

Politechnika Poznańska
Wydział Architektury Budownictwa
i Inżynierii Środowiska

ĆWICZENIE NR 4

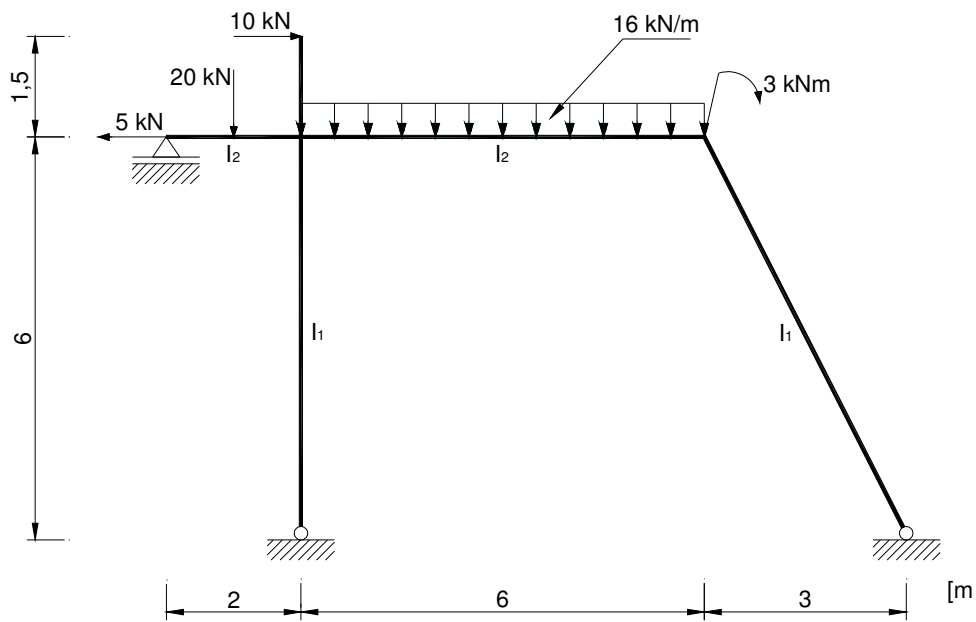
OBLICZENIE RAMY METODĄ PRZEMIESZCZEŃ **(wpływ obciążenia zewnętrznego)**

