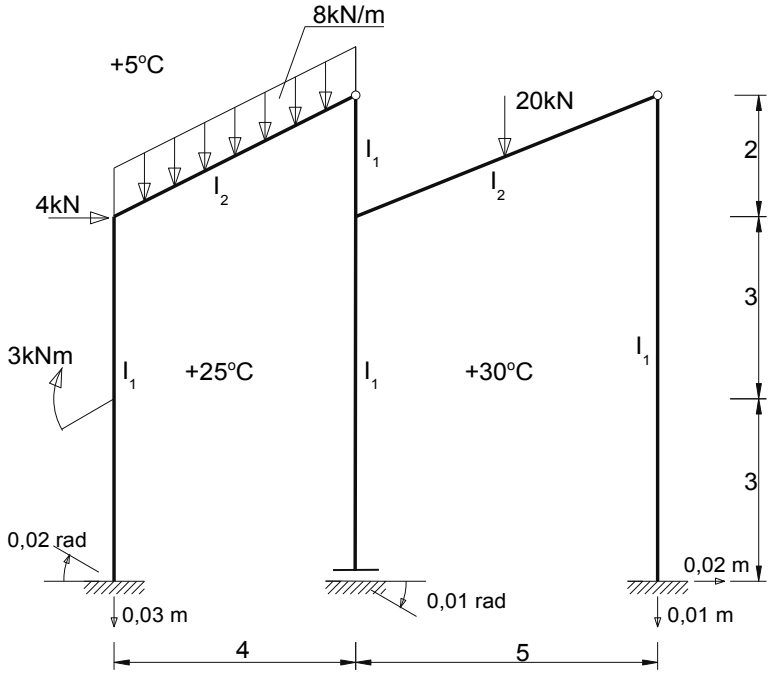
METODA PRZEMIESZCZEŃ
obciążenie siłami zewnętrznymi

wykonał: Krzysztof Kalisiak

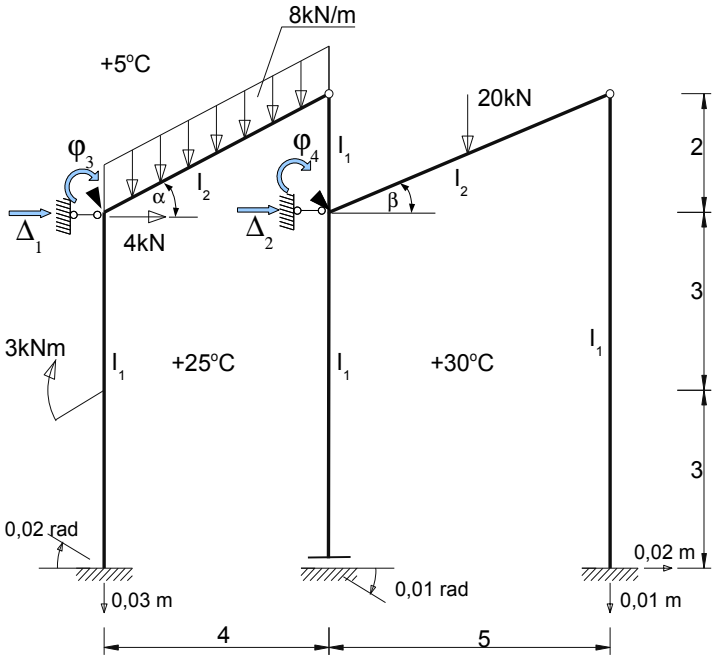
SCHEMAT KONSTRUKCJI:



STOPIEŃ GEOMETRYCZNEJ NIEWYZNACZALNOŚCI UKŁADU:

$$SGN = \sum \Delta + \sum \varphi \qquad \sum \Delta = 2 \qquad \sum \varphi = 2 \qquad SGN = 2 + 2 = 4$$

UKAD PODSTAWOWY:



$$\tan\alpha = \frac{2}{4} \Rightarrow \alpha = 26{,}565051$$

$$\sin\alpha = 0{,}447214$$

$$\cos\alpha = 0{,}894427$$

$$\tan\beta = \frac{2}{5} \Rightarrow \beta = 21{,}801409$$

$$\sin\beta = 0{,}371391$$

$$\cos\beta = 0{,}928477$$

UKŁAD RÓWNAŃ KANONICZNYCH:

$$r_{11}\cdot z_1+r_{12}\cdot z_2+r_{13}\cdot z_3+r_{14}\cdot z_4+r_{1P}=0$$
$$r_{21}\cdot z_1+r_{22}\cdot z_2+r_{23}\cdot z_3+r_{24}\cdot z_4+r_{2P}=0$$
$$r_{31}\cdot z_1+r_{32}\cdot z_2+r_{33}\cdot z_3+r_{34}\cdot z_4+r_{3P}=0$$
$$r_{41}\cdot z_1+r_{42}\cdot z_2+r_{43}\cdot z_3+r_{44}\cdot z_4+r_{4P}=0$$

NIEWIADOME:

$$\Delta_1=z_1$$
$$\Delta_2=z_2$$
$$\varphi_3=z_3$$
$$\varphi_4=z_4$$

WSPÓŁCZYNNIK PORÓWNAWCZY SZTYWNOŚCI:

I_1 – I220: $W_x=278{,}18\,cm^3$; $I_x=3060\,cm^4$

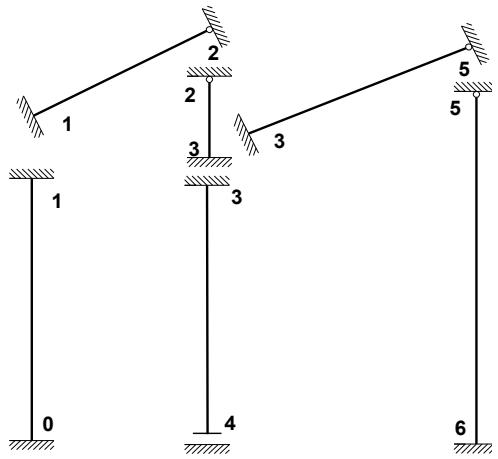
I_2 – I260: $W_x=441{,}54\,cm^3$; $I_x=5740\,cm^4$

$$EI_1=205\cdot10^6\cdot3060\cdot10^{-8}=6273\,kN/m^2$$
$$EI_2=205\cdot10^6\cdot5740\cdot10^{-8}=11767\,kN/m^2$$

współczynnik porównawczy: $n=\frac{EI_2}{EI_1}=\frac{11767}{6273}=1{,}875817$

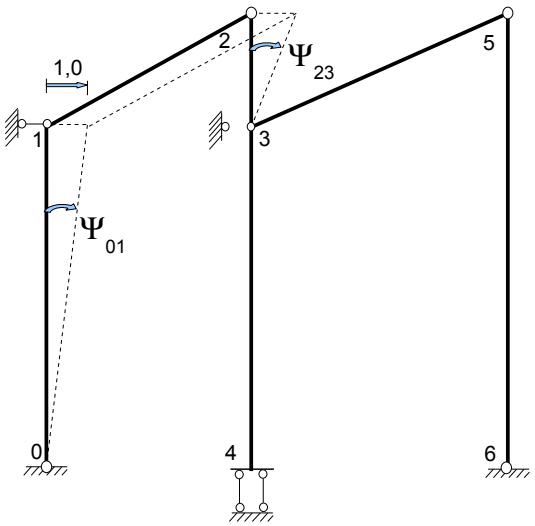
$$EI_2=n\cdot EI_1$$

WZORY TRANSFORMACYJNE:



