 869 \text{ cm}^4$
- I 200 PE; $I_2 = 1940 \text{ cm}^4$
- przyjęto: $I_2 = I_0 \Rightarrow I_1 = 0,4479 \cdot I_0$

4. Układ podstawowy:



5. Układ równań kanonicznych:

$$\begin{cases} r_{11} \cdot \varphi_1 + r_{12} \cdot \varphi_2 + r_{13} \cdot u_3 + r_{1P} = 0 \\ r_{21} \cdot \varphi_1 + r_{22} \cdot \varphi_2 + r_{23} \cdot u_3 + r_{2P} = 0 \\ r_{31} \cdot \varphi_1 + r_{32} \cdot \varphi_2 + r_{33} \cdot u_3 + r_{3P} = 0 \end{cases}$$

6. Stany jednostkowe:

6.1. Stan $\varphi_1 = 1$

$$M_{10} = \frac{3 \cdot 0,4479 EI}{5} (1 - 0)$$

$$\mathbf{M_{10} = 0,2687 EI}$$

$$M_{12} = \frac{3 EI}{4,8} (1 - 0)$$

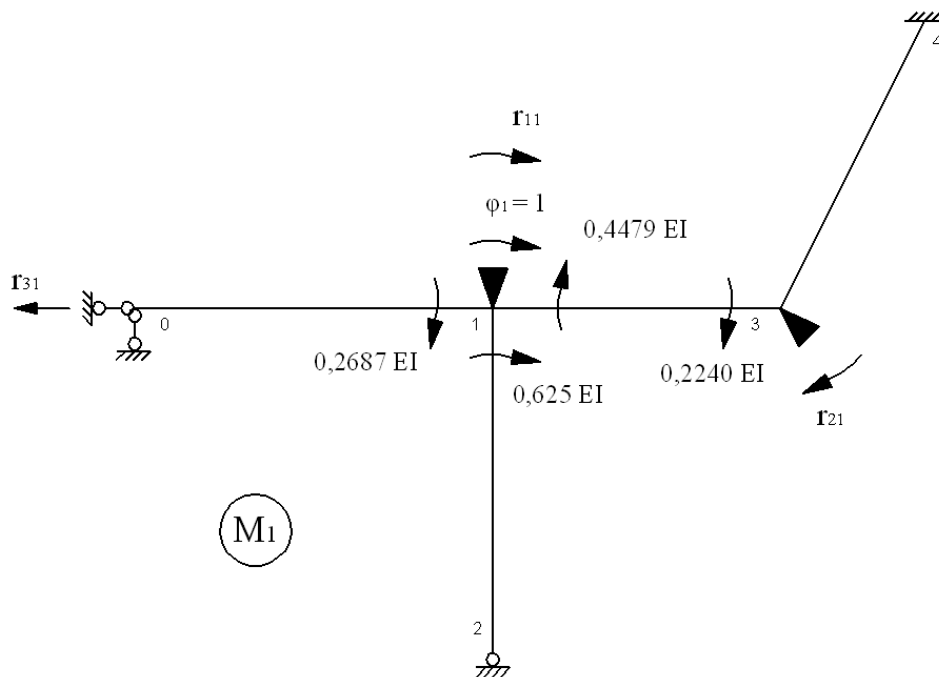
$$\mathbf{M_{12} = 0,625 EI}$$

$$M_{13} = \frac{2 \cdot 0,4479 EI}{4} (2 \cdot 1 + 0 - 3 \cdot 0)$$

$$\mathbf{M_{13} = 0,4479 EI}$$

$$M_{31} = \frac{2 \cdot 0,4479 EI}{4} (2 \cdot 0 + 1 - 3 \cdot 0)$$

$$\mathbf{M_{31} = 0,2240 EI}$$



6.2. Stan $\varphi_2 = 1$

$$M_{13} = \frac{2 \cdot 0,4479 EI}{4} (2 \cdot 0 + 1 - 3 \cdot 0)$$

$$M_{13} = \mathbf{0,2240 EI}$$

$$M_{31} = \frac{2 \cdot 0,4479 EI}{4} (2 \cdot 1 + 0 - 3 \cdot 0)$$

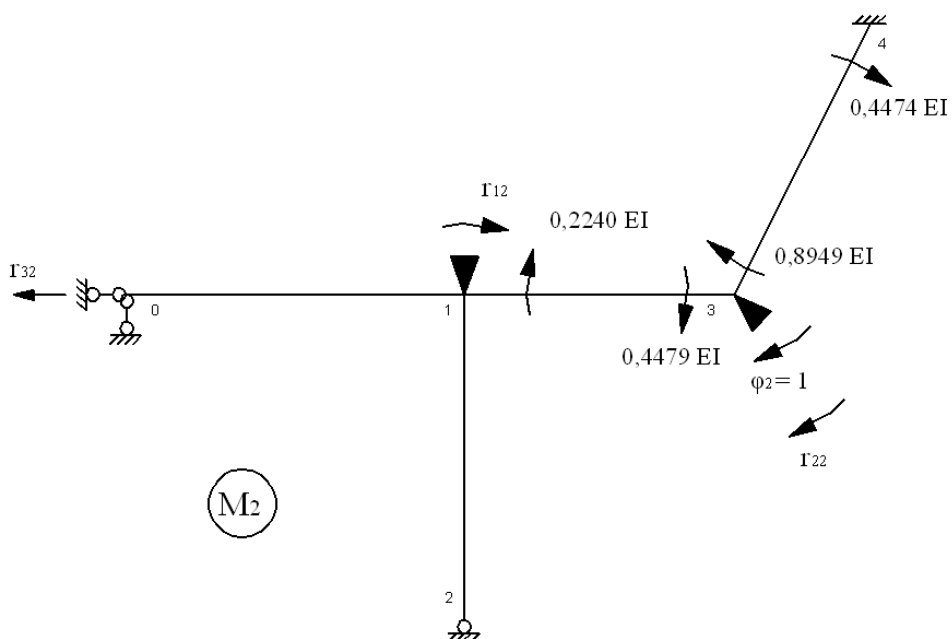
$$M_{31} = \mathbf{0,4479 EI}$$

$$M_{34} = \frac{2 EI}{4,47} (2 \cdot 1 + 0 - 3 \cdot 0)$$

$$M_{34} = \mathbf{0,8949 EI}$$

$$M_{43} = \frac{2 EI}{4,47} (2 \cdot 0 + 1 - 3 \cdot 0)$$

$$M_{43} = \mathbf{0,4474 EI}$$



6.3. Stan $u_3 = 1$

Równania łańcucha kinematycznego:

- 210 poziomo:

$$0 + \psi_{12} \cdot 4,8 + 0 = -1$$

$$\psi_{12} = -0,2083$$

- 2134 poziomo:

$$0 + \psi_{12} \cdot 4,8 + 0 + \psi_{34} \cdot 4,0 = 0$$

$$\psi_{34} = 0,25$$

- 2134 pionowo:

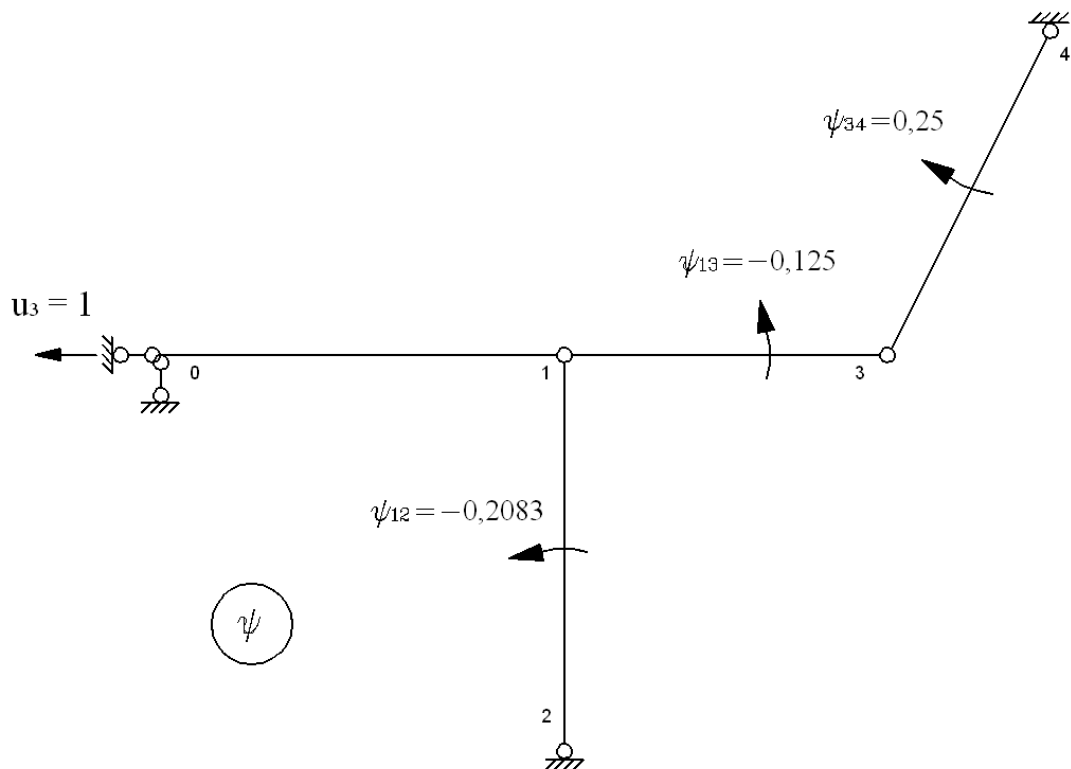
$$0 + 0 + \psi_{13} \cdot 4,0 + \psi_{34} \cdot 2,0 = 0$$

$$\psi_{13} = -0,125$$

- 4310 pionowo:

$$0 - \psi_{34} \cdot 2,0 - \psi_{13} \cdot 4,0 - \psi_{01} \cdot 5,0 = 0$$

$$\psi_{01} = 0,00$$



$$M_{10} = \frac{-3 \cdot 0,4479 EI}{5} \cdot 0$$

$$M_{10} = 0,00$$

$$M_{12} = \frac{-3 EI}{4,8} \cdot (-0,2083)$$

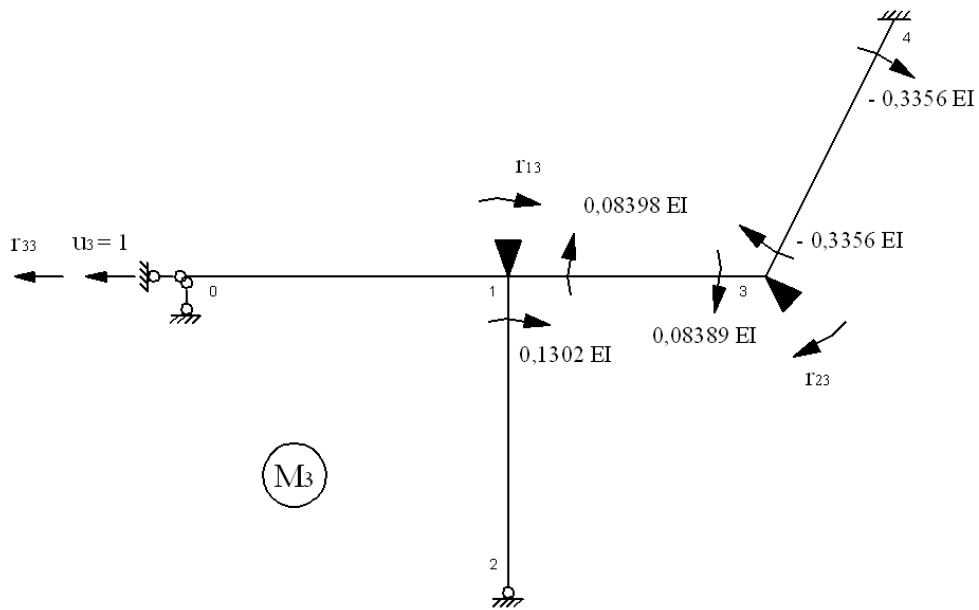
$$M_{12} = 0,1302 EI$$

$$M_{13} = M_{31} = \frac{-6 \cdot 0,4479 EI}{4} \cdot (-0,125)$$

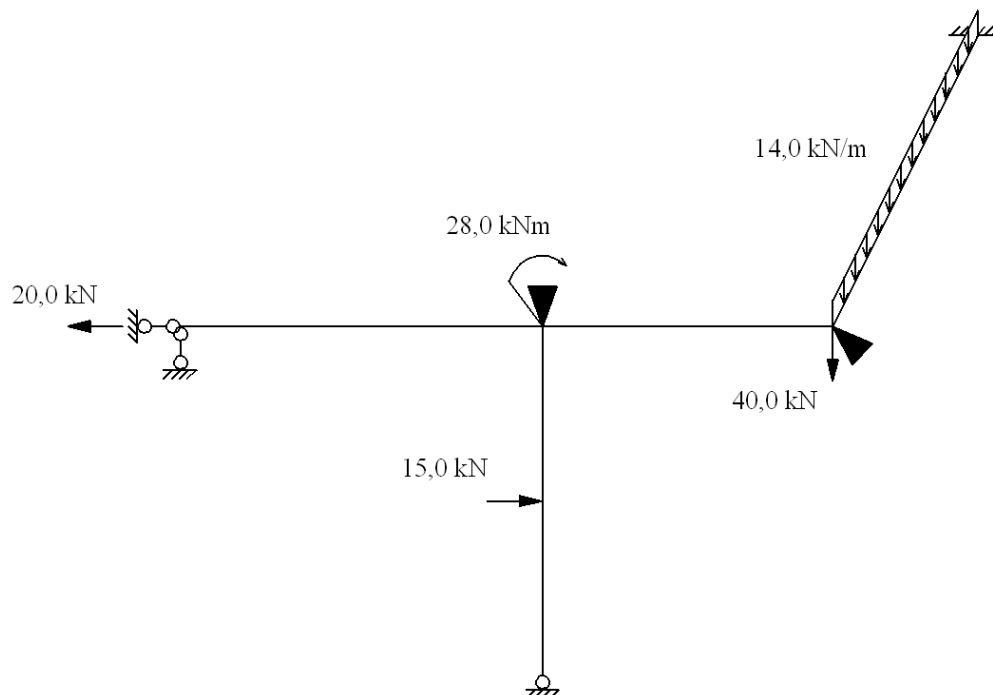
$$M_{13} = M_{31} = 0,08398 EI$$

$$M_{34} = M_{43} = \frac{-6 EI}{4,47} \cdot 0,25$$

$$M_{34} = M_{43} = -0,3356 EI$$



5.4. Stan „P“



- Belka prosta 12:

$$|M_{12}| = \frac{3}{16} PL$$

$$|M_{12}| = \frac{3}{16} \cdot 15,0 \cdot 4,8$$

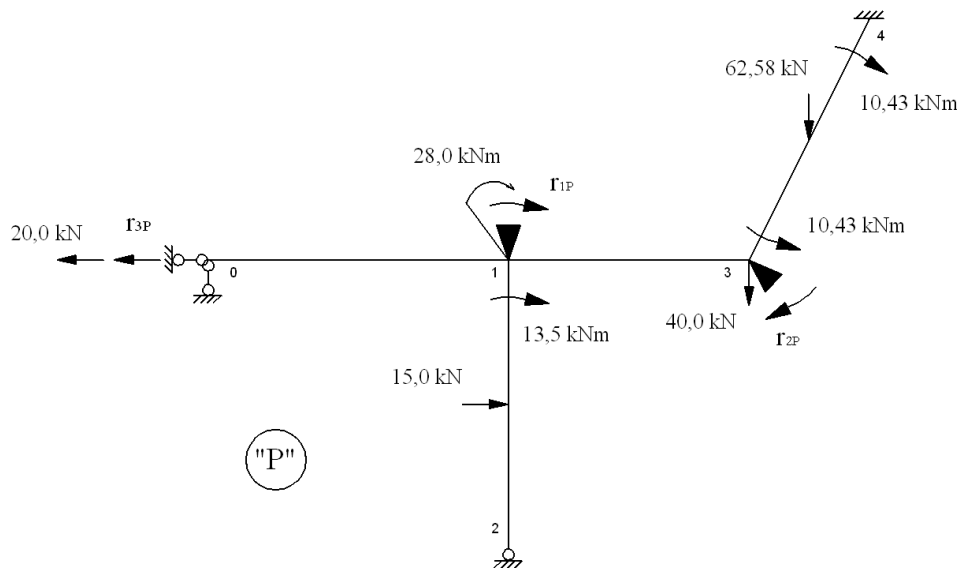
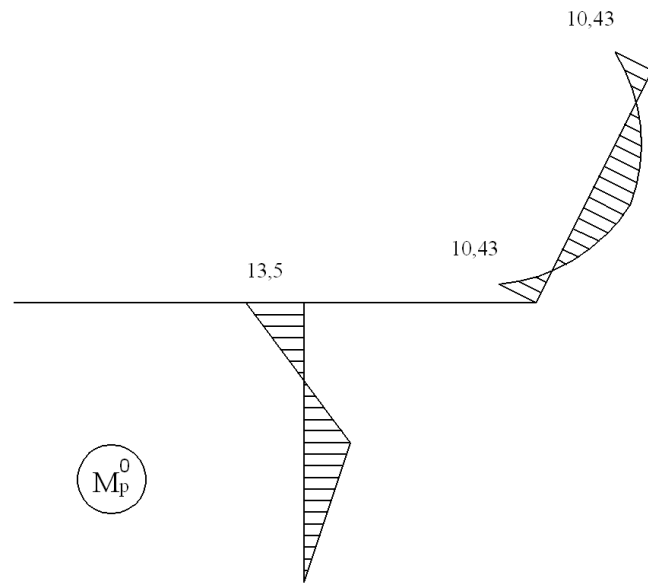
$$|M_{12}| = \mathbf{13,5 \text{ kNm}}$$