Sierocki Damian
rok studiów: III
semestr: VI
gr. 8

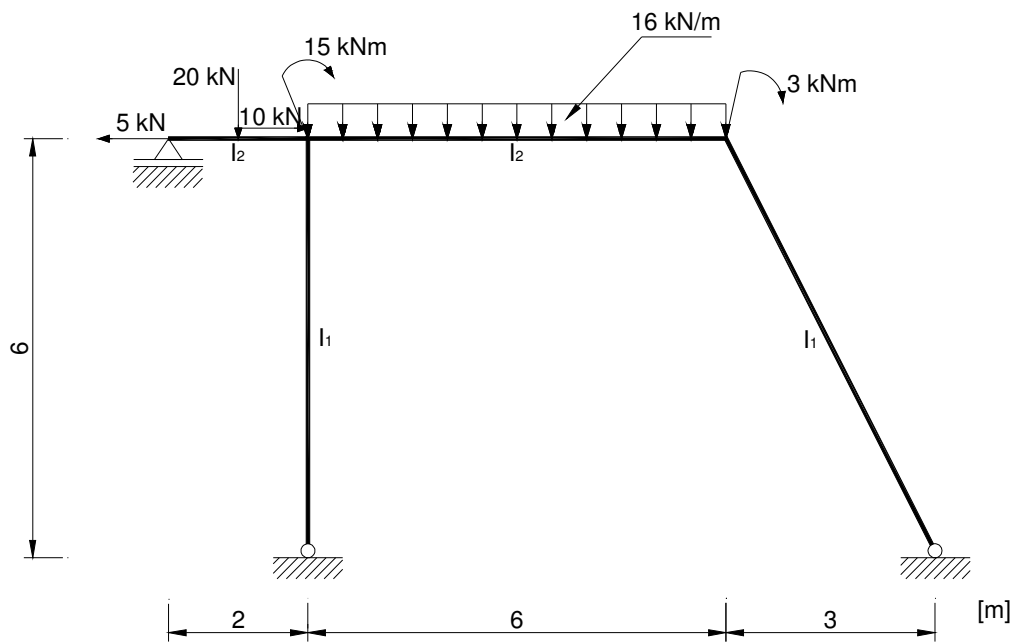
Poznań 2005

METODA PRZEMIESZCZEŃ – OBCIĄŻENIE ZEWNĘTRZNE

SCHEMAT KONSTRUKCJI



Sprowadzenie układu wyjściowego do równoważnego układu zastępczego:



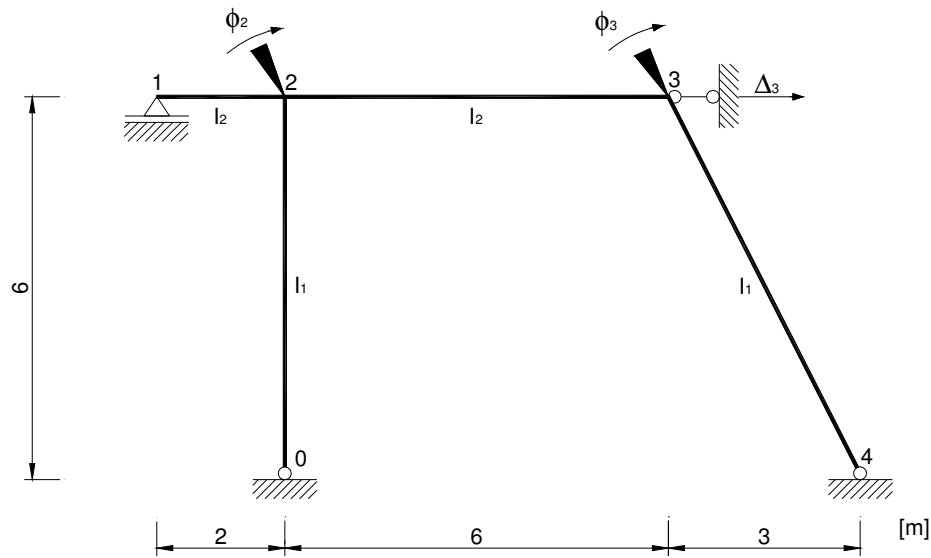
STOPIEŃ GEOMETRYCZNEJ NIEWYZNACZALNOŚCI UKŁADU:

$$SGN = \sum \Delta + \sum \varphi$$

$$\sum \Delta = 1$$

$$\sum \varphi = 2$$

$$SGN = 1 + 2 = 3$$

UKŁAD PODSTAWOWY**UKŁAD RÓWNAŃ KANONICZNYCH**

$$\begin{cases} r_{11} \cdot Z_1 + r_{12} \cdot Z_2 + r_{13} \cdot Z_3 + r_{1P} = 0 \\ r_{21} \cdot Z_1 + r_{22} \cdot Z_2 + r_{23} \cdot Z_3 + r_{2P} = 0 \\ r_{31} \cdot Z_1 + r_{32} \cdot Z_2 + r_{33} \cdot Z_3 + r_{3P} = 0 \end{cases}$$

