Politechnika Poznańska
Wydział Architektury Budownictwa
i Inżynierii Środowiska

ĆWICZENIE NR 4

OBLICZENIE RAMY METODĄ PRZEMIESZCZEŃ

(wpływ temperatury)

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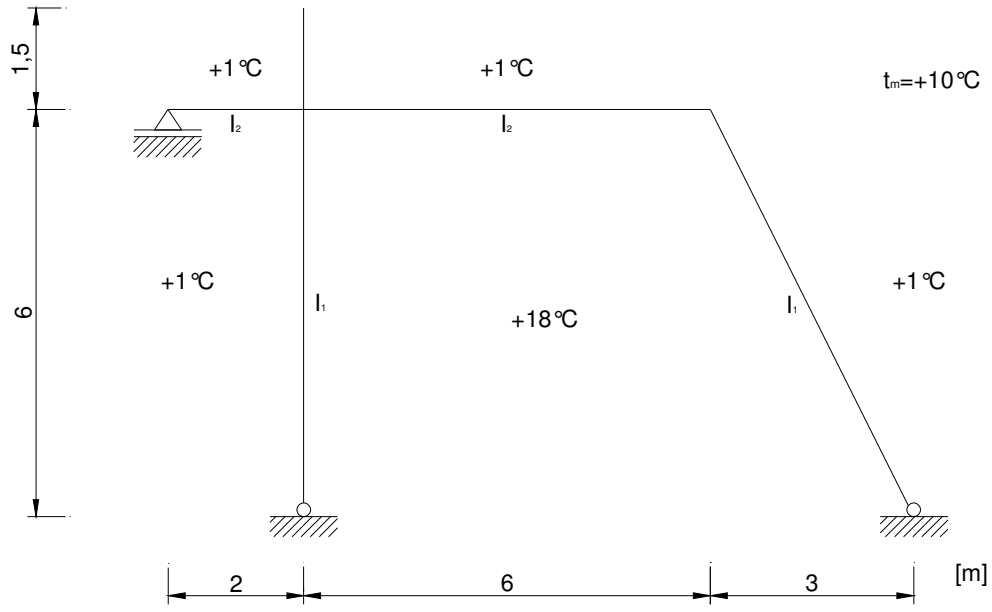
rok studiów: III
semestr: VI
gr. 8

Poznań 2005

METODA PRZEMIESZCZEŃ – ZMIANY TEMPERATURY

SCHEMAT KONSTRUKCJI



$t_w = +18^\circ C$ $t_z = +1^\circ C$ $t_m = +10^\circ C$

WYZNACZENIE Δt I t_0

$\Delta t = t_d - t_g$

$\Delta t^{12} = 1 - 1 = 0$

$\Delta t^{23} = \Delta t^{34} = \Delta t^{02} = 18 - 1 = 17^\circ C$

$$t_0 = \frac{t_d - t_g}{2} - t_m$$

$$t_0^{12} = \frac{1-1}{2} - 10 = -10^\circ C$$

$$t_0^{23} = t_0^{34} = t_0^{02} = \frac{18+1}{2} - 10 = -0{,}5^\circ C$$

UKŁAD RÓWNAŃ KANONICZNYCH

$$\begin{cases} r_{11} \cdot z_1 + r_{12} \cdot z_2 + r_{13} \cdot z_3 + r_{1t} = 0 \\ r_{21} \cdot z_1 + r_{22} \cdot z_2 + r_{23} \cdot z_3 + r_{2t} = 0 \\ r_{31} \cdot z_1 + r_{32} \cdot z_2 + r_{33} \cdot z_3 + r_{3t} = 0 \end{cases}$$

$$r_{it} = r_{it}^{\Delta t} + r_{it}^{t_o}$$

NIERÓWNOMIERNE OGRZANIE:

$\alpha_t = 1{,}2 \cdot 10^{-5} \frac{1}{^\circ C}$; $h_1 = 0{,}22m$; $h_2 = 0{,}26m$

MOMENTY PRZYWĘZŁOWE OD NIERÓWNOMIERNEGO OGRZANIA

$M_{12}^{\Delta t} = 0$

$$M_{21}^{\Delta t} = \frac{3}{2} \cdot \frac{\alpha_t \cdot \Delta t^{12}}{h_2} EI_0 = \frac{3}{2} \cdot \frac{1{,}2 \cdot 10^{-5} \cdot 0}{0{,}26} EI_0 = 0$$

$$M_{23}^{\Delta t} = -\frac{\alpha_t \cdot \Delta t^{23}}{h_2} EI_0 = -\frac{3}{2} \cdot \frac{1{,}2 \cdot 10^{-5} \cdot 17}{0{,}26} EI_0 = -9.2326\ kNm$$