$$M_{01}=\frac{2EI_1}{6}\cdot(2\cdot\varphi_0+\varphi_1-3\cdot\Psi_{01})$$
$$M_{10}=\frac{2EI_1}{6}\cdot(\varphi_0+2\cdot\varphi_1-3\cdot\Psi_{01})$$
$$M_{12}=\frac{3nEI_1}{\sqrt{20}}\cdot(\varphi_1-\Psi_{12})$$
$$M_{21}=0$$
$$M_{23}=0$$
$$M_{32}=\frac{3EI_1}{2}\cdot(\varphi_3-\Psi_{32})$$
$$M_{34}=\frac{EI_1}{6}\cdot(\varphi_3-\varphi_4)$$
$$M_{65}=\frac{3EI_1}{8}\cdot(\varphi_6-\Psi_{56})$$

- STAN $z_1=1 \quad (\Delta_2=\varphi_3=\varphi_4=0)$



ŁAŃCUCH KINEMATYCZNY:

$$\Psi_{34}^{(1)}=0$$

$01\rightarrow$

$$\Psi_{01}^{(1)}\cdot 6=1$$

$$\Psi_{01}^{(1)}=\frac{1}{6}$$

$123\downarrow$

$$\Psi_{12}^{(1)}\cdot 4=0$$

$$\Psi_{12}^{(1)}=0$$

$123\rightarrow$

$$1+\Psi_{12}^{(1)}\cdot 2+\Psi_{23}^{(1)}\cdot 2=0$$

$$\Psi_{23}^{(1)}=\frac{1}{2}$$

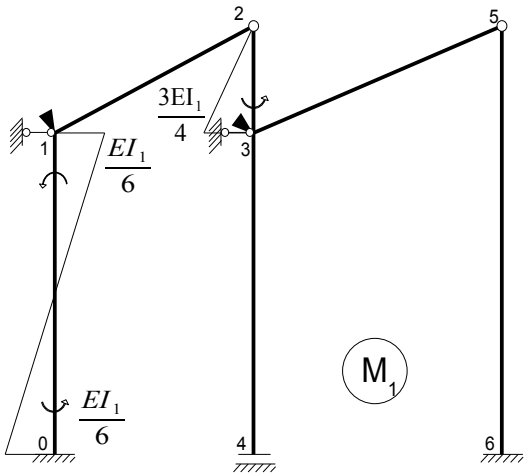
$4356\downarrow \Rightarrow \Psi_{35}^{(1)}=0$

$356\rightarrow \Rightarrow \Psi_{56}^{(1)}=0$

$$M_{01}^{(1)}=M_{10}^{(1)}=\frac{2\mathrm{EI}_1}{6}\cdot(2\cdot 0+0-\frac{3\cdot 1}{6})=-\frac{EI_1}{6}$$

$$M_{32}^{(1)}=\frac{3\mathrm{EI}_1}{2}\cdot(0-\frac{1}{2})=-\frac{3\mathrm{EI}_1}{4}$$

$$M_{12}^{(1)}=M_{21}^{(1)}=M_{23}^{(1)}=M_{34}^{(1)}=M_{43}^{(1)}=M_{35}^{(1)}=M_{53}^{(1)}=M_{56}^{(1)}=M_{65}^{(1)}=$$



RPW:

$$r_{11}\cdot\overline{1,0}+\Psi_{01}^{(1)}\cdot(M_{01}^{(1)}+M_{10}^{(1)})+\Psi_{23}^{(1)}\cdot M_{32}^{(1)}=0$$

$$r_{11}=-\frac{1}{6}\cdot(-\frac{1\mathrm{EI}_1}{6}-\frac{1\mathrm{EI}_1}{6})-\frac{1}{2}\cdot(\frac{-3\mathrm{EI}_1}{4})$$

$$\underline{r_{11}=\frac{31\mathrm{EI}_1}{72}}$$

$$r_{21}\cdot\overline{1,0}+\Psi_{23}^{(2)}\cdot M_{32}^{(1)}=0$$

$$r_{21}=-(-\frac{1}{2})(\frac{-3\mathrm{EI}_1}{4})$$

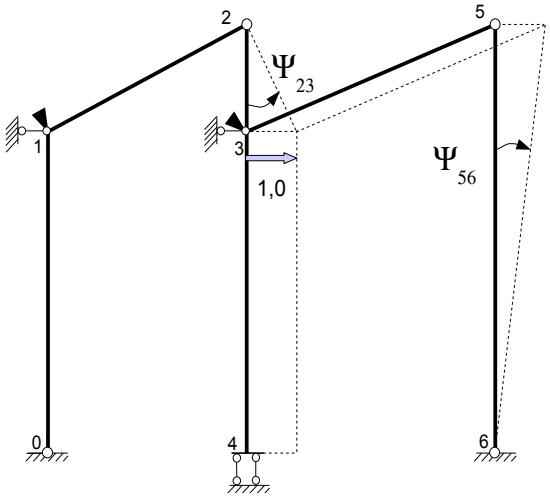
$$\underline{r_{21}=-\frac{3\mathrm{EI}_1}{8}}$$

RÓWNOWAGA WĘZŁÓW:

$$\underline{r_{31}=-\frac{EI_1}{6}}$$

$$\underline{r_{41}=-\frac{3\mathrm{EI}_1}{4}}$$

- STAN $z_2=1 \quad (\Delta_1=\varphi_3=\varphi_4=0)$



ŁAŃCUCH KINEMATYCZNY:

$$\Psi_{34}^{(2)}=0$$

$1234\downarrow$

$$\Psi_{12}^{(2)}\cdot 4=0$$

$$\Psi_{12}^{(2)}=0$$

$123\rightarrow$

$$\Psi_{12}^{(2)}\cdot 2-\Psi_{23}^{(2)}\cdot 2=1$$

$$\Psi_{23}^{(2)}=-\frac{1}{2}$$

$356\downarrow$

$$\Psi_{35}^{(2)}\cdot 5=0$$

$$\Psi_{35}^{(2)}=0$$

$123\rightarrow$

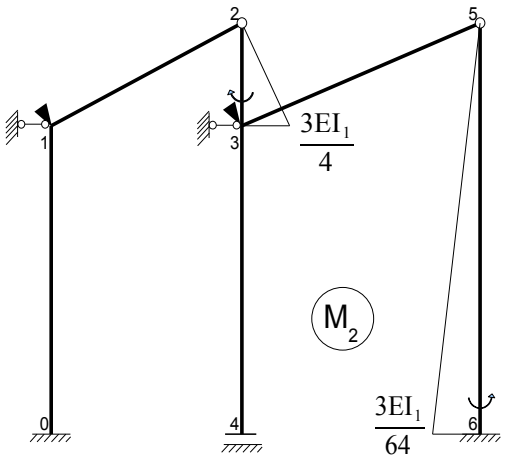
$$1+\Psi_{35}^{(2)}\cdot 2-\Psi_{56}^{(2)}\cdot 8=0$$

