

Politechnika Poznańska
Wydział Architektury Budownictwa
i Inżynierii Środowiska

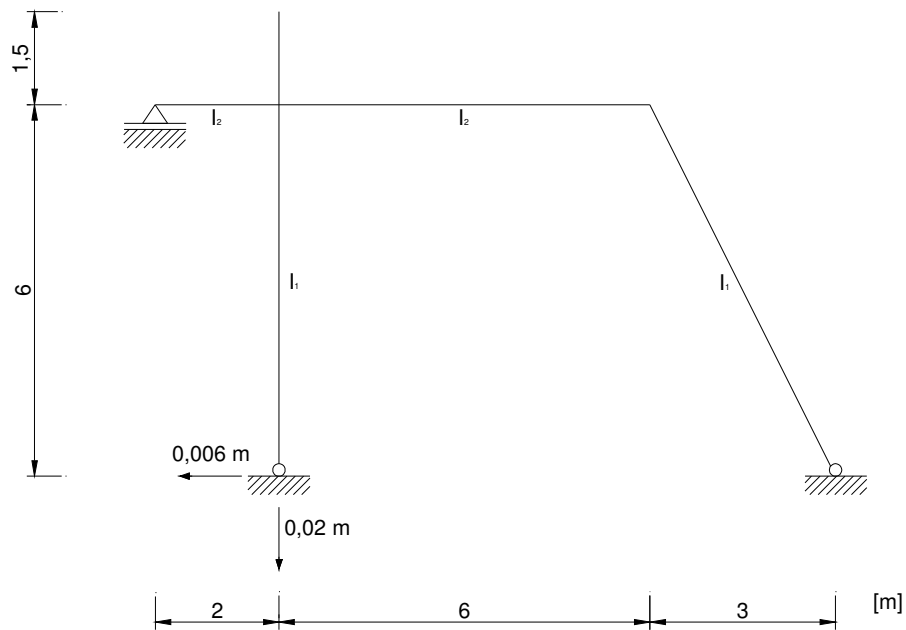
ĆWICZENIE NR 4

OBLICZENIE RAMY METODĄ PRZEMIESZCZEŃ **(wpływ osiadania podpór)**

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rok studiów: III
semestr: VI
gr. 8

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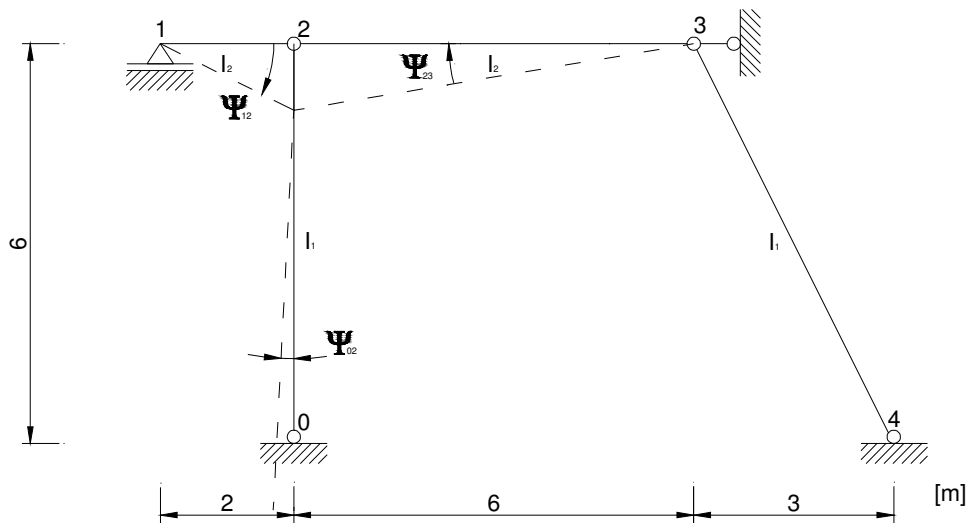
METODA PRZEMIESZCZEŃ - OSIADANIE PODPÓR
SCHEMAT KONSTRUKCJI