Niewiadome: $\varphi_2, \varphi_3, \Delta_3$

$$\varphi_2 = Z_1$$

$$\varphi_3 = Z_2$$

$$\Delta_3 = Z_3$$

WSPÓŁCZYNNIK PORÓWNAWCZY SZTYWNOŚCI

$$I_1 - I220 \quad (W_x = 278.18 \text{ cm}^3, I_x = 3060 \text{ cm}^4)$$

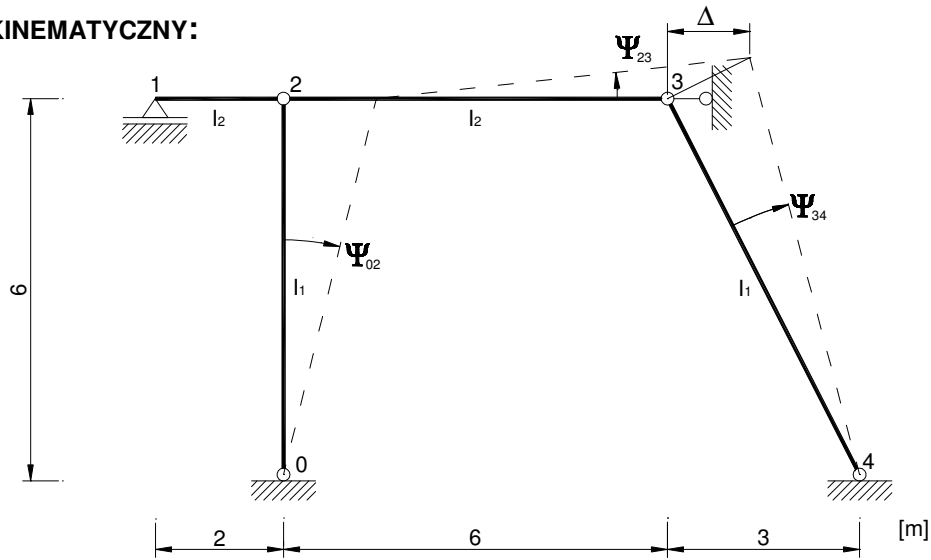
$$I_2 - I260 \quad (W_x = 441.54 \text{ cm}^3, I_x = 5740 \text{ cm}^4)$$

$$EI_1 = 205 \cdot 10^6 \cdot 3060 \cdot 10^{-8} = 6273 \text{ kN/m}$$

$$EI_2 = 205 \cdot 10^6 \cdot 5740 \cdot 10^{-8} = 11767 \text{ kN/m}$$

Przyjęto współczynnik porównawczy sztywności $EI_0 = EI_2 \rightarrow EI_1 = 0.5331 EI_0$

ŁAŃCUCH KINEMATYCZNY:



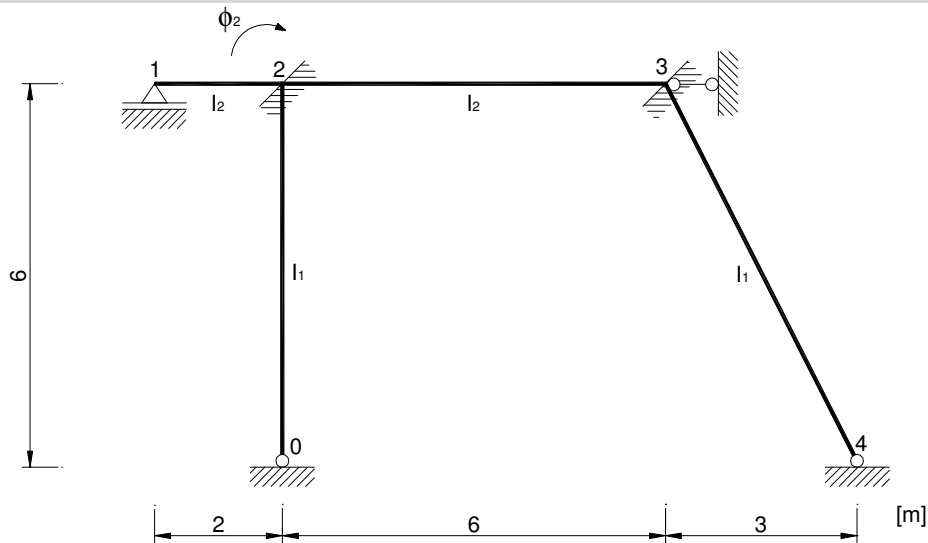
$$\rightarrow 023 \quad 6 \cdot \Psi_{02} = \Delta \Rightarrow \Psi_{02} = \frac{\Delta}{6}$$

$$\rightarrow 0234 \quad 6 \cdot \Psi_{02} - 6 \cdot \Psi_{34} = 0 \Rightarrow \Psi_{02} = \Psi_{34} \Rightarrow \Psi_{34} = \frac{\Delta}{6}$$

$$\downarrow 0234 \quad 6 \cdot \Psi_{23} - 3 \cdot \Psi_{34} = 0 \Rightarrow \Psi_{34} = -2\Psi_{23} \Rightarrow \Psi_{23} = -\frac{\Delta}{12}$$