- Belka prosta 34:

$$|M_{34}| = |M_{43}| = \frac{qL^2}{12}$$

$$|M_{34}| = |M_{43}| = \frac{14,0 \cdot 2,0 \cdot 4,47}{12}$$

$$|M_{34}| = |M_{43}| = \mathbf{10,43 \text{ kNm}}$$



7. Obliczenie wartości r_{ik} :

$$r_{11} = 0,2687 EI + 0,4479 EI + 0,625 EI$$

$$\mathbf{r_{11} = 1,3416 EI}$$

$$\mathbf{r_{21} = 0,2240 EI}$$

$$\mathbf{r_{12} = 0,2240 EI}$$

$$r_{22} = 0,8949 EI + 0,4479 EI$$

$$\mathbf{r_{22} = 1,3428 EI}$$

$$r_{13} = 0,08398 EI + 0,1302 EI$$

$$\mathbf{r_{13} = 0,21418 EI}$$

$$r_{23} = 0,08398 EI - 0,3356 EI$$

$$\mathbf{r_{23} = - 0,2516 EI}$$

$$r_{33} \cdot 1 + 0,1302 EI \cdot (- 0,2083) + 0,08398 EI \cdot (- 0,125) \cdot 2 - 0,3356 EI \cdot 0,25 \cdot 2 = 0$$

$$\mathbf{r_{33} = 0,2159 EI}$$

$$r_{31} \cdot 1 + 0,2687 EI \cdot 0 + 0,625 EI \cdot (- 0,2083) + (0,4479 EI + 0,2240 EI) \cdot (- 0,125) = 0$$

$$\mathbf{r_{31} = 0,21418 EI}$$

$$r_{32} \cdot 1 + (0,2240 EI + 0,4479 EI) \cdot (- 0,125) + (0,8949 EI + 0,4474 EI) \cdot 0,25 = 0$$

$$\mathbf{r_{32} = - 0,2516 EI}$$

$$r_{1P} + 28,0 - 13,5 = 0$$

$$\mathbf{r_{1P} = - 14,5 \text{ kNm}}$$

$$r_{2P} + 10,43 = 0$$

$$\mathbf{r_{2P} = - 10,43 \text{ kNm}}$$

$$r_{3P} \cdot \bar{1} + 20,0 \cdot \bar{1} + 13,5 \cdot (- \overline{0,2083}) + 28,0 \cdot 0 + (-10,43 + 10,43) \cdot \overline{0,25} + 15,0 \cdot \bar{\delta}_A^H + 62,58 \cdot \bar{\delta}_B^V + 40,0 \cdot \bar{\delta}_C^V = 0$$

Przemieszczenia, na których pracują siły skupione wyznaczamy z równań łańcucha kinematycznego:

- 01A poziomo:

$$- 1 + 0 - \psi_{12} \cdot 2,4 = \bar{\delta}_A^H$$

$$\bar{\delta}_A^H = - 0,5$$

- 4B pionowo:

$$0 - \psi_{34} \cdot 1,0 = \bar{\delta}_B^V$$

$$\bar{\delta}_B^V = - 0,25$$

- 43 pionowo:

$$0 - \psi_{34} \cdot 2,0 = \bar{\delta}_C^V$$

$$\bar{\delta}_C^V = - 0,5$$

$$\mathbf{r_{3P} = 25,95705 \text{ kN}}$$

8. Obliczenie wartości φ_1 , φ_2 oraz u_3 :

$$\begin{cases} 1,3416 EI \cdot \varphi_1 + 0,2240 EI \cdot \varphi_2 + 0,21418 EI \cdot u_3 - 14,5 = 0 \\ 0,2240 EI \cdot \varphi_1 + 1,3428 EI \cdot \varphi_2 - 0,2516 EI \cdot u_3 - 10,43 = 0 \\ 0,21418 EI \cdot \varphi_1 - 0,2516 EI \cdot \varphi_2 + 0,2159 EI \cdot u_3 + 25,95705 = 0 \end{cases}$$

$$\begin{cases} \varphi_1 = \frac{53,7258}{EI} \\ \varphi_2 = \frac{-43,1168}{EI} \\ u_3 = \frac{-223,7386}{EI} \end{cases}$$

9. Wyznaczenie rzeczywistych momentów przywęzłowych:

$M_{01}^n = 0,00 \text{ kNm}$

$$M_{10}^n = 0,2687 EI \cdot \frac{53,7258}{EI} + 0 + 0 + 0$$

$M_{10}^n = 14,4361 \text{ kNm}$

$$M_{12}^n = 0,625 EI \cdot \frac{53,7258}{EI} + 0 + 0,1302 EI \cdot \left(-\frac{223,7386}{EI}\right) + 13,5$$

$M_{12}^n = 17,9479 \text{ kNm}$

$M_{21}^n = 0,00 \text{ kNm}$

$$M_{13}^n = 0,4479 EI \cdot \frac{53,7258}{EI} + 0,2240 EI \cdot \left(-\frac{43,1168}{EI}\right) + 0,08398 EI \cdot \left(-\frac{223,7386}{EI}\right) + 0$$

$M_{13}^n = -4,3839 \text{ kNm}$

$$M_{31}^n = 0,2240 EI \cdot \frac{53,7258}{EI} + 0,4479 EI \cdot \left(-\frac{43,1168}{EI}\right) + 0,08398 EI \cdot \left(-\frac{223,7386}{EI}\right) + 0$$