UKŁAD RÓWNAŃ KANONICZNYCH

$$\begin{cases} r_{11} \cdot Z_1 + r_{12} \cdot Z_2 + r_{13} \cdot Z_3 + r_{1\Delta} = 0 \\ r_{21} \cdot Z_1 + r_{22} \cdot Z_2 + r_{23} \cdot Z_3 + r_{2\Delta} = 0 \\ r_{31} \cdot Z_1 + r_{32} \cdot Z_2 + r_{33} \cdot Z_3 + r_{3\Delta} = 0 \end{cases}$$

ŁAŃCUCH KINEMATYCZNY:



$$\downarrow 120 \quad 2 \cdot \Psi_{12}^{\Delta} = 0,02 \Rightarrow \Psi_{12}^{\Delta} = \frac{0,02}{2} \Rightarrow \Psi_{12}^{\Delta} = 0,0100$$

$$\rightarrow 023 \quad -0,006 + 6 \cdot \Psi_{02}^{\Delta} + 0 \cdot \Psi_{23}^{\Delta} = 0 \Rightarrow \Psi_{02}^{\Delta} = \frac{0,006}{6} \Rightarrow \Psi_{02}^{\Delta} = 0,0010$$

$$\downarrow 321 \quad -6 \cdot \Psi_{23}^{\Delta} - 2 \cdot \Psi_{12}^{\Delta} = 0 \Rightarrow \Psi_{23}^{\Delta} = -\frac{2 \cdot 0,0100}{6} \Rightarrow \Psi_{23}^{\Delta} = -0,0033$$

$$\downarrow 34 \quad 3 \cdot \Psi_{34}^{\Delta} = 0 \Rightarrow \Psi_{34}^{\Delta} = 0$$

MOMENTY PRZYWĘZŁOWE OD OSIADANIA

$$M_{12}^{\Delta} = 0$$

$$M_{21}^{\Delta} = \frac{3EI_0}{l} (\varphi_2 - \Psi_{12}) = \frac{3EI_0}{2} (0 - 0.01) = -176.5050 \text{ kNm}$$

$$M_{23}^{\Delta} = \frac{2EI_0}{l} (2\varphi_2 + \varphi_3 - 3\Psi_{23}) = \frac{2EI_0}{6} (2 \cdot 0 + 0 - 3 \cdot (-0.0033)) = 39.2233 \text{ kNm}$$