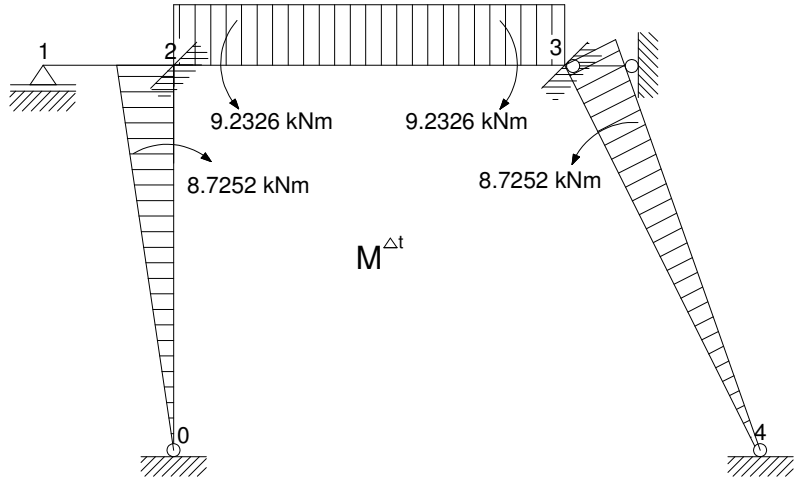
$$M_{32}^{\Delta t} = \frac{\alpha_t \cdot \Delta t^{23}}{h_2} EI_0 = \frac{3}{2} \cdot \frac{1{,}2 \cdot 10^{-5} \cdot 17}{0{,}26} EI_0 = 9.2326\ kNm$$

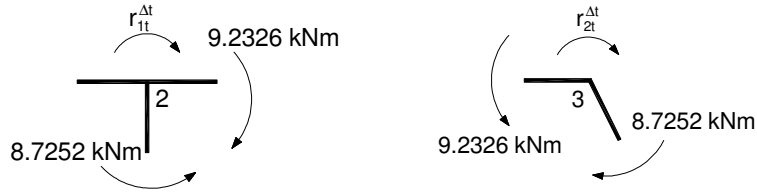
$$M_{34}^{\Delta t} = -\frac{3}{2} \cdot \frac{\alpha_t \cdot \Delta t^{34}}{h_1} \cdot 0.5331 EI_0 = -\frac{3}{2} \cdot \frac{1{,}2 \cdot 10^{-5} \cdot 17}{0{,}22} \cdot 0.5331 EI_0 = -8.7252\ kNm$$

$$M_{43}^{\Delta t} = 0$$

$$M_{02}^{\Delta t} = 0$$

$$M_{20}^{\Delta t} = \frac{3}{2} \cdot \frac{\alpha_t \cdot \Delta t^{02}}{h_1} \cdot 0.5331 EI_0 = \frac{3}{2} \cdot \frac{1{,}2 \cdot 10^{-5} \cdot 17}{0{,}22} \cdot 0.5331 EI_0 = 8.7252\,\text{kNm}$$