$M_{31}^n = -26,07 \text{ kNm}$

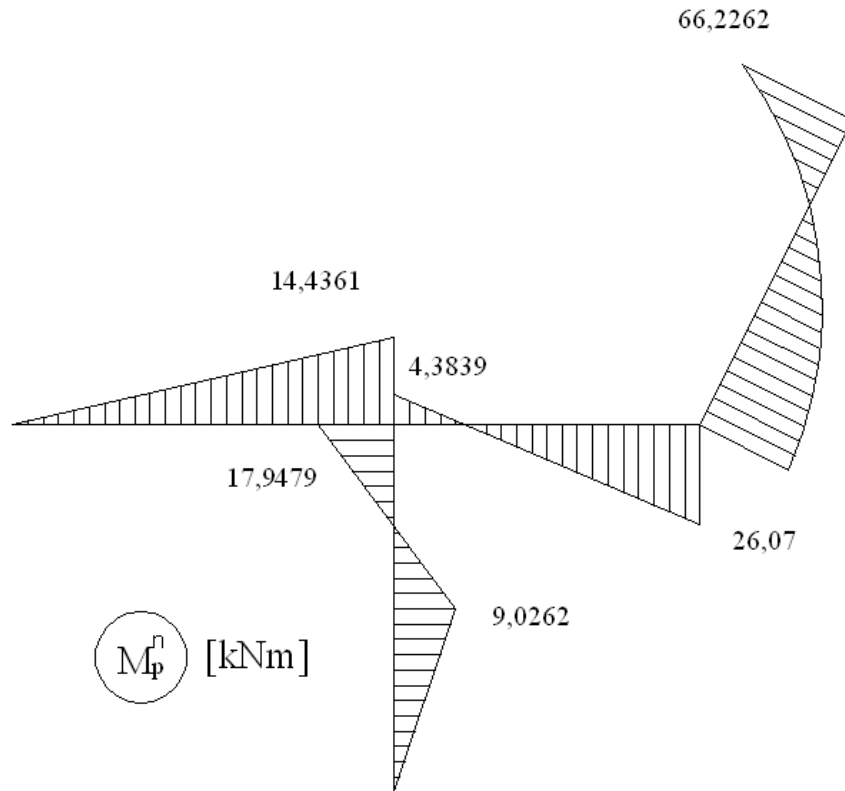
$$M_{34}^n = 0 + 0,8949 EI \cdot \left(-\frac{43,1168}{EI}\right) - 0,3356 EI \cdot \left(-\frac{223,7386}{EI}\right) - 10,43$$

$M_{34}^n = 26,07 \text{ kNm}$

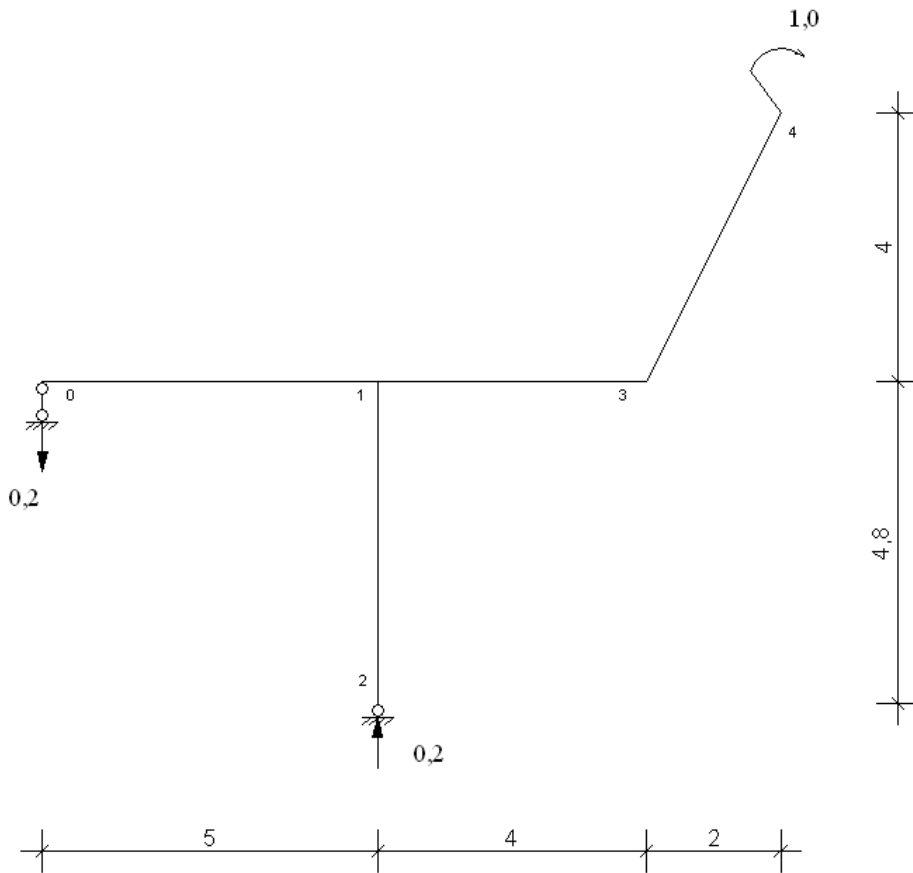
$$M_{43}^n = 0 + 0,4474 EI \cdot \left(-\frac{43,1168}{EI}\right) - 0,3356 EI \cdot \left(-\frac{223,7386}{EI}\right) + 10,43$$