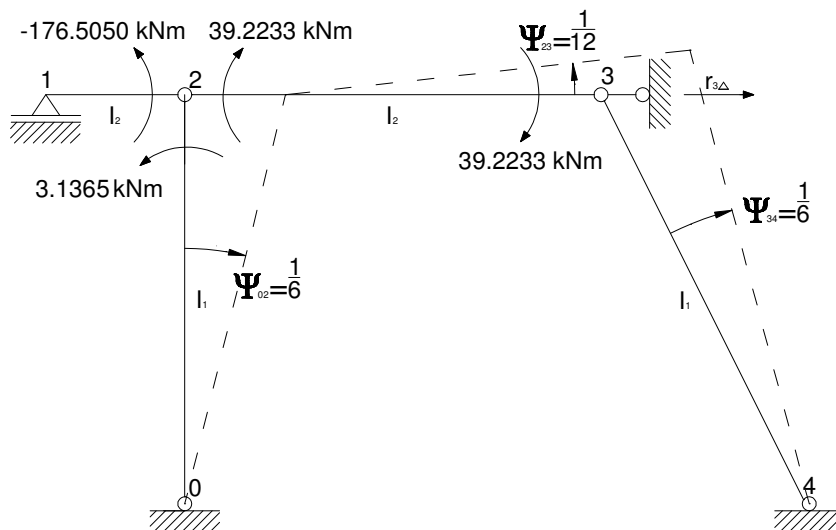
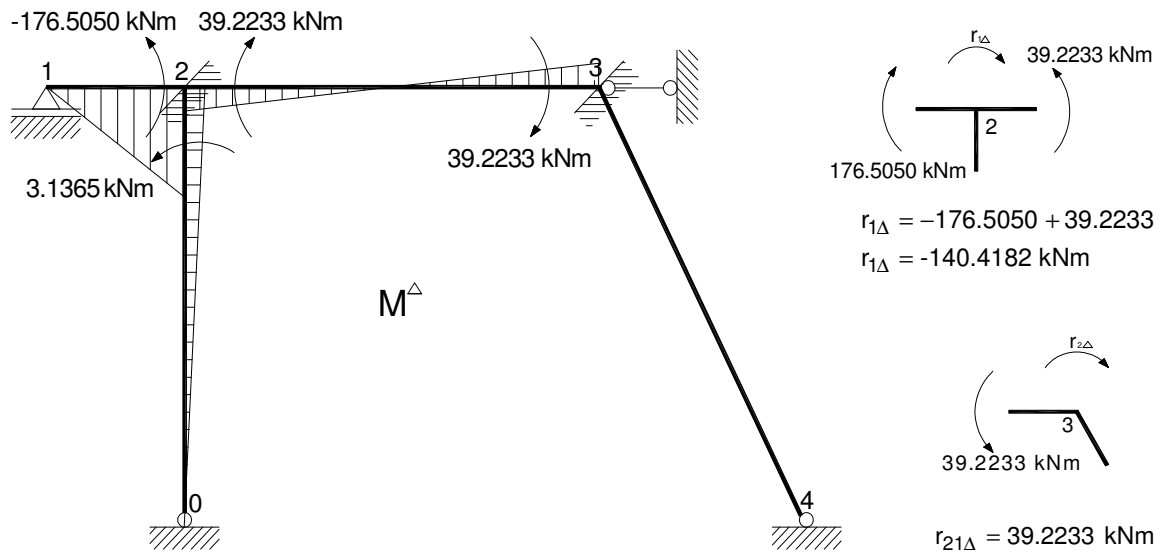
$$M_{32}^{\Delta} = M_{23}^{\Delta} = 39.2233 \text{ kNm}$$

$$M_{34}^{\Delta} = \frac{3EI_0}{l} (\varphi_3 - \Psi_{34}) = \frac{3 \cdot 0.5331 \cdot EI_0}{6.7082} (0 - 0) = 0$$

$$M_{43}^{\Delta} = 0$$

$$M_{02}^{\Delta} = 0$$

$$M_{20}^{\Delta} = \frac{3EI_0}{l} (\varphi_2 - \Psi_{02}) = \frac{3 \cdot 0.5331 \cdot EI_0}{6} (0 - 0.001) = -3.1365 \text{ kNm}$$



R.P.W.

$$\overline{1.0} \cdot r_{3\Delta} - 3.1365 \cdot \frac{1}{6} - (39.2233 + 39.2233) \cdot \frac{1}{12} = 0 \Rightarrow r_{3\Delta} = 7.0600 \text{ kNm}$$

ROZWIĄZANIE UKŁADU RÓWNAŃ KANONICZNYCH

$$\begin{cases} r_{11} \cdot z_1 + r_{12} \cdot z_2 + r_{13} \cdot z_3 + r_{1\Delta} = 0 \\ r_{21} \cdot z_1 + r_{22} \cdot z_2 + r_{23} \cdot z_3 + r_{2\Delta} = 0 \\ r_{31} \cdot z_1 + r_{32} \cdot z_2 + r_{33} \cdot z_3 + r_{3\Delta} = 0 \end{cases}$$