$$\Psi_{56}^{(2)}=\frac{1}{8}$$

$01\rightarrow \quad \Rightarrow \Psi_{01}^{(2)}=0$

$$M_{01}^{(2)}=M_{10}^{(2)}=0$$
$$M_{12}^{(2)}=M_{21}^{(2)}=0$$
$$M_{23}^{(2)}=0$$
$$M_{32}^{(2)}=\frac{3EI_1}{2}\cdot(0-(-\frac{1}{2}))=\frac{3EI_1}{4}$$

$$M_{34}^{(2)}=M_{43}^{(2)}=0$$
$$M_{35}^{(2)}=M_{53}^{(2)}=0$$
$$M_{56}^{(2)}=0$$
$$M_{65}^{(2)}=\frac{3EI_1}{8}\cdot(0-\frac{1}{8})=-\frac{3EI_1}{64}$$



RPW:

$$r_{12}\cdot\overline{1,0}+\Psi_{23}^{(1)}\cdot M_{32}^{(2)}=0$$

$$r_{22}\cdot\overline{1,0}+\Psi_{23}^{(1)}\cdot M_{32}^{(2)}+\Psi_{56}^{(1)}\cdot M_{65}^{(2)}=0$$

$$r_{12}=-\frac{1}{2}\cdot\frac{3EI_1}{4}$$

$$r_{22}=-(-\frac{1}{2})\cdot\frac{3EI_1}{4}-\frac{1}{8}\cdot(\frac{-3EI_1}{64})$$

$$\underline{r_{12}=-\frac{3EI_1}{8}}$$

$$\underline{r_{22}=\frac{195EI_1}{512}}$$

RÓWNOWAGA WĘZŁÓW:

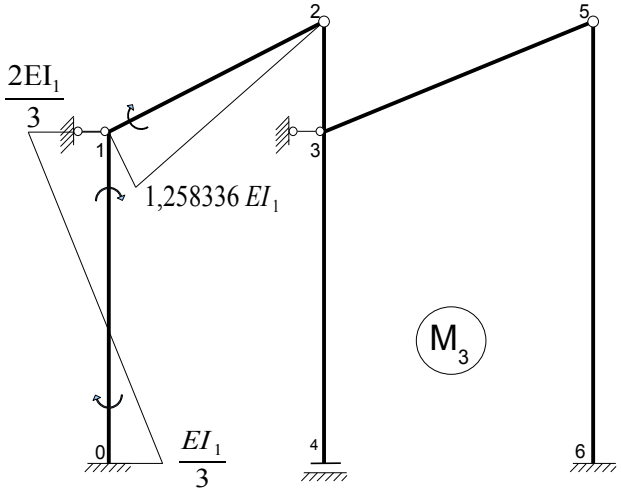
$$\underline{r_{32}=0}$$

$$\underline{r_{42}=\frac{3EI_1}{4}}$$

- STAN $z_3=1 \quad (\Delta_1=\Delta_2=\varphi_4=0)$

$$M_{01}^{(3)}=\frac{2EI_1}{6}\cdot(2\cdot 0+1-3\cdot 0)=\frac{EI_1}{3}$$
$$M_{10}^{(3)}=\frac{2EI_1}{6}\cdot(0+2\cdot 1-3\cdot 0)=\frac{2EI_1}{3}$$
$$M_{12}^{(3)}=\frac{3nEI_1}{\sqrt{20}}\cdot(1-0)=\frac{3\cdot 1{,}875817\,EI_1}{\sqrt{20}}=1{,}258336\,EI_1$$

$$M_{21}^{(3)}=0$$
$$M_{23}^{(3)}=M_{32}^{(3)}=0$$
$$M_{34}^{(3)}=M_{43}^{(3)}=0$$
$$M_{35}^{(3)}=M_{53}^{(3)}=0$$
$$M_{56}^{(3)}=M_{65}^{(3)}=0$$



RPW:

$$r_{13}\cdot\overline{1,0}+\Psi_{01}^{(1)}\cdot(M_{01}^{(3)}+M_{10}^{(3)})=0$$
$$r_{13}=-\frac{1}{6}\cdot(\frac{EI_1}{3}+\frac{2EI_1}{3})$$
$$\underline{r_{13}=-\frac{EI_1}{6}}$$
$$\underline{r_{23}=0}$$

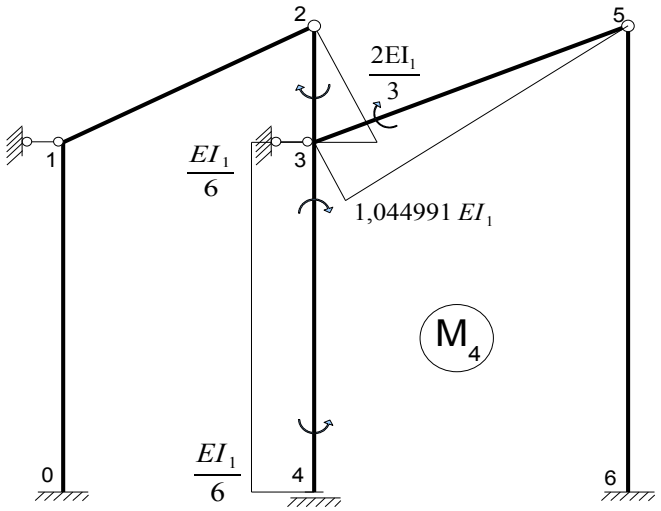
RÓWNOWAGA WĘZŁÓW:

$$r_{33}=\frac{3nEI_1}{\sqrt{20}}+\frac{2EI_1}{3}=\frac{3\cdot 1{,}875817\,EI_1}{\sqrt{20}}+\frac{2EI_1}{3}$$
$$\underline{r_{33}=1{,}925003\,EI_1}$$
$$\underline{r_{43}=0}$$

- STAN $z_4=1 \quad (\Delta_1=\Delta_2=\varphi_3=0)$

$$M_{01}^{(4)}=M_{10}^{(4)}=0$$
$$M_{12}^{(4)}=M_{21}^{(4)}=0$$
$$M_{23}^{(4)}=0$$
$$M_{32}^{(4)}=\frac{3EI_1}{2}\cdot(1-0)=\frac{3EI_1}{2}$$
$$M_{34}^{(4)}=\frac{EI_1}{6}\cdot(1-0)=\frac{EI_1}{6}$$

$$M_{43}^{(4)}=\frac{EI_1}{6}\cdot(-1+0)=-\frac{EI_1}{6}$$
$$M_{35}^{(4)}=\frac{3nEI_1}{\sqrt{29}}\cdot(1-0)=\frac{3\cdot 1{,}875817\,EI_1}{\sqrt{29}}=1{,}044991\,EI_1$$
$$M_{53}^{(4)}=0$$
$$M_{56}^{(4)}=M_{65}^{(4)}=0$$



RPW:

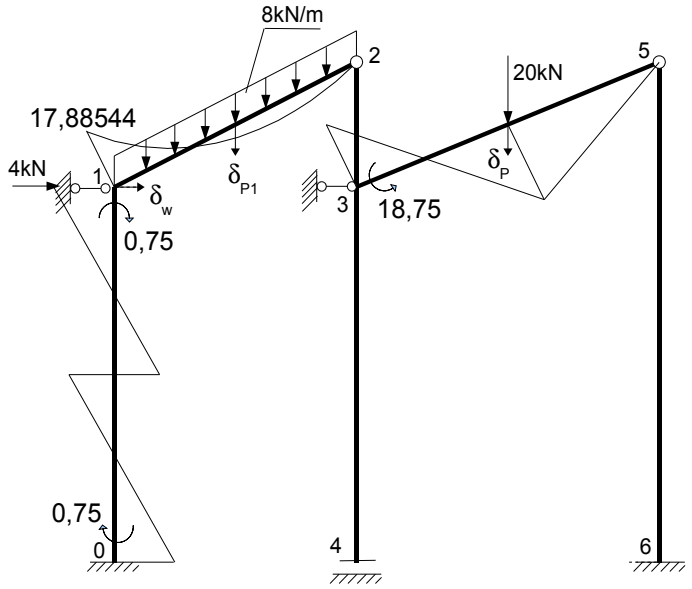
$$r_{14}\cdot\overline{1,0}+\Psi_{32}^{(1)}\cdot(M_{32}^{(4)})=0$$
$$r_{14}=-\frac{1}{2}\cdot\frac{3EI_1}{2}$$
$$\underline{r_{13}=-\frac{3EI_1}{4}}$$

$$r_{24}\cdot\overline{1,0}+\Psi_{32}^{(2)}\cdot(M_{32}^{(4)})=0$$
$$r_{24}=-(-\frac{1}{2})\cdot\frac{3EI_1}{2}$$
$$\underline{r_{24}=\frac{3EI_1}{4}}$$

RÓWNOWAGA WĘZŁÓW:

$$\underline{r_{34}=0}$$
$$r_{44}=\frac{3nEI_1}{\sqrt{29}}+\frac{3EI_1}{2}+\frac{EI_1}{6}=(\frac{3\cdot 1{,}875817}{\sqrt{29}}+\frac{3}{2}+\frac{1}{6})\cdot EI_1=\underline{2{,}711658\,EI_1}$$

- STAN $'P'$ $(\Delta_1=\Delta_2=\varphi_3=0)$



$$M_{01}^{(P)}=\frac{3}{4}[kNm]$$
$$M_{10}^{(P)}=\frac{3}{4}[kNm]$$
$$M_{12}^{(P)}=\frac{8\cdot 4\cdot\sqrt{20}}{8}=-17{,}888544[kNm]$$
$$M_{21}^{(P)}=0[kNm]$$
$$M_{23}^{(P)}=M_{32}^{(P)}=0[kNm]$$
$$M_{34}^{(P)}=M_{43}^{(P)}=0[kNm]$$
$$M_{35}^{(P)}=\frac{-60\sqrt{29}\cos\beta}{16}=-18{,}75[kNm]$$
$$M_{53}^{(P)}=0$$
$$M_{56}^{(P)}=M_{65}^{(P)}=0$$

WYZNACZENIE r_{1P} – RPW:

$$r_{1P}\cdot\overline{1{,}0}+\Psi_{01}^{(1)}\cdot(M_{01}^{(P)}+M_{10}^{(P)})+\Psi_{01}^{(1)}\cdot M+P_1\cdot\delta_{P1}^{(1)}+ql\cdot\delta_W^{(1)}+P\cdot\delta_P^{(1)}=0$$

- wyznaczenie przemieszczeń pod siłami skupionymi i wypadkową obciążenia ciągłego:

0W↓

$\Psi_{12}^{(1)}\cdot 2=\delta_W^{(1)}$ $\quad 0\cdot 2=\delta_W^{(1)}$ $\quad \delta_W^{(1)}=0$

01→

$\Psi_{01}^{(1)}\cdot 6=\delta_{P1}^{(1)}$ $\quad \frac{1}{6}\cdot 6=\delta_{P1}^{(1)}$ $\quad \delta_{P1}^{(1)}=1$

0P↓

$\Psi_{35}^{(1)}\cdot 2{,}5=\delta_P^{(1)}$ $\quad 0\cdot 2{,}5=\delta_P^{(1)}$ $\quad \delta_P^{(1)}=0$

$$r_{1P}\cdot\overline{1{,}0}=-\frac{1}{6}\cdot(\frac{3}{4}+\frac{3}{4})-\frac{1}{6}\cdot 3+4\cdot 1+8\sqrt{20}\cdot 0+20\cdot 0$$

$$\underline{r_{1P}=-4{,}75\,kN}$$

WYZNACZENIE r_{2P} – RPW:

$$r_{2P}\cdot\overline{1{,}0}+\Psi_{01}^{(2)}\cdot(M_{01}^{(P)}+M_{10}^{(P)})+\Psi_{01}^{(2)}\cdot M+P_1\cdot\delta_{P1}^{(2)}+ql\cdot\delta_W^{(2)}+P\cdot\delta_P^{(2)}=0$$

- wyznaczenie przemieszczeń:

0W↓

$\Psi_{12}^{(2)}\cdot 2=\delta_W^{(2)}$ $\quad 0\cdot 2=\delta_W^{(2)}$ $\quad \delta_W^{(2)}=0$

01→

$\Psi_{01}^{(2)}\cdot 6=\delta_{P1}^{(2)}$ $\quad 0\cdot 6=\delta_{P1}^{(2)}$ $\quad \delta_{P1}^{(2)}=0$

0P↓

$\Psi_{35}^{(2)}\cdot 2{,}5=\delta_P^{(2)}$ $\quad 0\cdot 2{,}5=\delta_P^{(2)}$ $\quad \delta_P^{(2)}=0$

$$r_{2P}\cdot\overline{1{,}0}=0\cdot(\frac{3}{4}+\frac{3}{4})-0\cdot 3+4\cdot 0+8\sqrt{20}\cdot 0+20\cdot 0$$

$$\underline{r_{2P}=0}$$

WYZNACZENIE r_{3P} *oraz* r_{4P} - RÓWNOWAGA WĘZŁÓW:

$$r_{3P}=\frac{3}{4}-20\cos\alpha=\frac{3}{4}-20\cdot 0{,}894427 \qquad r_{4P}=-\frac{60\sqrt{29}\cos\beta}{16}=-\frac{60\sqrt{29}\cdot 0{,}928477}{16}$$

$$\underline{r_{3P}=-17{,}13854\,kNm} \qquad \underline{r_{4P}=-18{,}75\,kNm}$$

WYZNACZANIE NIEWIADOMYCH UKŁADU RÓWNAŃ KANONICZNYCH:

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & r_{32} & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{bmatrix}\cdot\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}+\begin{bmatrix} r_{1P} \\ r_{2P} \\ r_{3P} \\ r_{4P} \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{31EI_1}{72} & -\frac{3EI_1}{8} & -\frac{EI_1}{6} & -\frac{3EI_1}{4} \\ -\frac{3EI_1}{8} & \frac{195EI_1}{512} & 0 & \frac{3EI_1}{4} \\ -\frac{EI_1}{6} & 0 & 1{,}925003EI_1 & 0 \\ -\frac{3EI_1}{4} & \frac{3EI_1}{4} & 0 & 2{,}711658EI_1 \end{bmatrix}\cdot\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}=\begin{bmatrix} 4{,}75 \\ 0 \\ 17{,}13854 \\ 18{,}75 \end{bmatrix}$$

$$\boxed{\begin{aligned} EI_1 z_1&=136{,}984097\\ EI_1 z_2&=102{,}452187\\ EI_1 z_3&=20{,}763200\\ EI_1 z_4&=16{,}465547 \end{aligned}}$$

MOMENTY $M_P^{(n)}$:

KORZYSTAJĄC Z ZASADY SUPERPOZYCJI: $M_P^{(n)}=M^{(1)}z_1+M^{(2)}z_2+M^{(3)}z_3+M^{(4)}z_4+M^{(P)}$

$$M_{01}^{(n)}=M_{01}^{(1)}z_1+M_{01}^{(2)}z_2+M_{01}^{(3)}z_3+M_{01}^{(4)}z_4+M_{01}^{(P)}$$

$$M_{01}^{(n)}=-\frac{EI_1}{6}\cdot\frac{136{,}984097}{EI_1}+0\cdot\frac{102{,}452187}{EI_1}+\frac{EI_1}{3}\cdot\frac{20{,}763200}{EI_1}+0\cdot\frac{16{,}465547}{EI_1}+\frac{3}{4}$$

$$\underline{M_{01}^{(n)}=-15{,}159616[kNm]}$$

$$M_{10}^{(n)}=M_{10}^{(1)}z_1+M_{10}^{(2)}z_2+M_{10}^{(3)}z_3+M_{10}^{(4)}z_4+M_{10}^{(P)}$$

$$M_{10}^{(n)}=-\frac{EI_1}{6}\cdot\frac{136{,}984097}{EI_1}+0\cdot\frac{102{,}452187}{EI_1}+\frac{2\mathrm{EI}_1}{3}\cdot\frac{20{,}763200}{EI_1}+0\cdot\frac{16{,}465547}{EI_1}+\frac{3}{4}$$

$$\underline{M_{10}^{(n)}=-8{,}238550[kNm]}$$

$$M_{12}^{(n)}=M_{12}^{(1)}z_1+M_{12}^{(2)}z_2+M_{12}^{(3)}z_3+M_{12}^{(4)}z_4+M_{12}^{(P)}$$

$$M_{12}^{(n)}=0\cdot\frac{136{,}984097}{EI_1}+0\cdot\frac{102{,}452187}{EI_1}+1{,}258336\,EI_1\cdot\frac{20{,}763200}{EI_1}+0\cdot\frac{16{,}465547}{EI_1}-17{,}888544$$

$$\underline{M_{12}^{(n)}=8{,}238538[kNm]}$$

$$M_{21}^{(n)}=M_{21}^{(1)}z_1+M_{21}^{(2)}z_2+M_{21}^{(3)}z_3+M_{21}^{(4)}z_4+M_{21}^{(P)}$$

$$\underline{M_{21}^{(n)}=0[kNm]}$$

$$M_{23}^{(n)}=M_{23}^{(1)}z_1+M_{23}^{(2)}z_2+M_{23}^{(3)}z_3+M_{23}^{(4)}z_4+M_{23}^{(P)}$$

$$\underline{M_{23}^{(n)}=0[kNm]}$$

$$M_{32}^{(n)}=M_{32}^{(1)}z_1+M_{32}^{(2)}z_2+M_{32}^{(3)}z_3+M_{32}^{(4)}z_4+M_{32}^{(P)}$$
$$M_{32}^{(n)}=-\frac{3EI_1}{4}\cdot\frac{136,984097}{EI_1}+\frac{3EI_1}{4}\cdot\frac{102,452187}{EI_1}+0\cdot\frac{20,763200}{EI_1}+\frac{3EI_1}{2}\cdot\frac{16,465547}{EI_1}+0$$
$$\underline{M_{32}^{(n)}=-1,200612\,[kNm]}$$

$$M_{34}^{(n)}=M_{34}^{(1)}z_1+M_{34}^{(2)}z_2+M_{34}^{(3)}z_3+M_{34}^{(4)}z_4+M_{34}^{(P)}$$
$$M_{34}^{(n)}=0\cdot\frac{136,984097}{EI_1}+0\cdot\frac{102,452187}{EI_1}+0\cdot\frac{20,763200}{EI_1}+\frac{EI_1}{6}\cdot\frac{16,465547}{EI_1}+0$$
$$\underline{M_{34}^{(n)}=2,744258\,[kNm]}$$

$$M_{43}^{(n)}=M_{43}^{(1)}z_1+M_{43}^{(2)}z_2+M_{43}^{(3)}z_3+M_{43}^{(4)}z_4+M_{43}^{(P)}$$
$$M_{43}^{(n)}=0\cdot\frac{136,984097}{EI_1}+0\cdot\frac{102,452187}{EI_1}+0\cdot\frac{20,763200}{EI_1}+-\frac{EI_1}{6}\cdot\frac{16,465547}{EI_1}+0$$
$$\underline{M_{43}^{(n)}=-2,744258\,[kNm]}$$

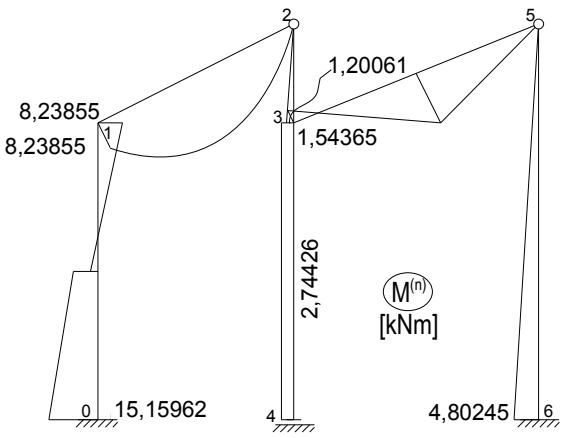
$$M_{35}^{(n)}=M_{35}^{(1)}z_1+M_{35}^{(2)}z_2+M_{35}^{(3)}z_3+M_{35}^{(4)}z_4+M_{35}^{(P)}$$
$$M_{35}^{(n)}=0\cdot\frac{136,984097}{EI_1}+0\cdot\frac{102,452187}{EI_1}+0\cdot\frac{20,763200}{EI_1}+1,044991\,EI_1\cdot\frac{16,465547}{EI_1}-18,75$$
$$\underline{M_{35}^{(n)}=-1,543652\,[kNm]}$$

$$M_{53}^{(n)}=M_{53}^{(1)}z_1+M_{53}^{(2)}z_2+M_{53}^{(3)}z_3+M_{53}^{(4)}z_4+M_{53}^{(P)}$$
$$\underline{M_{53}^{(n)}=0\,[kNm]}$$

$$M_{56}^{(n)}=M_{56}^{(1)}z_1+M_{56}^{(2)}z_2+M_{56}^{(3)}z_3+M_{56}^{(4)}z_4+M_{56}^{(P)}$$
$$\underline{M_{56}^{(n)}=0\,[kNm]}$$

$$M_{65}^{(n)}=M_{65}^{(1)}z_1+M_{65}^{(2)}z_2+M_{65}^{(3)}z_3+M_{65}^{(4)}z_4+M_{65}^{(P)}$$
$$M_{65}^{(n)}=0\cdot\frac{136,984097}{EI_1}-\frac{3EI_1}{64}\cdot\frac{102,452187}{EI_1}+0\cdot\frac{20,763200}{EI_1}+0\cdot16,465547+0$$
$$\underline{M_{65}^{(n)}=4,802446\,[kNm]}$$