$$\downarrow 021 \quad 2 \cdot \Psi_{12} = 0 \Rightarrow \Psi_{12} = 0$$

Stan $z_1 = 1$ ($\varphi_2 = 1$)



$$M_{12} = 0$$

$$M_{21} = \frac{3EI_0}{l} (\varphi_2 - \Psi_{12}) = \frac{3EI_0}{2} (1 - 0) = 1.5000 EI_0$$

$$M_{23} = \frac{2EI_0}{l} (2\varphi_2 + \varphi_3 - 3\Psi_{23}) = \frac{2EI_0}{6} (2 \cdot 1 + 0 - 3 \cdot 0) = 0.6667 EI_0$$

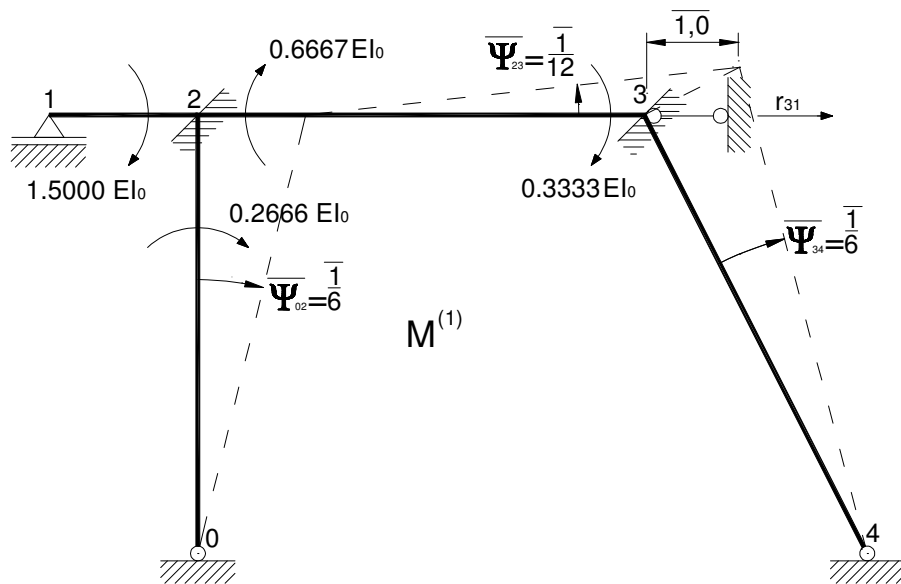
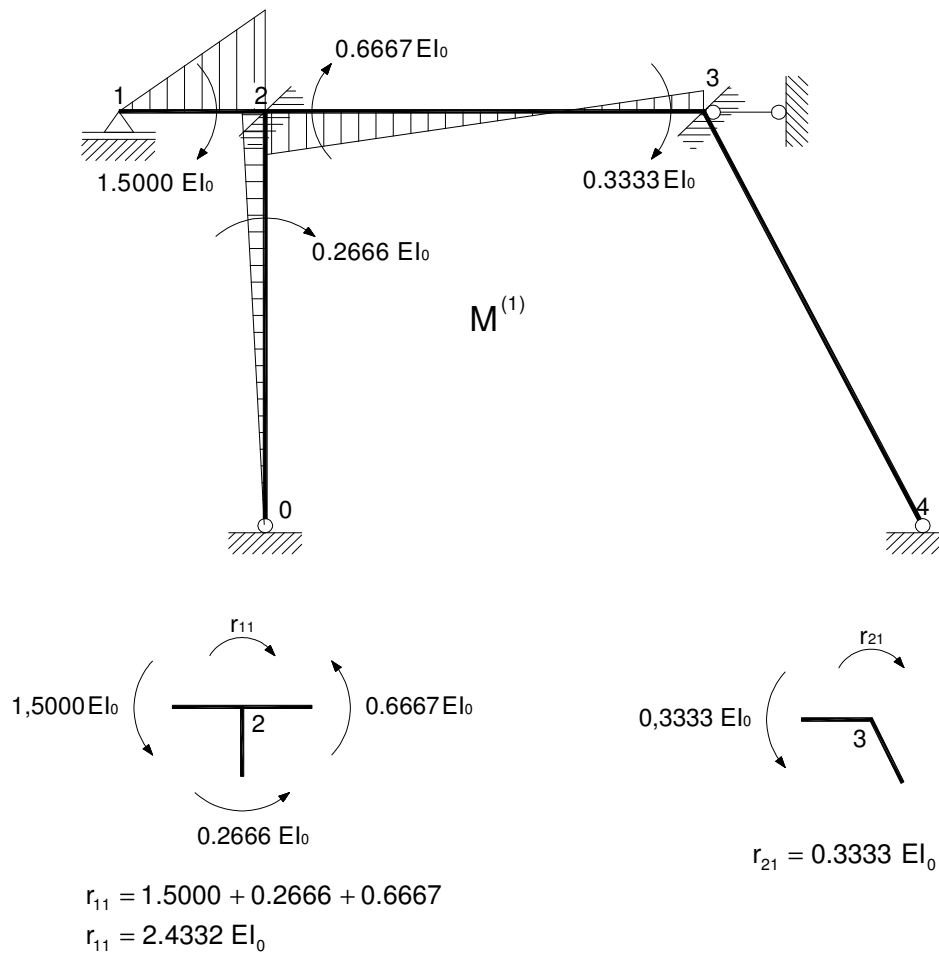
$$M_{32} = \frac{2EI_0}{l} (\varphi_2 + 2\varphi_3 - 3\Psi_{23}) = \frac{2EI_0}{6} (1 + 2 \cdot 0 - 3 \cdot 0) = 0.3333 EI_0$$