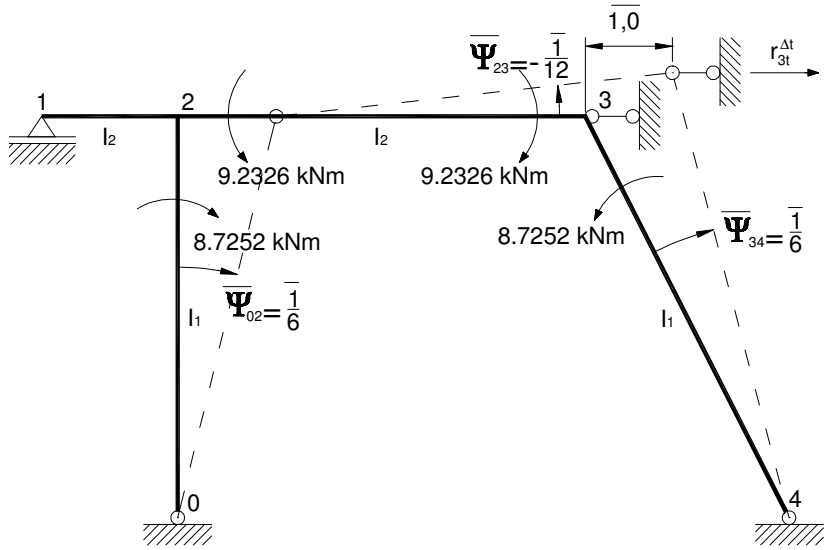


$$r_{1t}^{\Delta t} = 8.7252 - 9.2326$$

$$r_{1t}^{\Delta t} = -0.5074\ \text{kNm}$$

$$r_{2t}^{\Delta t} = 9.2326 - 8.7252$$

$$r_{2t}^{\Delta t} = 0.5074\ \text{kNm}$$



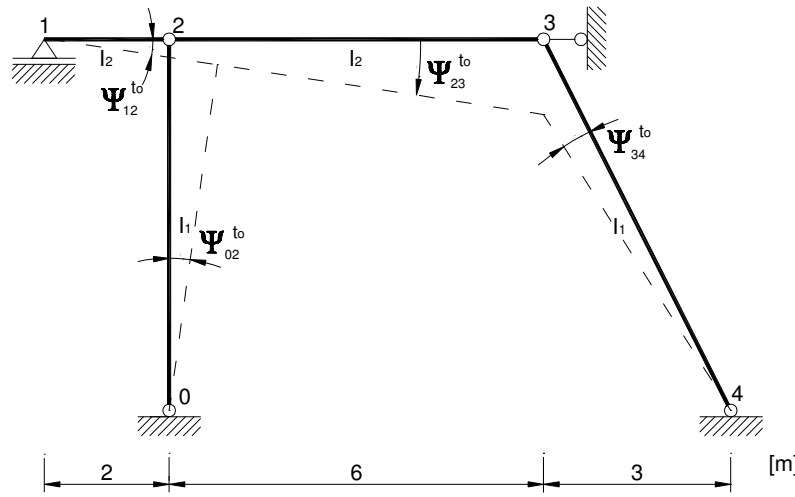
R.P.W.

$$\overline{1{,}0} \cdot r_{3t}^{\Delta t} - 8.7252 \cdot \frac{\overline{1}}{6} + (9.2326 - 9.2326) \cdot \frac{\overline{1}}{12} - 8.7252 \cdot \frac{\overline{1}}{6} = 0$$

$$r_{3t}^{\Delta t} = 0$$

RÓWNOMIERNE OGRZANIE:

ŁAŃCUCH KINEMATYCZNY:



$\downarrow 120 \qquad 2\cdot\Psi_{12}^{t_\circ} - 6\cdot\alpha_t\cdot 0{,}5 = 0 \;\Rightarrow\; \Psi_{12}^{t_\circ} = 0.0000180$

$\rightarrow 023 \qquad 6\cdot\Psi_{02}^{t_\circ} - 6\cdot\alpha_t\cdot 0{,}5 = 0 \;\Rightarrow\; \Psi_{02}^{t_\circ} = 0.0000060$

$\rightarrow 43 \qquad 6\cdot\Psi_{34}^{t_\circ} + 3\cdot\alpha_t\cdot 0{,}5 = 0 \;\Rightarrow\; \Psi_{34}^{t_\circ} = -0.0000030$

$\downarrow 0234 \qquad 6\cdot\Psi_{23}^{t_\circ} - 6\cdot\alpha_t\cdot 0{,}5 + 3\cdot\Psi_{34}^{t_\circ} + 6\cdot\alpha_t\cdot 0{,}5 = 0 \;\Rightarrow\; \Psi_{23}^{t_\circ} = 0.0000015$

MOMENTY PRZYWĘZŁOWE OD RÓWNOMIERNEGO OGRZANIA

$M_{12}^{t_\circ} = 0\,; \qquad M_{43}^{t_\circ} = 0\,; \qquad M_{02}^{t_\circ} = 0$

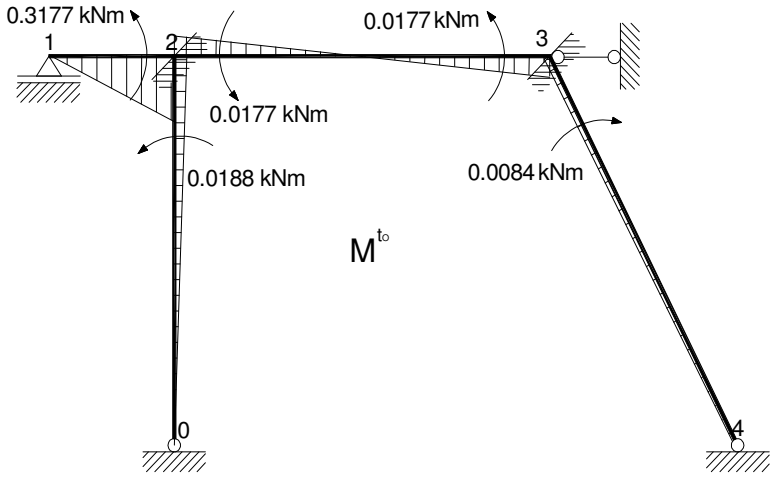
$M_{21}^{t_\circ} = \frac{3EI_0}{l}(\varphi_2 - \Psi_{12}) = \frac{3EI_0}{2}(0 - 0.0000180) = -0.3177\ \text{kNm}$