WYKRES OSTATECZNY MOMENTÓW:

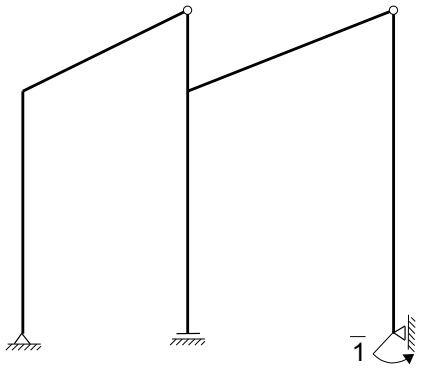


SPRAWDZENIE KINEMATYCZNE:

Obrót węzła 6 (korzystamy z twierdzenia redukcyjnego):

$$\bar{1}\cdot\varphi_6=\sum\int_s\frac{M^{(n)}\cdot M^{(n)}}{EI}ds=\sum\int_s\frac{M^{(n)}\cdot M^{(0)}}{EI}ds$$

- układ podstawowy:



Wyznaczenie wartości momentów zginających pod obciążeniem skupionym na przęśle 01:

$M_1=M_{01}^{(n)}+T_{01}^{(n)}\cdot 3$

$M_2=M_{01}^{(n)}+T_{01}^{(n)}\cdot 3+3$

- siły poprzeczne:

$\sum M^0=0$

$M_{01}^{(n)}-3-T_{10}^{(n)}+M_{10}^{(n)}=0$

$15{,}159616-3-T_{10}^{(n)}\cdot 6+8{,}238550=0$

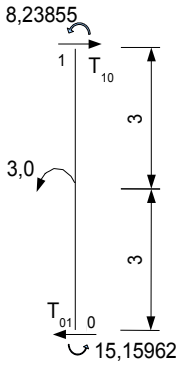
$T_{10}^{(n)}=3{,}399694\,kN$

$\sum M^1=0$

$M_{10}^{(n)}-3-T_{01}^{(n)}\cdot l+M_{01}^{(n)}=0$

$8{,}238550-3-T_{01}^{(n)}\cdot 6+15{,}159616=0$

$T_{01}^{(n)}=3{,}399694\,kN$



- stąd:

$M_1=-15{,}159616+3{,}399694\cdot 3$

$M_1=-4{,}960534\,kNm$

$M_2=-15{,}159616+3{,}399694\cdot 3+3$

$M_2=-1{,}960534\,kNm$

Wyznaczenie wartości momentów zginających pod obciążeniem skupionym na przęśle 35:

$M_{65}^{(n)}+T_{65}^{(n)}\cdot 7+N_{65}^{(n)}\cdot 2{,}5=M_{max}$

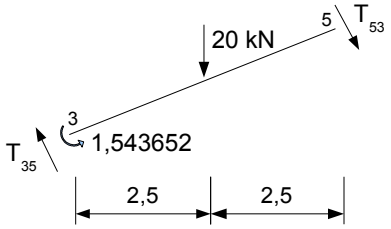
- siły poprzeczne:

$\sum M^5=0$

$T_{35}^{(n)}\cdot l-M_{35}^{(n)}-P\cdot 2{,}5=0$

$T_{35}^{(n)}\cdot\sqrt{29}-1{,}543652-20\cdot 2{,}5=0$

$T_{35}^{(n)}=9{,}571419\,[kN]$



$\sum M^3=0$

$T_{53}^{(n)}\cdot l+P\cdot 2{,}5+M_{35}^{(n)}=0$

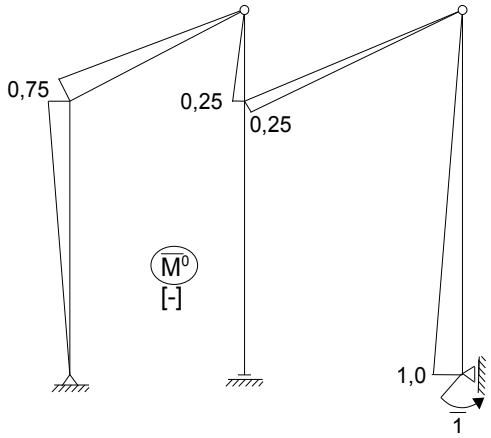
$T_{53}^{(n)}\cdot\sqrt{29}+20\cdot 2{,}5+1{,}543652=0$

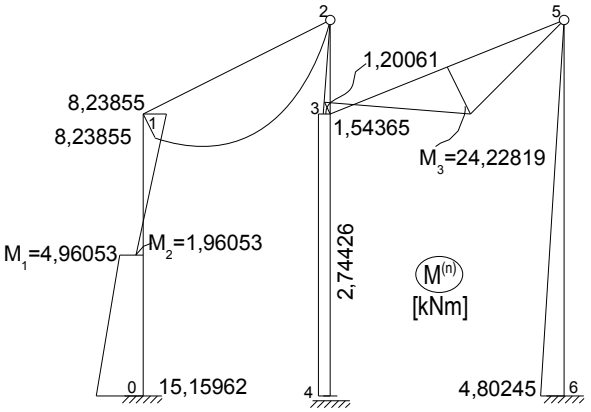
$T_{53}^{(n)}=-8{,}998121\,kN$

- stąd:

$M_3=-4{,}802446+0{,}600306\cdot 7-9{,}451155\cdot 2{,}5$

$M_3=24{,}2281915\,kNm$



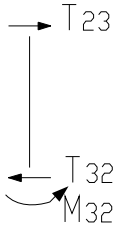


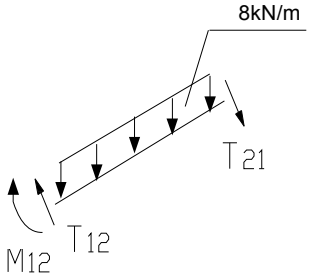
Obrót węzła 6:

$$\bar{1}\cdot\varphi_6=\frac{1}{EI_1}\cdot\left[0{,}5\cdot3\cdot\frac{0{,}75}{2}\cdot\left(\frac{1}{3}\cdot15{,}15962+\frac{2}{3}\cdot4{,}96053\right)+0{,}5\cdot3\cdot\frac{0{,}75}{2}\cdot\left(\frac{2}{3}\cdot1{,}96053-\frac{1}{3}\cdot8{,}23855\right)\right]+$$
$$+\frac{1}{EI_1}\left[0{,}5\cdot3\cdot0{,}75\cdot\left(\frac{1}{3}\cdot1{,}96053-\frac{2}{3}\cdot8{,}23855\right)\right]+\frac{1}{1{,}87582\,EI_1}\cdot\left(-0{,}5\cdot\sqrt{20}\cdot8{,}23855\cdot\frac{2}{3}\cdot0{,}75\right)+$$
$$+\frac{1}{1{,}87582\,EI_1}\cdot\left[-\frac{2}{3}\cdot\frac{8\cdot\sqrt{20}\cdot4}{8}\cdot\sqrt{20}\cdot\frac{0{,}75}{2}-0{,}5\cdot\frac{\sqrt{29}}{2}\cdot1{,}54365\cdot\left(\frac{2}{3}\cdot0{,}25+\frac{1}{3}\cdot\frac{0{,}25}{2}\right)\right]+$$
$$+\frac{1}{1{,}87582\,EI_1}\left[0{,}5\cdot24{,}22819\cdot\frac{\sqrt{29}}{2}\cdot\left(\frac{1}{3}\cdot0{,}25+\frac{2}{3}\cdot\frac{0{,}25}{2}\right)+0{,}5\cdot24{,}22819\cdot\frac{\sqrt{29}}{2}\cdot\frac{0{,}25}{2}\cdot\frac{2}{3}\right]+$$
$$+\frac{1}{EI_1}\left[0.5\cdot2\cdot0{,}25\cdot\frac{2}{3}\cdot1{,}20061+0{,}5\cdot8\cdot4{,}80245\cdot\frac{2}{3}\cdot1\right]=-\frac{0{,}000023}{EI_1}$$

$$\left|\frac{-0{,}000023}{EI_1}\right|<\frac{1}{EI}$$

SIŁY POPRZECZNE:





$\sum M^2=0$

$T_{32}^{(n)}\cdot l-M_{32}^{(n)}=0$

$T_{32}^{(n)}\cdot2-1{,}20061=0$ $\qquad T_{23}^{(n)}=T_{32}^{(n)}$

$T_{32}^{(n)}=0{,}60031\,kN$ $\qquad T_{23}^{(n)}=0{,}60031\,kN$

$\sum M^1=0$

$T_{21}^{(n)}\cdot l+M_{12}^{(n)}+q\cdot l\cdot\frac{l_x}{2}=0$

$T_{21}^{(n)}\cdot\sqrt{20}+8{,}23855+8(\sqrt{20})\cdot\frac{4}{2}=0$

$T_{21}^{(n)}=-17{,}84219\,kN$

$\sum M^2=0$

$M_{12}^{(n)}+T_{12}^{(n)}\cdot l-q\cdot l\cdot\frac{l_x}{2}=0$

$T_{21}^{(n)}\cdot\sqrt{20}+8{,}23855-8(\sqrt{20})\cdot\frac{4}{2}=0$

$T_{12}^{(n)}=14{,}15780\,kN$

$\sum M^5=0$

$T_{65}^{(n)}\cdot l-M_{65}^{(n)}=0$

$T_{65}^{(n)}\cdot8-4{,}80245=0$

$T_{65}^{(n)}=0{,}60031\,kN$

$\sum M^6=0$

$T_{56}^{(n)}\cdot l-M_{65}^{(n)}=0$

$T_{56}^{(n)}\cdot8-4{,}80245=0$

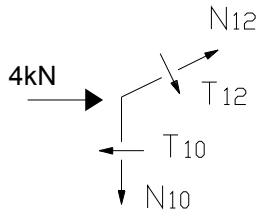
$T_{56}^{(n)}=0{,}60031\,kN$

$T_{34}^{(n)}=T_{43}^{(n)}=0\,kN$

- na przęsłach 01 i 35 - siły poprzeczne są już wyznaczone;

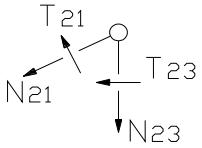
SIŁY NORMALNE:

$$\sum X = 0$$
$$N_{12}^{(n)} \cdot \cos\alpha + T_{12}^{(n)} \cdot \sin\alpha - T_{10}^{(n)} + 4 = 0$$
$$N_{12}^{(n)} \cdot 0{,}89443 + 14{,}15780 \cdot 0{,}44721 - 3{,}39969 + 4 = 0$$
$$N_{12}^{(n)} = -7{,}75007\,kN$$



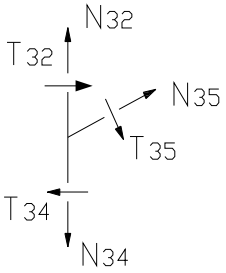
$$\sum Y = 0$$
$$N_{10}^{(n)} + T_{12}^{(n)} \cdot \cos\alpha - N_{12}^{(n)} \cdot \sin\alpha = 0$$
$$N_{10}^{(n)} + 14{,}15780 \cdot 0{,}89443 + 7{,}75007 \cdot 0{,}44721 = 0$$
$$N_{10}^{(n)} = -16{,}12906\,kN$$

$$\sum X = 0$$
$$N_{21}^{(n)} - N_{12}^{(n)} - q \cdot l \cdot \sin\alpha = 0$$
$$N_{21}^{(n)} + 7{,}75007 - 8 \cdot \sqrt{20} \cdot 0{,}44721 = 0$$
$$N_{21}^{(n)} = 8{,}24994\,kN$$



$$\sum Y = 0$$
$$N_{23}^{(n)} + N_{21}^{(n)} \cdot \sin\alpha - T_{21}^{(n)} \cdot \cos\alpha = 0$$
$$N_{23}^{(n)} + 8{,}24994 \cdot 0{,}44721 + 17{,}84219 \cdot 0{,}89443 = 0$$
$$N_{23}^{(n)} = -19{,}64803\,kN$$

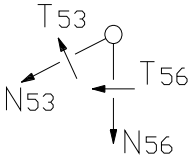
$$\sum X = 0$$
$$T_{32}^{(n)} - T_{34}^{(n)} + T_{35}^{(n)} \cdot \sin\beta + N_{35}^{(n)} \cdot \cos\beta = 0$$
$$0{,}60031 - 0 + 9{,}57142 \cdot 0{,}37139 + N_{35}^{(n)} \cdot 0{,}92848 = 0$$
$$N_{35}^{(n)} = -4{,}47512\,kN$$



$$\sum Y = 0$$
$$N_{34}^{(n)} - N_{32}^{(n)} - N_{35}^{(n)} \cdot \sin\beta + T_{35}^{(n)} \cdot \cos\beta = 0$$
$$N_{34}^{(n)} + 19{,}64803 + 4{,}47512 \cdot 0{,}37139 + 9{,}57142 \cdot 0{,}92848 = 0$$
$$N_{34}^{(n)} = -30{,}19689\,kN$$

$$\sum Y = 0$$
$$N_{43}^{(n)} = N_{34}^{(n)}$$
$$N_{43}^{(n)} = -30{,}19689\,kN$$