$$M_{34} = \frac{3EI_0}{l} (\varphi_3 - \Psi_{34}) = \frac{3 \cdot 0.5331 \cdot EI_0}{6.7082} (0 - 0) = 0,0 EI_0$$

$$M_{43} = 0$$

$$M_{02} = 0$$

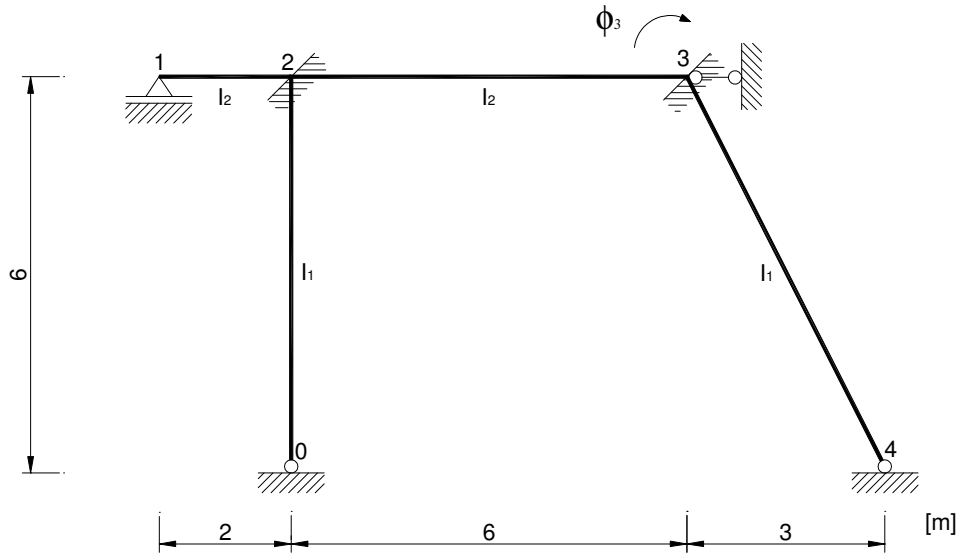
$$M_{20} = \frac{3EI_0}{l} (\varphi_2 - \Psi_{02}) = \frac{3 \cdot 0.5331 \cdot EI_0}{6} (1 - 0) = 0.2666 EI_0$$



R.P.W.

$$1,0 \cdot r_{31} + 0.2666 El_0 \cdot \frac{1,0}{6} - (0.6667 + 0.3333) El_0 \cdot \frac{1,0}{12} = 0 \quad \Rightarrow \quad r_{31} = 0.0389 El_0$$