$$\begin{cases} 2.4332E_{l_0} \cdot z_1 + 0.3333E_{l_0} \cdot z_2 + 0.0389E_{l_0} \cdot z_3 = 140.4182 \\ 0.3333E_{l_0} \cdot z_1 + 0.9051E_{l_0} \cdot z_2 + 0.0436E_{l_0} \cdot z_3 = -39.2233 \\ 0.0389E_{l_0} \cdot z_1 + 0.0436E_{l_0} \cdot z_2 + 0.0279E_{l_0} \cdot z_3 = -7.0600 \end{cases}$$

$$z_1 = \frac{69.6179}{E_{l_0}} \Rightarrow \varphi_2 = \frac{69.6179}{E_{l_0}}$$

$$z_2 = \frac{-56.3602}{E_{l_0}} \Rightarrow \varphi_3 = \frac{-56.3602}{E_{l_0}}$$

$$z_3 = \frac{-261.9138}{E_{l_0}} \Rightarrow \Delta_3 = \frac{-261.9138}{E_{l_0}}$$

WYZNACZENIE WARTOŚCI MOMENTÓW ZGINAJĄCYCH – METODA SUPERPOZYCJI:

$$M_P^{(n)} = M_1 \cdot z_1 + M_2 \cdot z_2 + M_3 \cdot z_3 + M_\Delta$$

$$M_{12} = 0$$

$$M_{21} = 1.50E_{l_0} \cdot \frac{69.6179}{E_{l_0}} + 0 \cdot \frac{-56.3602}{E_{l_0}} + 0 \cdot \frac{-261.9138}{E_{l_0}} - 176.5050 = -72.0781 \text{ kNm}$$

$$M_{23} = 0.6667E_{l_0} \cdot \frac{69.6179}{E_{l_0}} + 0.3333E_{l_0} \cdot \frac{-56.3602}{E_{l_0}} + 0.0833E_{l_0} \cdot \frac{-261.9138}{E_{l_0}} + 39.2233 = 45.0224 \text{ kNm}$$

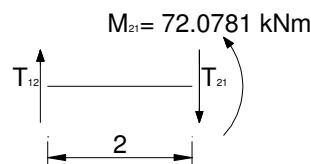
$$M_{32} = 0.3333E_{l_0} \cdot \frac{69.6179}{E_{l_0}} + 0.6667E_{l_0} \cdot \frac{-56.3602}{E_{l_0}} + 0.0833E_{l_0} \cdot \frac{-261.9138}{E_{l_0}} + 39.2233 = 3.0297 \text{ kNm}$$

$$M_{34} = 0 \cdot \frac{69.6179}{E_{l_0}} + 0.2384E_{l_0} \cdot \frac{-56.3602}{E_{l_0}} - 0.0397E_{l_0} \cdot \frac{-261.9138}{E_{l_0}} + 0 = -3.0297 \text{ kNm}$$

$$M_{43} = 0$$

$$M_{02} = 0$$

$$M_{20} = 0.2666E_{l_0} \cdot \frac{69.6179}{E_{l_0}} + 0 \cdot \frac{-56.3602}{E_{l_0}} - 0.0444E_{l_0} \cdot \frac{-261.9138}{E_{l_0}} - 3.1365 = 27.0557 \text{ kNm}$$

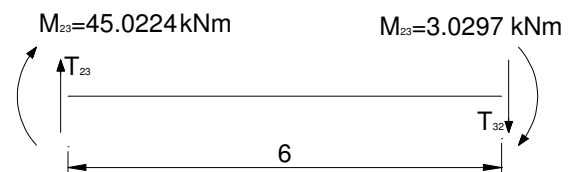
WYZNACZENIE WARTOŚCI SIŁ TNĄCYCH:

$$\sum M_1 = 0$$

$$T_{21} \cdot 2 - 72.0781 = 0$$

$$T_{21} = 36.0391 \text{ kN}$$

$$T_{12} = T_{21} = 36.0391 \text{ kN}$$

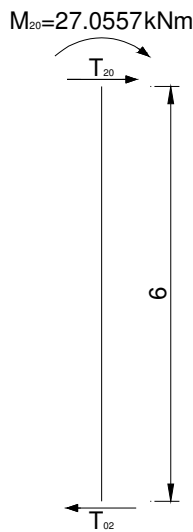


$$\sum M_2 = 0$$

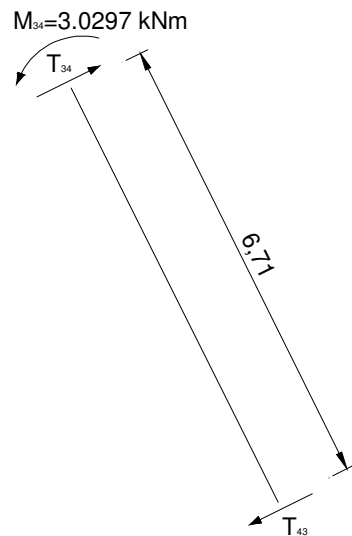
$$T_{32} \cdot 6 + 45.0224 + 3.0297 = 0$$

$$T_{32} = -8.0087 \text{ kN}$$

$$T_{23} = T_{32} = -8.0087 \text{ kN}$$



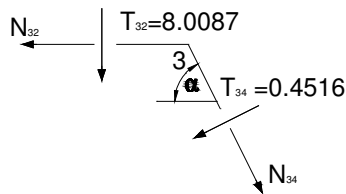
$$\begin{aligned} \sum M_0 = 0 & \quad T_{20} \cdot 6 + 27.0557 = 0 \\ & \quad T_{20} = -4.5093 \text{ kN} \\ & \quad T_{02} = T_{20} = -4.5093 \text{ kN} \end{aligned}$$



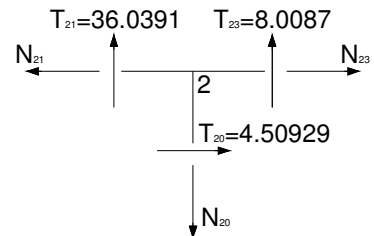
$$\begin{aligned} \sum M_3 = 0 & \quad T_{43} \cdot 6.7082 - 3.0297 = 0 \\ & \quad T_{43} = 0.45164 \text{ kN} \\ & \quad T_{34} = T_{43} = 0.45164 \text{ kN} \end{aligned}$$

WYZNACZENIE WARTOŚCI SIŁ NORMALNYCH:

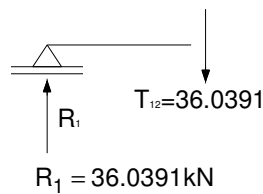
$\sin \alpha = 0.8944$; $\cos \alpha = 0.4472$



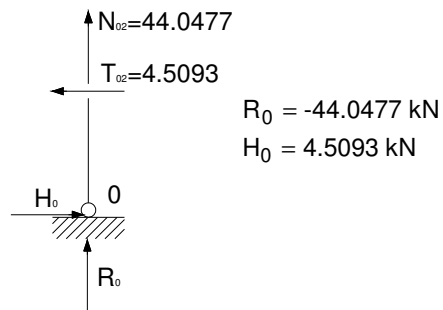
$$\begin{aligned} \sum Y = 0 & \quad -N_{34} \cdot \sin \alpha - 8.0087 - 0.4516 \cdot \cos \alpha = 0 \\ & \quad N_{34} = -9.1798 \text{ kN} \\ \sum X = 0 & \quad -N_{32} + 0.4516 \cdot \sin \alpha + N_{34} \cdot \cos \alpha = 0 \\ & \quad N_{32} = -4.5093 \text{ kN} \end{aligned}$$