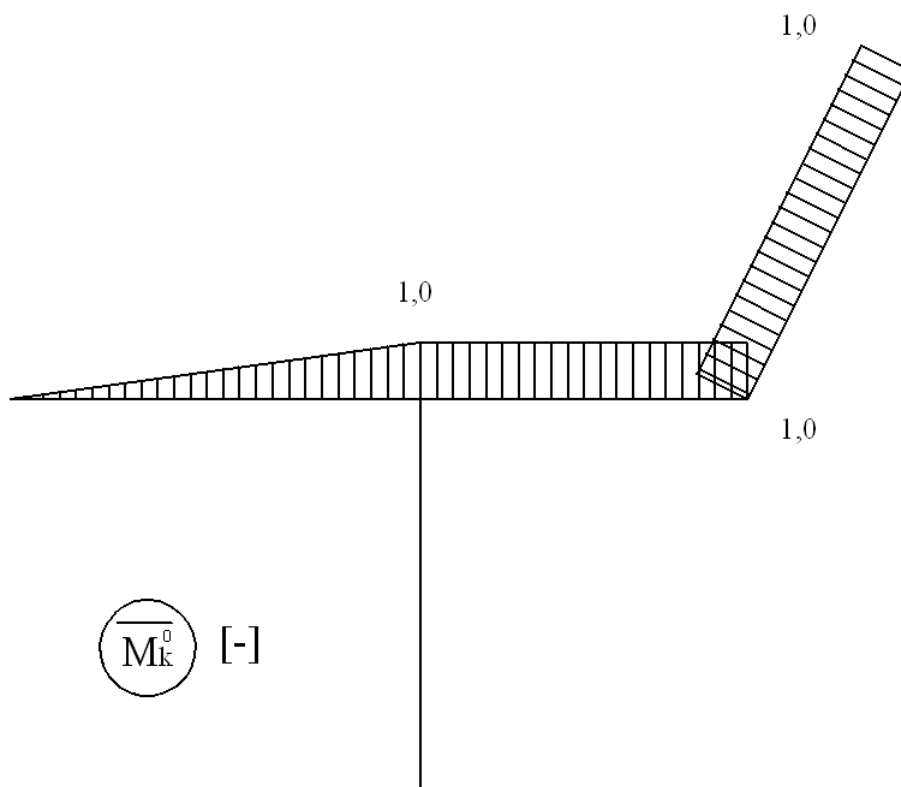
$M_{43}^n = 66,2262 \text{ kNm}$

Moment zginający pod siłą skupioną obliczono po wyznaczeniu sił tnących.



10. Sprawdzenie kinematyczne:





$$\delta_4 = \sum \int \frac{M_p^n \cdot \overline{M_k^0}}{EI} dx$$

$$\delta_4 = \frac{1}{0,4479 EI} \left[\frac{1}{2} \cdot 14,4361 \cdot 5,0 \cdot \frac{2}{3} \cdot 1,0 + \frac{1}{2} \cdot 4,3839 \cdot 4,0 \cdot 1,0 - \frac{1}{2} \cdot 26,07 \cdot 4,0 \cdot 1,0 \right] +$$

$$+ \frac{1}{EI} \left[\frac{1}{2} \cdot 66,2262 \cdot 4,47 \cdot 1,0 - \frac{1}{2} \cdot 26,07 \cdot 4,47 \cdot 1,0 - \frac{2}{3} \cdot \frac{14,0 \cdot 2,0 \cdot 4,47}{8} \cdot 4,47 \cdot 1,0 \right]$$

$$\delta_4 = \frac{0,0102}{EI} \quad |\delta_4| < \frac{1}{EI} \cong 0$$

11. Wyznaczenie sił tnących:

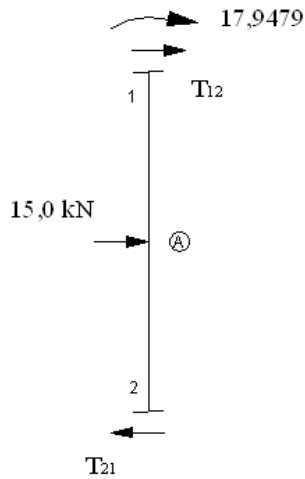


$$\sum M_0 = T_{10} \cdot 5,0 + 14,4361 = 0$$

$$\mathbf{T_{10} = -2,8872 \text{ kN}}$$

$$\sum M_1 = T_{01} \cdot 5,0 + 14,4361 = 0$$

$$\mathbf{T_{01} = -2,8872 \text{ kN}}$$

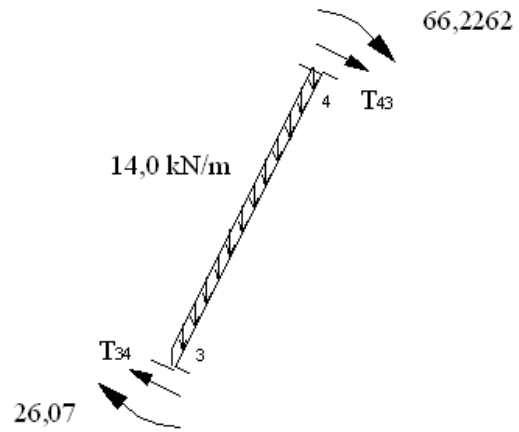


$$\begin{aligned} \sum M_1 &= T_{21} \cdot 4,8 - 15,0 \cdot 2,4 + 17,9479 = 0 \\ \mathbf{T_{21} &= 3,7609 \text{ kN}} \\ \sum M_2 &= T_{12} \cdot 4,8 + 15,0 \cdot 2,4 + 17,9479 = 0 \\ \mathbf{T_{12} &= -11,239 \text{ kN}} \end{aligned}$$

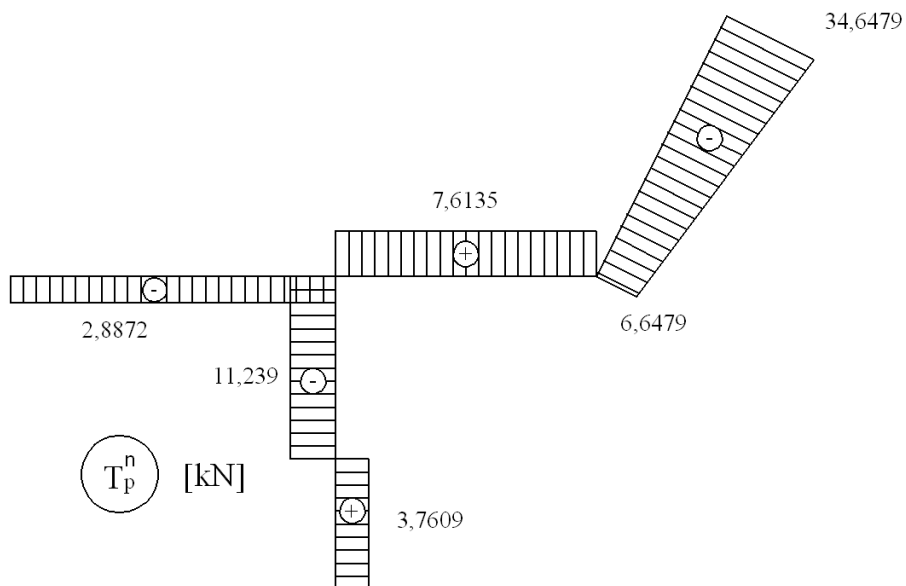
Moment pod siłą skupioną:
 $M_A = T_{21} \cdot 2,4 = 3,7609 \cdot 2,4$
 $\mathbf{M_A = 9,0262 \text{ kNm}}$