Stan $z_2 = 1$ ($\varphi_3 = 1$)



$$M_{12} = 0$$

$$M_{21} = \frac{3EI_0}{l} (\varphi_2 - \Psi_{12}) = \frac{3EI_0}{2} (0 - 0) = 0$$

$$M_{23} = \frac{2EI_0}{l} (2\varphi_2 + \varphi_3 - 3\Psi_{23}) = \frac{2EI_0}{6} (2 \cdot 0 + 1 - 3 \cdot 0) = 0.3333 EI_0$$

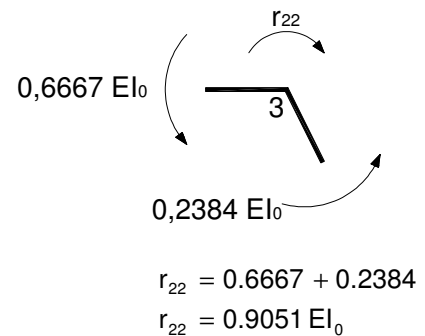
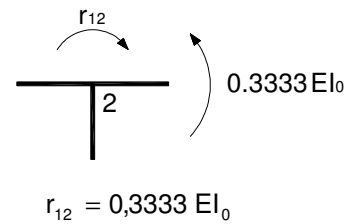
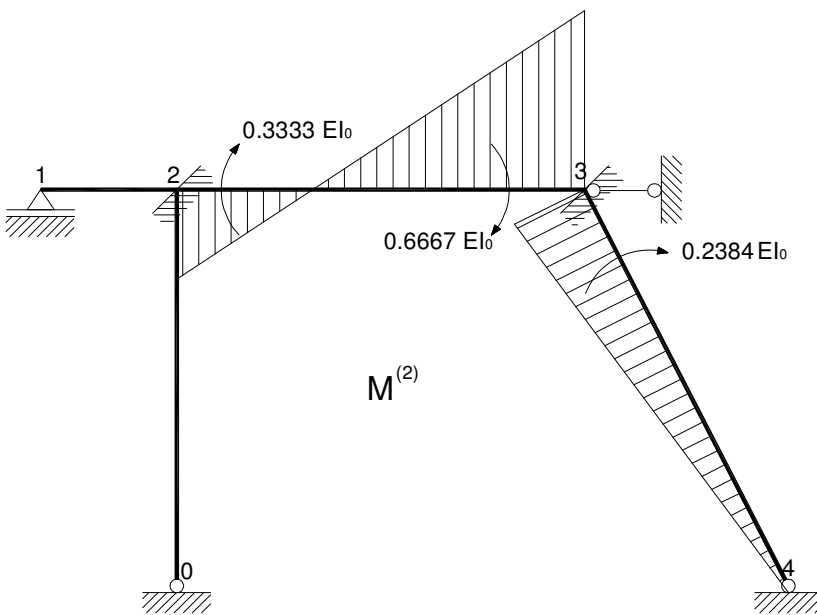
$$M_{32} = \frac{2EI_0}{l} (\varphi_2 + 2\varphi_3 - 3\Psi_{23}) = \frac{2EI_0}{6} (0 + 2 \cdot 1 - 3 \cdot 0) = 0.6667 EI_0$$

$$M_{34} = \frac{3EI_0}{l} (\varphi_3 - \Psi_{34}) = \frac{3 \cdot 0.5331 \cdot EI_0}{6.7082} (1 - 0) = 0.2384 EI_0$$