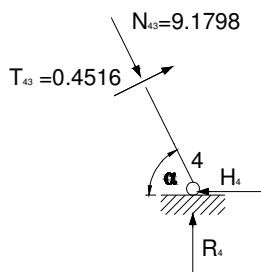
$$\begin{aligned} \sum Y = 0 & \quad -N_{20} + 36.0391 + 8.0087 = 0 \\ & \quad N_{20} = 44.0477 \text{ kN} \\ \sum X = 0 & \quad -N_{21} + 4.50929 + N_{23} = 0 \\ & \quad N_{21} = 0 \end{aligned}$$



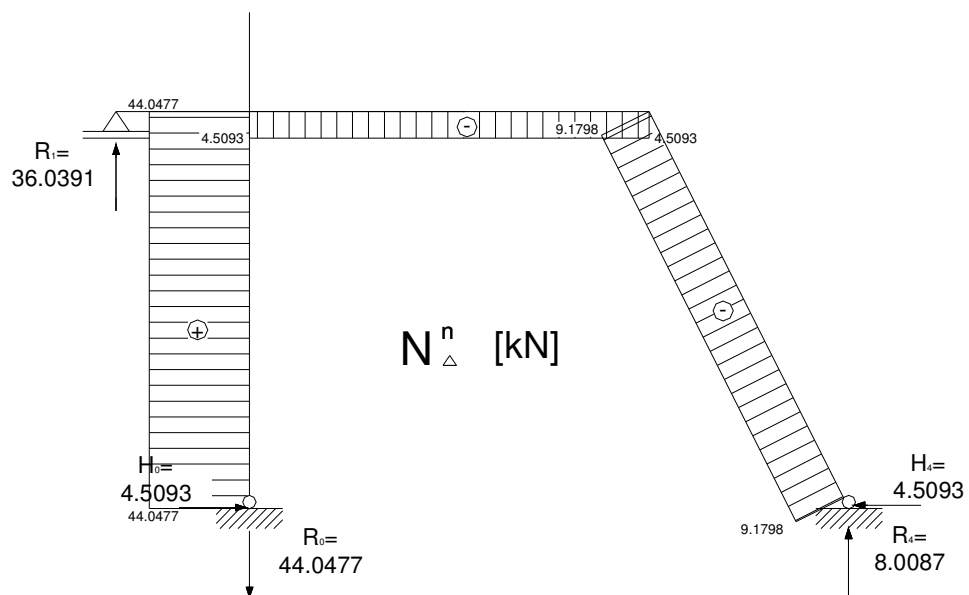
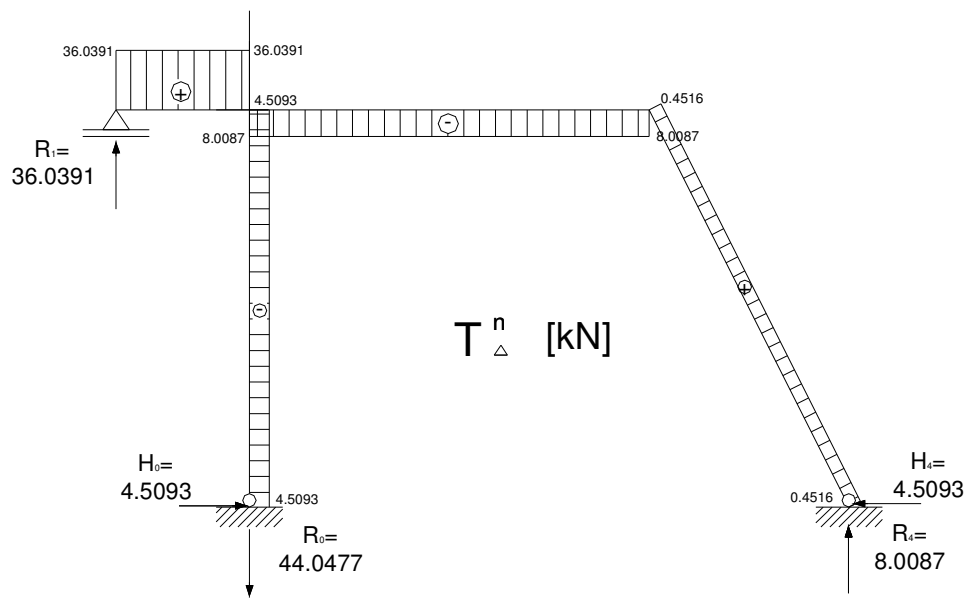
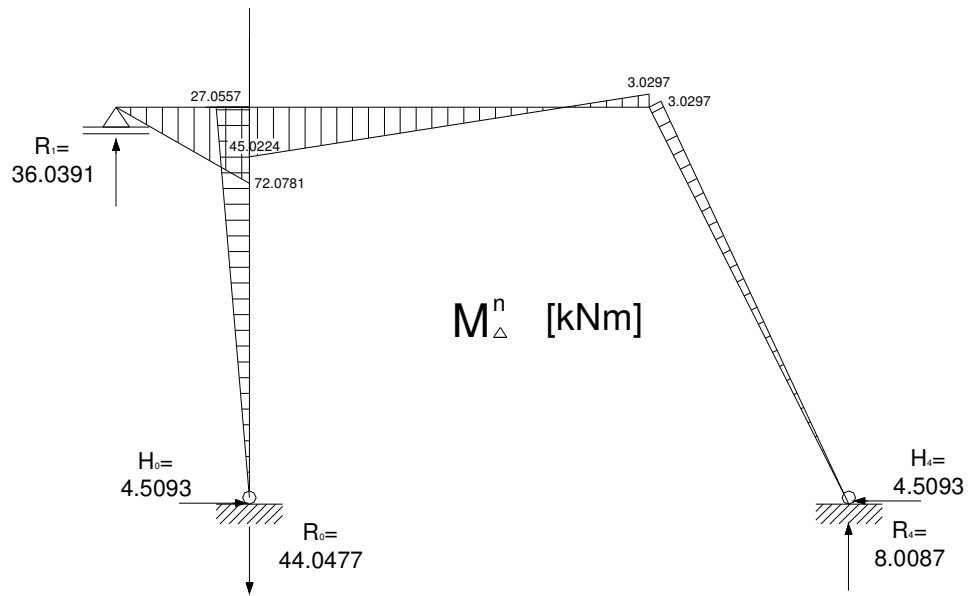
$R_1 = 36.0391 \text{ kN}$