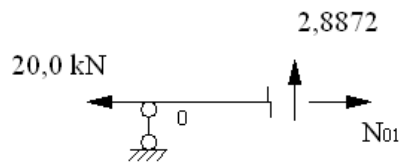
$$\begin{aligned} \sum M_1 &= T_{31} \cdot 4,0 - 26,07 - 4,3839 = 0 \\ \mathbf{T_{31} &= 7,6135 \text{ kN}} \\ \sum M_3 &= T_{13} \cdot 4,0 - 4,3839 - 26,07 = 0 \\ \mathbf{T_{13} &= 7,6135 \text{ kN}} \end{aligned}$$



$$\begin{aligned} \sum M_3 &= \\ &= T_{43} \cdot 4,47 + 26,07 + 66,2262 + 14,0 \cdot 4,47 \cdot 1,0 = 0 \\ \mathbf{T_{43} &= -34,6479 \text{ kN}} \\ \sum M_4 &= \\ &= T_{34} \cdot 4,47 + 26,07 + 66,2262 - 14,0 \cdot 4,47 \cdot 1,0 = 0 \\ \mathbf{T_{13} &= -6,6479 \text{ kN}} \end{aligned}$$



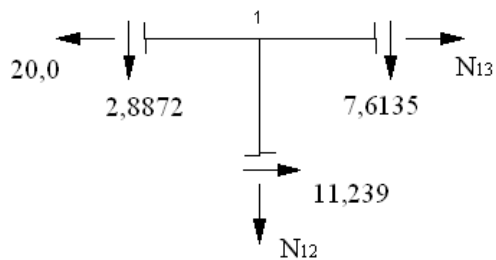
12. Wyznaczenie sił normalnych:



$$\sum X = N_{01} - 20,0 = 0$$

$$N_{01} = 20,0 \text{ kN}$$

$$N_{10} = N_{01} = 20,0 \text{ kN}$$



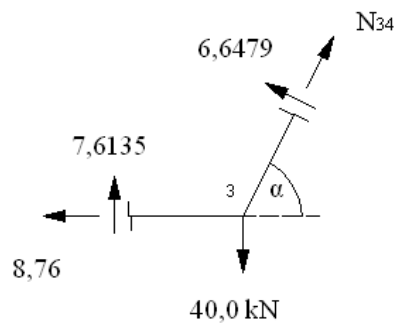
$$\sum Y = -N_{12} - 7,6135 - 2,8872 = 0$$

$$N_{12} = -10,5 \text{ kN}$$

$$\sum X = N_{13} - 20,0 + 11,239 = 0$$

$$N_{13} = 8,76 \text{ kN}$$

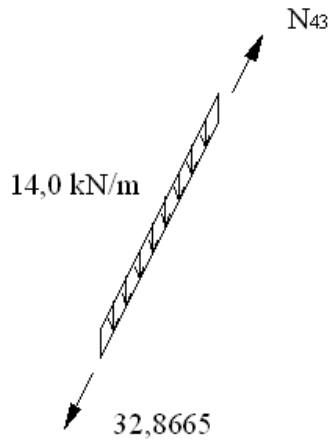
$$N_{31} = N_{13}$$



$$\sum Y = N_{34} \cdot 0,8949 + 6,6479 \cdot 0,4474 + 7,6135 - 40,0 = 0$$

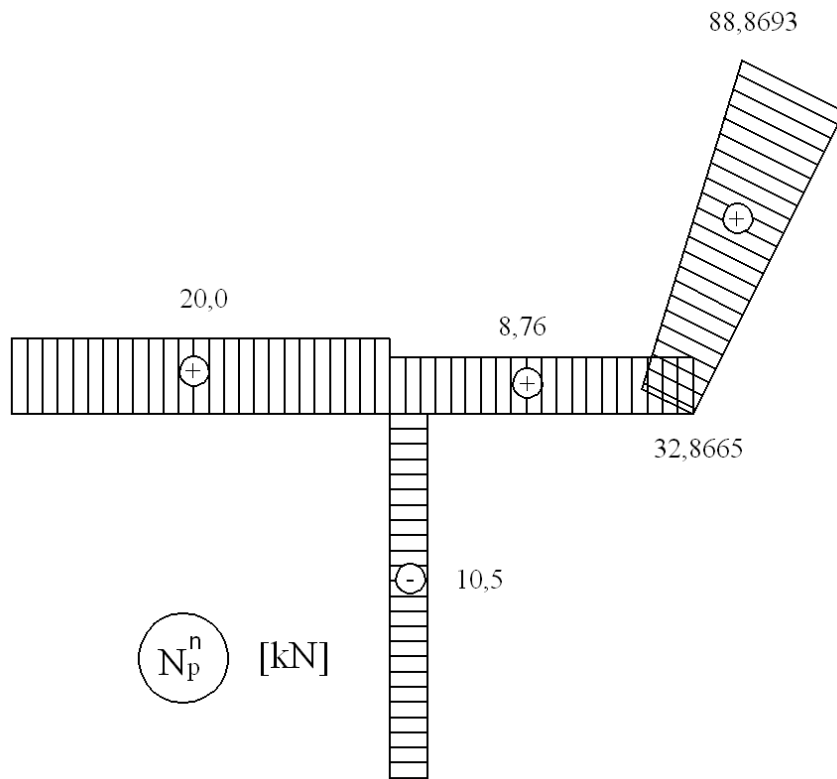
$$N_{34} = 32,8665 \text{ kN}$$

Sprawdzenie: $\sum X = -8,76 - 6,6479 \cdot 0,8949 + 32,8665 \cdot 0,4474 = 0$
 $-0,00473 \cong 0$