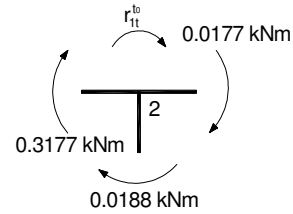
$M_{23}^{t_\circ} = \frac{2EI_0}{l}(2\varphi_2 + \varphi_3 - 3\Psi_{23}) = \frac{2EI_0}{6}(2\cdot 0 + 0 - 3\cdot 0.0000015) = -0.0177\ \text{kNm}$

$M_{32}^{t_\circ} = M_{23}^{t_\circ} = -0.0177\ \text{kNm}$

$M_{34}^{t_\circ} = \frac{3EI_0}{l}(\varphi_3 - \Psi_{34}) = \frac{3\cdot 0.5331EI_0}{6.7082}(0 - (-0.0000030)) = 0.0084\ \text{kNm}$

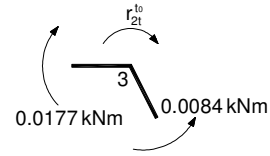
$M_{20}^{t_\circ} = \frac{3EI_0}{l}(\varphi_2 - \Psi_{02}) = \frac{3\cdot 0.5331\cdot EI_0}{6}(0 - 0.0000060) = -0.0188\ \text{kNm}$





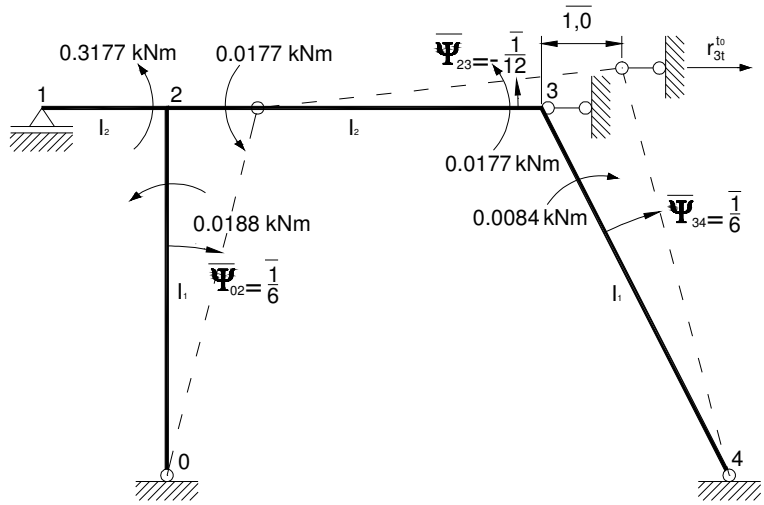
$r_{1t}^{t_\circ} = 0.3177 + 0.0188 + 0.0177$

$r_{1t}^{t_\circ} = -0.3542\,\text{kNm}$



$r_{2t}^{t_\circ} = -0.0177 + 0.0084$

$r_{2t}^{t_\circ} = -0.0092\,\text{kNm}$



R.P.W.

$$\overline{1,0}\cdot r_{3t}^{t_o}+(0.0177+0.0177)\cdot\frac{1}{12}+0.0084\cdot\frac{1}{6}-0.0188\cdot\frac{1}{6}=0$$

$$r_{3t}^{t_o}=-0.0012\,\text{kNm}$$

ROZWIĄZANIE UKŁADU RÓWNAŃ KANONICZNYCH

$$\begin{cases} r_{11}\cdot z_1+r_{12}\cdot z_2+r_{13}\cdot z_3+r_{1t}=0 \\ r_{21}\cdot z_1+r_{22}\cdot z_2+r_{23}\cdot z_3+r_{2t}=0 \\ r_{31}\cdot z_1+r_{32}\cdot z_2+r_{33}\cdot z_3+r_{3t}=0 \end{cases}$$

$$\begin{cases} 2.4332EI_0\cdot z_1+0.3333EI_0\cdot z_2+0.0389EI_0\cdot z_3=-(-0.5074-0.3542) \\ 0.3333EI_0\cdot z_1+0.9051EI_0\cdot z_2+0.0436EI_0\cdot z_3=-(0.5074-0.0092) \\ 0.0389EI_0\cdot z_1+0.0436EI_0\cdot z_2+0.0279EI_0\cdot z_3=-(0-0.0012) \end{cases}$$

$$z_1=\frac{0.4466}{EI_0}\quad\Rightarrow\quad\varphi_2=\frac{0.4466}{EI_0}$$

$$z_2=\frac{-0.7429}{EI_0}\quad\Rightarrow\quad\varphi_3=\frac{-0.7429}{EI_0}$$

$$z_3=\frac{0.5811}{EI_0}\quad\Rightarrow\quad\Delta_3=\frac{0.5811}{EI_0}$$

WYZNACZENIE WARTOŚCI MOMENTÓW ZGINAJĄCYCH – METODA SUPERPOZYCJI:

$$M_P^{(n)}=M_1\cdot Z_1+M_2\cdot Z_2+M_3\cdot Z_3+M_\Delta$$

$$M_{12}=0$$

$$M_{21}=1.50EI_0\cdot\frac{0.4466}{EI_0}+0\cdot\frac{-0.7429}{EI_0}+0\cdot\frac{0.5811}{EI_0}-0.3177=0.3521\text{kNm}$$

$$M_{23}=0.6667EI_0\cdot\frac{0.4466}{EI_0}+0.3333EI_0\cdot\frac{-0.7429}{EI_0}+0.0833EI_0\cdot\frac{0.5811}{EI_0}-9.2502=-9.1517\,\text{kNm}$$

$$M_{32}=0.3333EI_0\cdot\frac{0.4466}{EI_0}+0.6667EI_0\cdot\frac{-0.7429}{EI_0}+0.0833EI_0\cdot\frac{0.5811}{EI_0}+9.2149=8.9170\,\text{kNm}$$

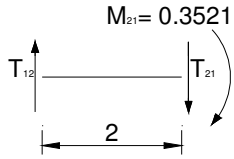
$$M_{34}=0\cdot\frac{0.4466}{EI_0}+0.2384EI_0\cdot\frac{-0.7429}{EI_0}-0.0397EI_0\cdot\frac{0.5811}{EI_0}-8.7168=-8.9170\,\text{kNm}$$

$$M_{43}=0$$

$$M_{02}=0$$

$$M_{20}=0.2666EI_0\cdot\frac{0.4466}{EI_0}+0\cdot\frac{-0.7429}{EI_0}-0.0444EI_0\cdot\frac{0.5811}{EI_0}+8.7064=8.7996\text{kNm}$$

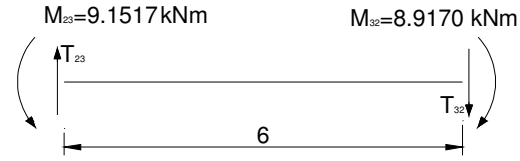
WYZNACZENIE WARTOŚCI SIŁ TNĄCYCH:



$\sum M_1 = 0 \qquad T_{21} \cdot 2 + 0.3521 = 0$

$T_{21} = -0.1761\,kN$

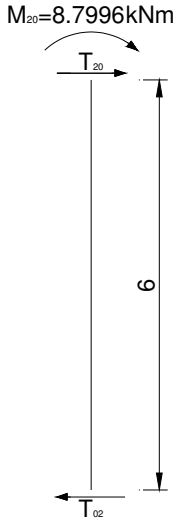
$T_{12} = T_{21} = -0.1761 kN$



$\sum M_2 = 0 \qquad T_{32} \cdot 6 - 9.1517 + 8.9170 = 0$

$T_{32} = 0.0391\,kN$

$T_{23} = T_{32} = 0.0391 kN$

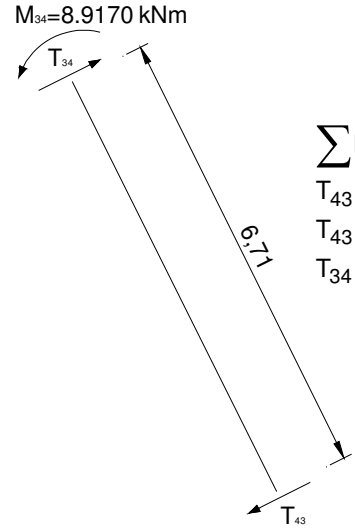


$\sum M_0 = 0$

$T_{20} \cdot 6 + 8.7996 = 0$

$T_{20} = -1.4666\ kN$

$T_{02} = T_{20} = -1.4666\ kN$



$\sum M_3 = 0$

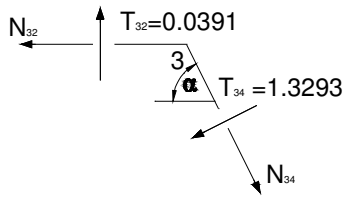
$T_{43} \cdot 6.7082 - 8.9170 = 0$

$T_{43} = 1.3293 kN$

$T_{34} = T_{43} = 1.3293\ kN$

WYZNACZENIE WARTOŚCI SIŁ NORMALNYCH:

$\sin\alpha = 0.8944\ ; \qquad \cos\alpha = 0.4472$

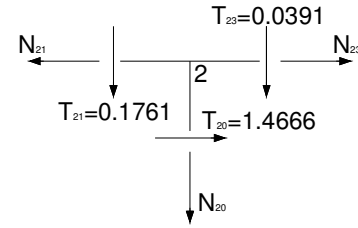


$\sum Y = 0 \qquad -N_{34} \cdot \sin\alpha + 0.0391 - 1.3293 \cdot \cos\alpha = 0$

$N_{34} = -0.6209 kN$

$\sum X = 0 \qquad -N_{32} - 1.33293 \cdot \sin\alpha + N_{34} \cdot \cos\alpha = 0$

$N_{32} = -1.4666\,kN$

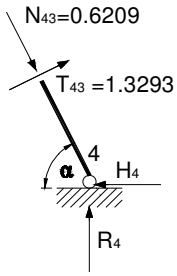


$\sum Y = 0 \qquad -N_{20} - 0.1761 - 0.0391 = 0$

$N_{20} = -0.2152\,kN$

$\sum X = 0 \qquad -N_{21} + 1.4666 + N_{23} = 0$

$N_{21} = 0$



$\sum X = 0$

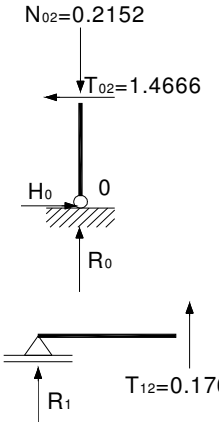
$-H_4 + 1.3293 \cdot \sin\alpha + 0.6209 \cdot \cos\alpha = 0$