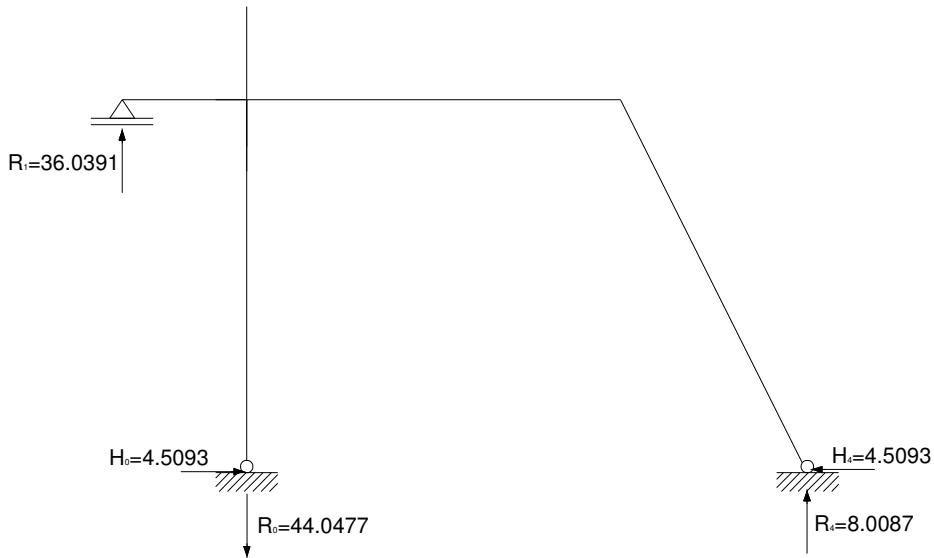
$R_0 = -44.0477 \text{ kN}$
 $H_0 = 4.5093 \text{ kN}$