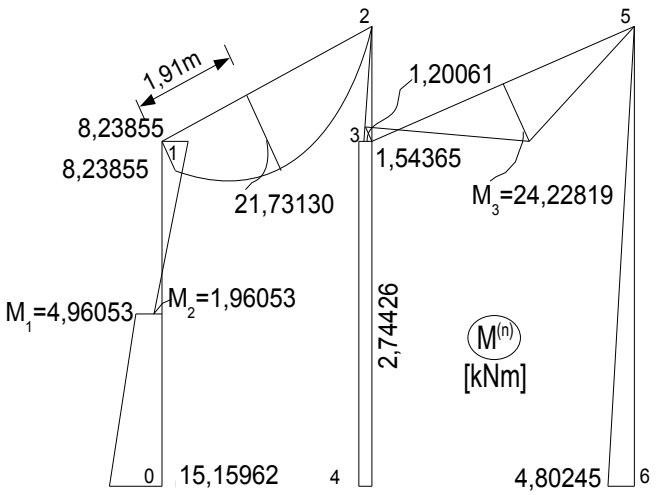
$$\sum X = 0$$
$$N_{35}^{(n)} - N_{53}^{(n)} + P\sin\beta = 0$$
$$-4{,}47512 - N_{53}^{(n)} + 20 \cdot 0{,}37139 = 0$$
$$N_{53}^{(n)} = 2{,}95270\,kN$$

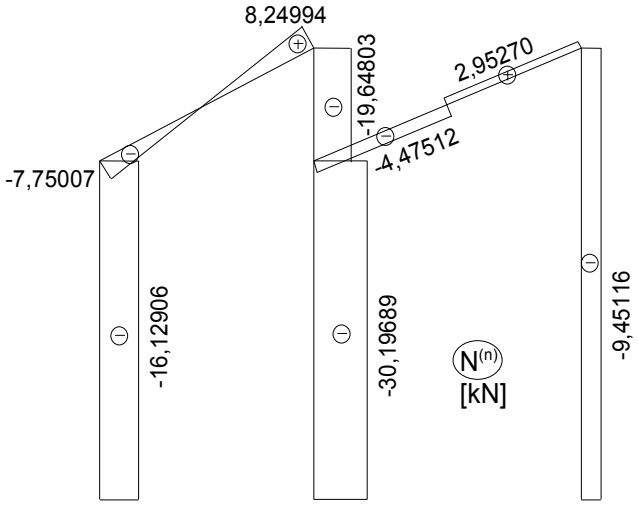


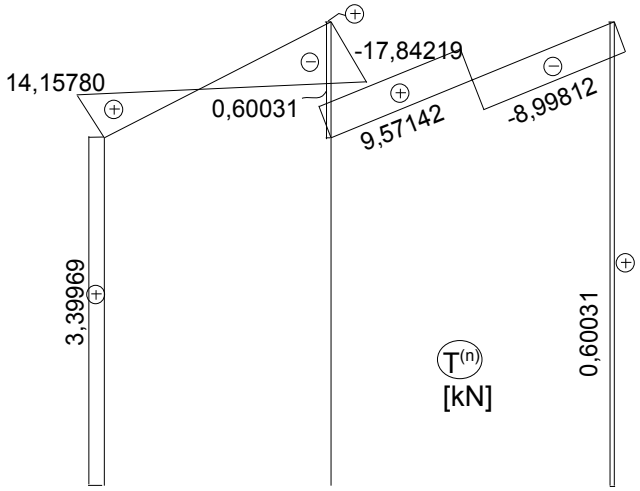
$$\sum X = 0$$
$$N_{56}^{(n)} + N_{53}^{(n)} \cdot \sin\beta - T_{53}^{(n)} \cos\beta = 0$$
$$N_{56}^{(n)} + 2{,}95270 \cdot 0{,}37139 + 8{,}99812 \cdot 0{,}92848 = 0$$
$$N_{56}^{(n)} = -9{,}45116\,kN$$

ZEBRANIE WYNIKÓW (znakowanie momentów zginających wg metody przemieszczeń):

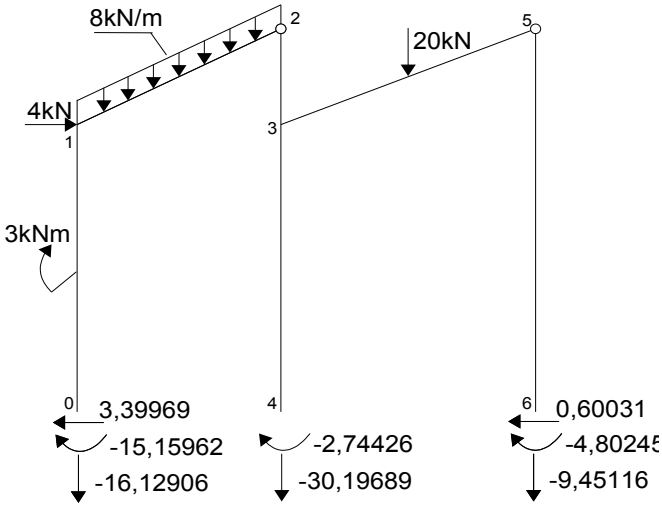
Nr przęsła	M [kNm]	N [kN]	T [kN]
01	-15,15962	-16,12906	3,39969
10	8,23855	-16,12906	3,39969
12	8,23855	-7,75007	14,15780
21	0	8,24994	-17,84219
23	0	-19,64803	0,60031
32	-1,20061	-19,64803	0,60031
34	2,74426	-30,19689	0
43	-2,74426	-30,19689	0
35	-1,54365	-4,47512	9,57142
53	0	2,95270	-8,99812
56	0	-9,45116	0,60031
65	-4,80245	-9,45116	0,60031







SPRAWDZENIE STATYCZNE



$$\sum Y = 0$$

$$N_{01}^{(n)} + N_{43}^{(n)} + N_{65}^{(n)} + 8\sqrt{20} + 20 = 0$$

$$-16,12906 - 30,19689 - 9,45116 +$$

$$+8\sqrt{20} + 20 = -0,00002\,kN \approx 0\,kN$$

$$\sum X = 0$$

$$T_{01}^{(n)} + T_{43}^{(n)} + T_{65}^{(n)} - 4 = 0$$

$$3,39969 + 0,60031 - 4 = 0,0\,kN$$

$$\sum M^{(2)} = 0$$

$$M_{01}^{(n)} + M_{43}^{(n)} + M_{65}^{(n)} - N_{01}^{(n)} \cdot 4 + N_{65}^{(n)} \cdot 5 + T_{01}^{(n)} \cdot 8 + T_{43}^{(n)} \cdot 8 + T_{65}^{(n)} \cdot 8 - 4 \cdot 2 - 8 \cdot \sqrt{20} \cdot 2 + 20 \cdot 2,5 = 0$$

$$-15,15962 - 2,74426 - 4,80245 + 16,12906 \cdot 4 - 9,45116 \cdot 5 + 3,39969 \cdot 8 + 0,60031 \cdot 8 -$$

$$-4 \cdot 2 - 8 \cdot \sqrt{20} \cdot 2 + 20 \cdot 2,5 + 3 = -0,00007\,kNm \approx 0\,kNm$$