$H_4 = 1.4666\ kN$

$\sum Y = 0$

$R_4 + 1.3293 \cdot \cos\alpha - 0.6209 \cdot \sin\alpha = 0$

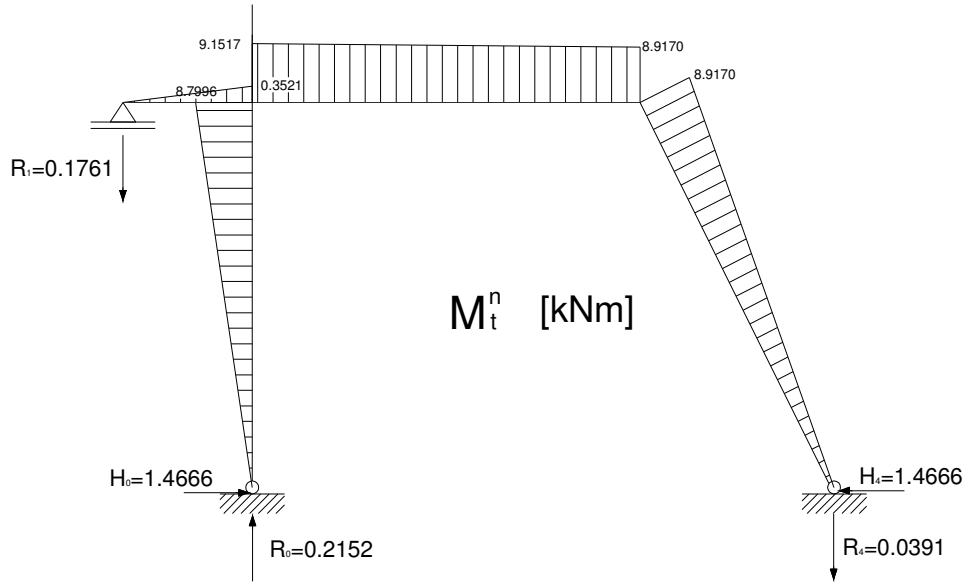
$R_4 = -0.0391\,kN$

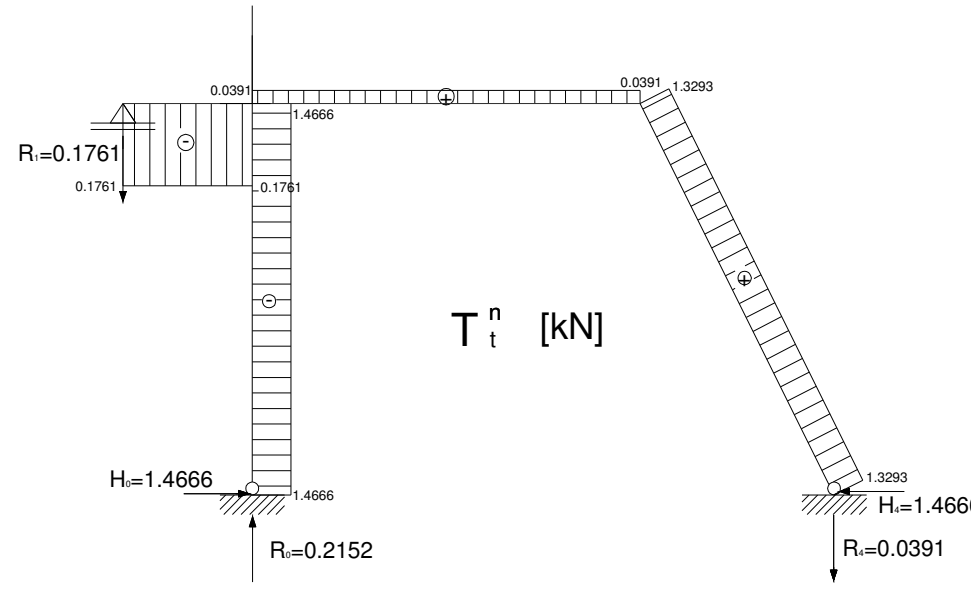


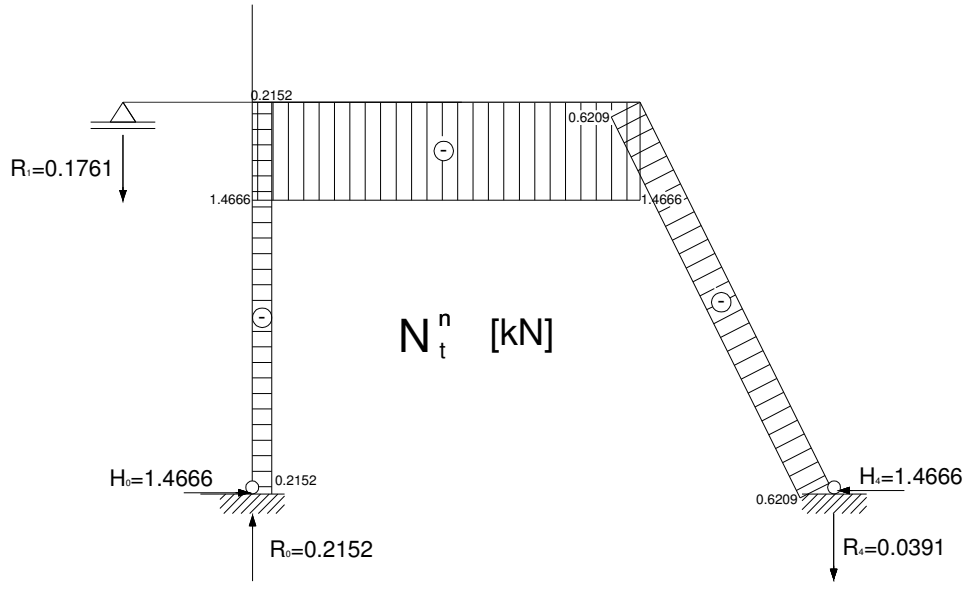
$R_0 = 0.2152\ kN$

$H_0 = 1.4666\ kN$

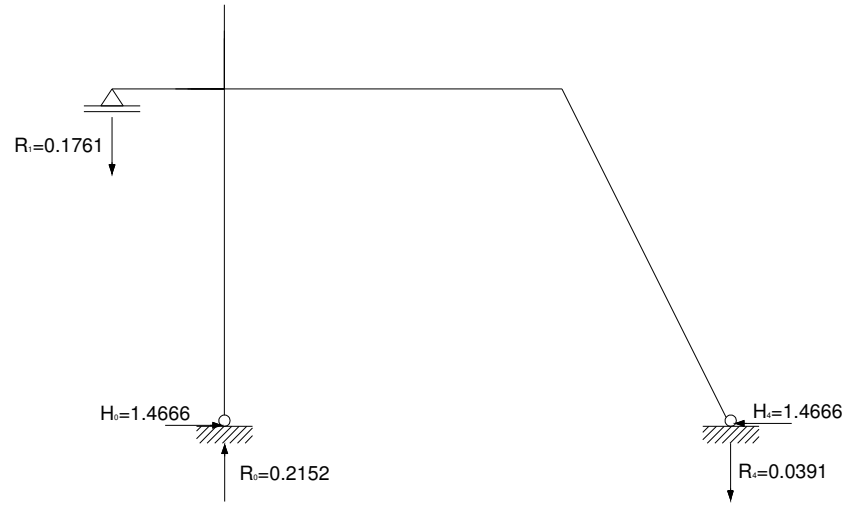
$R_1 = 0.1761\ kN$


9.1517
8.9170
8.7996
0.3521
8.9170
R1=0.1761
M_t^n [kNm]
H0=1.4666
H4=1.4666
R0=0.2152
R4=0.0391


0.0391
0.0391
1.3293
1.4666
R1=0.1761
0.1761
0.1761
T_t^n [kN]
H0=1.4666
1.3293
H4=1.4666
1.4666
R0=0.2152
R4=0.0391