$$\begin{aligned} \sum Y = 0 & \quad R_4 + 0.45164 \cdot \cos \alpha - 9.1798 \cdot \sin \alpha = 0 \\ & \quad R_4 = 8.0087 \text{ kN} \\ \sum X = 0 & \quad -H_4 + 0.45164 \cdot \sin \alpha + 9.1798 \cdot \cos \alpha = 0 \\ & \quad H_4 = 4.5093 \text{ kN} \end{aligned}$$



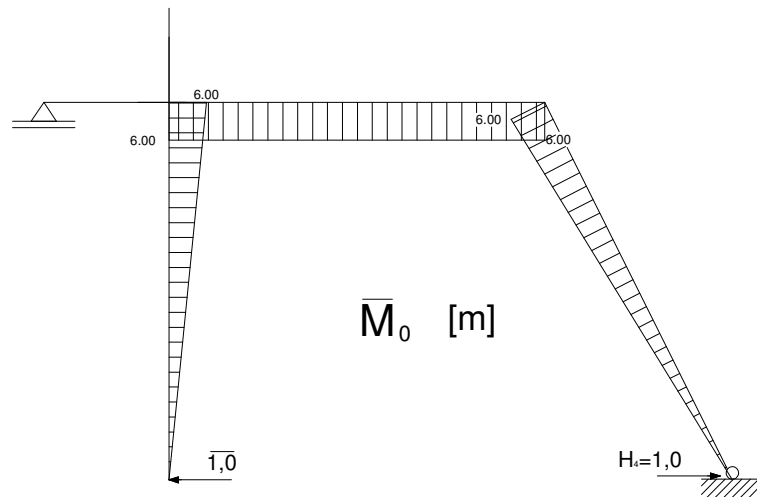
KONTROLA STATYCZNA



$$\begin{aligned} \sum X = 0 & \quad 4.5093 - 4.50929 = 0 \Rightarrow \underline{0 = 0} \\ \sum Y = 0 & \quad 36.0391 - 44.0477 + 8.0087 = 0 \Rightarrow \underline{0 = 0} \\ \sum M_0 = 0 & \quad 36.0391 \cdot 2 - 8.0087 \cdot 9 = 0 \Rightarrow \underline{0 = 0} \end{aligned}$$

KONTROLA KINEMATYCZNA

$$H_0 \cdot \bar{1}_0 = \sum \int \frac{M_{\Delta}^n \cdot \bar{M}}{EI} ds - \sum_n \bar{R}_n \cdot \Delta_n$$



$$H_0 \cdot \bar{1}_0 = 0,006m$$

$$\begin{aligned} H_0 \cdot \bar{1}_0 &= \frac{1}{EI_0} \left[\frac{6}{6} (2 \cdot 45.0224 \cdot 6 - 2 \cdot 3.0297 \cdot 6 + 45.0224 \cdot 6 - 3.0297 \cdot 6) \right] + \\ &+ \frac{1}{0.5331EI_0} \left[-\frac{1}{2} \cdot 6 \cdot 27.0557 \cdot \frac{2}{3} \cdot 6 + \frac{1}{2} \cdot 6.7082 \cdot 3.0297 \cdot \frac{2}{3} \cdot 6 \right] \end{aligned}$$

$$H_0 \cdot \bar{1}_0 = \frac{755.86867}{EI_0} + \frac{-365.3164}{0.5331EI_0} = 0.0642 - 0.0582 = 0,006m$$