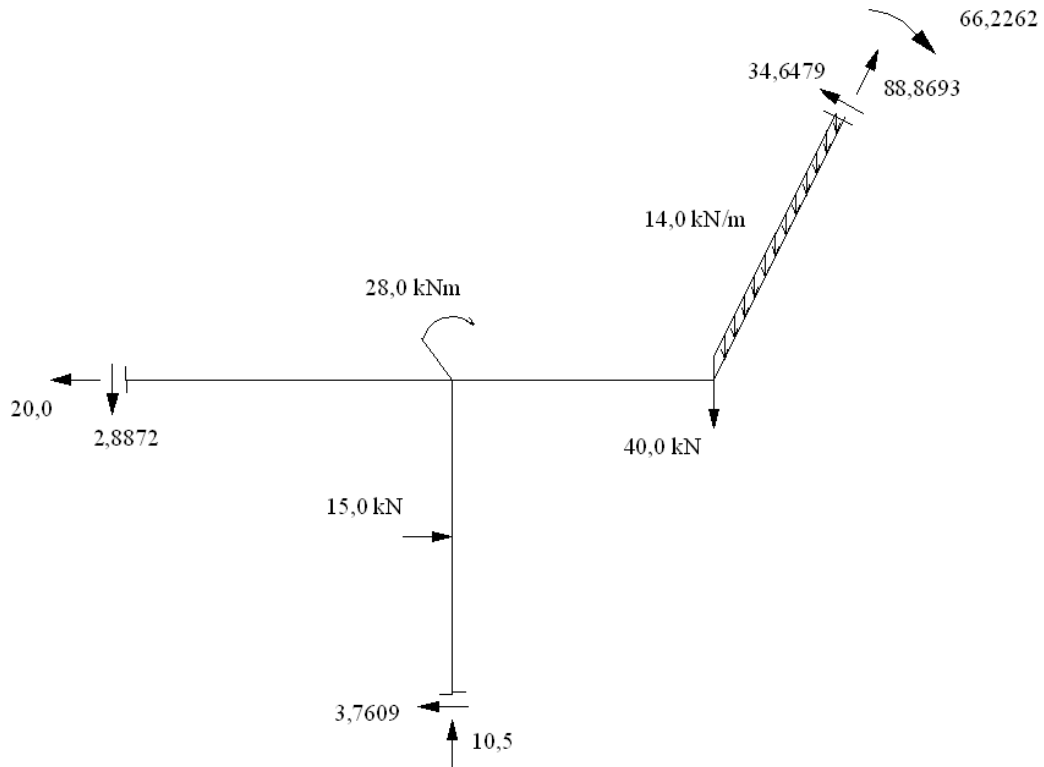


$$\sum X = N_{43} - 32,8665 - 14,0 \cdot 4,47 \cdot 0,8949 = 0$$

$$N_{43} = 88,8693 \text{ kN}$$



13. Sprawdzenie statyczne:



$$\sum X = -20,0 + 15,0 - 3,7609 - 34,6479 \cdot 0,8949 + 88,8693 \cdot 0,4474 = 0$$

$$\sum X = -0,00718 \cong 0$$

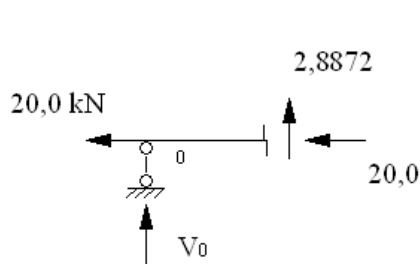
$$\sum Y = -2,8872 + 10,5 - 40,0 - 14,0 \cdot 4,47 + 88,8693 \cdot 0,8949 + 34,6479 \cdot 0,4474 = 0$$

$$\sum Y = 0,0638 \cong 0$$

$$\sum M_3 = -2,8872 \cdot 9,0 + 28,0 - 15,0 \cdot 2,4 + 3,7609 \cdot 4,8 + 10,5 \cdot 4,0 + 14,0 \cdot 4,47 \cdot 1,0 - 34,6479 \cdot 4,47 + 66,2262 = 0$$

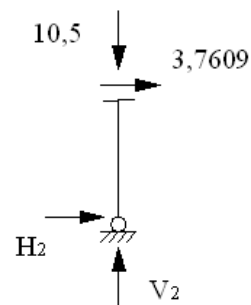
$$\sum M_3 = -0,00239 \cong 0$$

14. Wyznaczenie reakcji podporowych:



$$\sum Y = V_0 + 2,8872 = 0$$

$$V_0 = -2,8872 \text{ kN}$$

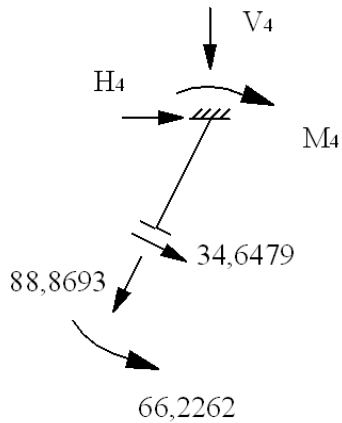


$$\sum X = H_2 + 3,7609 = 0$$

$$H_2 = -3,7609 \text{ kN}$$

$$\sum Y = V_2 - 10,5 = 0$$

$$V_2 = 10,5 \text{ kN}$$



$$\begin{aligned}\sum X &= H_4 + 34,6479 \cdot 0,8949 - \\ &88,8693 \cdot 0,4474 = 0\end{aligned}$$

$$\mathbf{H_4 = 8,7537 \text{ kN}}$$

$$\begin{aligned}\sum Y &= -V_4 - 88,8693 \cdot 0,8949 - \\ &34,6479 \cdot 0,4474 = 0\end{aligned}$$

$$\mathbf{V_4 = -95,0306 \text{ kN}}$$

$$\sum M_4 = M_4 - 66,2262 = 0$$

$$\mathbf{M_4 = 66,2262 \text{ kNm}}$$

15. Sprawdzenie naprężeń:

$$\sigma = \frac{M_{max}}{W}$$

15.1. Przekrój I₁:

$$\sigma = \frac{2607}{109}$$

$$\mathbf{\sigma = 239,17 \text{ MPa} > 215 \text{ MPa}}$$

15.2. Przekrój I₂:

$$\sigma = \frac{6622,62}{194}$$

$$\mathbf{\sigma = 341,37 \text{ MPa} > 215 \text{ MPa}}$$

15.3. Wnioski:

Naprężenia w obu grupach przekrojów I₁ oraz I₂ przekraczają wartości dopuszczalne naprężeń (215 MPa). Należałoby przyjąć większe przekroje i ponownie wykonać obliczenia.