0.2152
0.6209
R1=0.1761
1.4666
1.4666
N_t^n [kN]
H0=1.4666
0.2152
H4=1.4666
0.6209
R0=0.2152
R4=0.0391

KONTROLA STATYCZNA



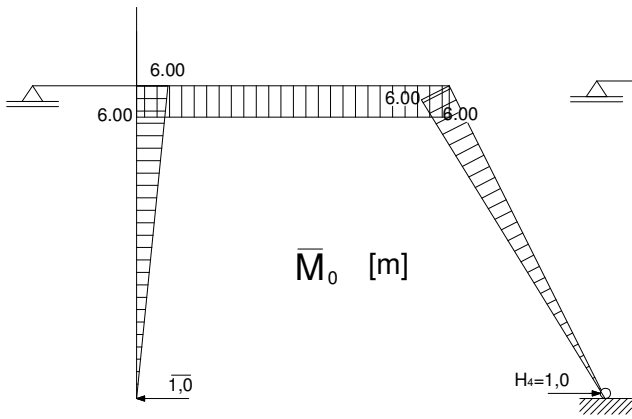
$\sum X = 0 \qquad 1.4666 - 1.4666 = 0 \quad \Rightarrow \quad \underline{0 = 0}$

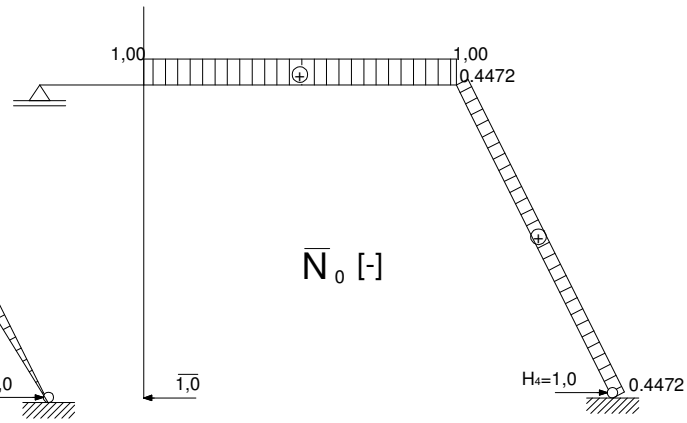
$\sum Y = 0 \qquad -0.1761 + 0.2152 - 0.0391 = 0 \quad \Rightarrow \quad \underline{0 = 0}$

$\sum M_0 = 0 \qquad -0.1761 \cdot 2 + 0.0391 \cdot 9 = 0 \quad \Rightarrow \quad \underline{0 = 0}$

KONTROLA KINEMATYCZNA

$$H_0 \cdot \overline{1,0} = \sum \int \frac{M_t^n \cdot \overline{M}}{EI} ds + \sum \int \overline{M} \frac{\Delta t \cdot \alpha_t}{h} ds + \sum \int \overline{N} \cdot t_\circ \cdot \alpha_t ds$$





$$H_0 \cdot \overline{1,0} = \frac{1}{EI_\circ}\left[\frac{6}{6}\left(-2 \cdot 9.1517 \cdot 6 - 2 \cdot 8.9170 \cdot 6 - 9.1517 \cdot 6 - 8.9170 \cdot 6\right)\right] +$$
$$+ \frac{1}{0.5331EI_0}\left[-\frac{1}{2} \cdot 6 \cdot 8.7996 \cdot \frac{2}{3} \cdot 6 + \frac{1}{2} \cdot 6.7082 \cdot 8.9170 \cdot \frac{2}{3} \cdot 6\right] +$$
$$+ \left[\frac{1}{2} \cdot 6 \cdot 6 \cdot \frac{17 \cdot 1{,}2 \cdot 10^{-5}}{0{,}26} + \frac{1}{2} \cdot 6 \cdot 6 \cdot \frac{17 \cdot 1{,}2 \cdot 10^{-5}}{0{,}22} + \frac{1}{2} \cdot 6.7082 \cdot 6 \cdot \frac{17 \cdot 1{,}2 \cdot 10^{-5}}{0{,}22}\right] +$$
$$+ \left[6 \cdot 1 \cdot 1{,}2 \cdot 10^{-5} \cdot (-0{,}5) + 6.7082 \cdot 1 \cdot 1{,}2 \cdot 10^{-5} \cdot (-0{,}5)\right]$$

$$H_0 \cdot \overline{1,0} = \frac{-325.23591}{EI_0} + \frac{-225.23}{0.5331EI_0} + 0.0282 + 0.0167 + 0.0187 - 0.000036 - 0.000018 = 1.23735E - 17 \approx 0{,}0$$