$$M_{43} = 0$$

$$M_{02} = 0$$

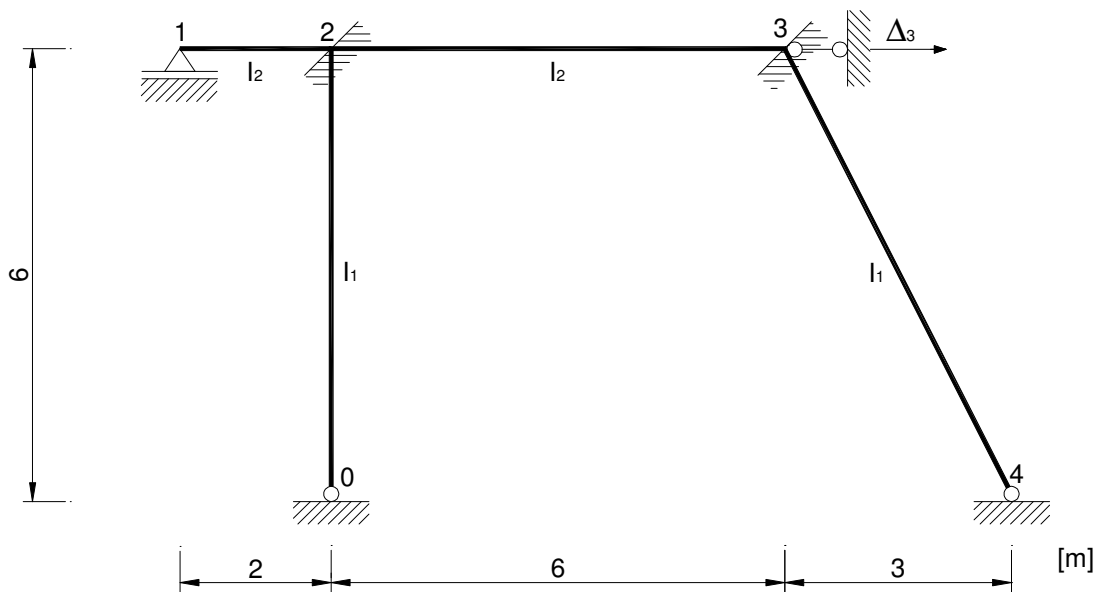
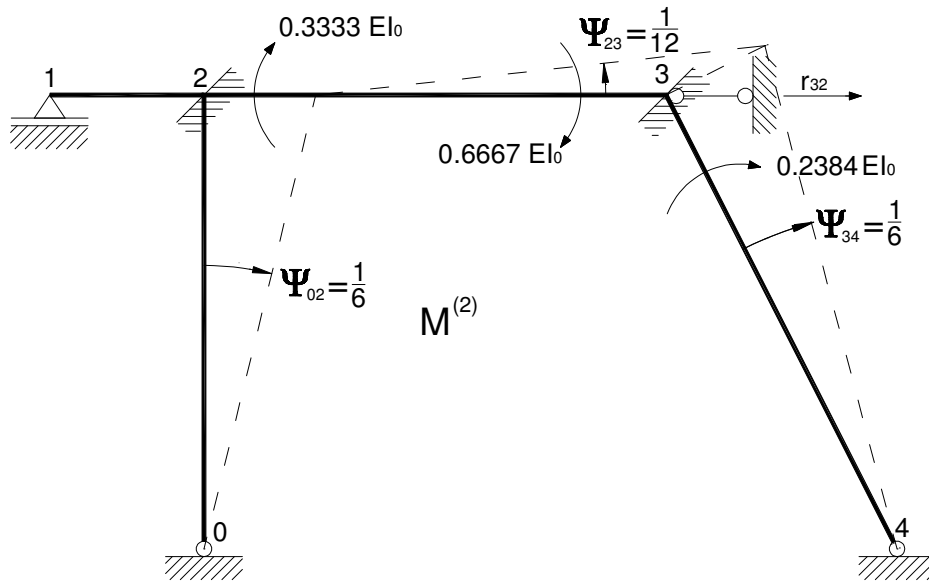
$$M_{20} = \frac{3EI_0}{l} (\varphi_2 - \Psi_{02}) = \frac{3 \cdot 0.5331 \cdot EI_0}{6} (0 - 0) = 0$$



R.P.W

$$1.0 \cdot r_{32} - (0.6667 + 0.3333) EI_0 \cdot \frac{1.0}{12} + 0.2384 EI_0 \cdot \frac{1.0}{6} = 0 \quad \rightarrow \quad r_{32} = 0.0436 EI_0$$

Stan $z_3 = 1$ ($\Delta_3 = 1$)



$$M_{12} = 0$$

$$M_{21} = \frac{3El_0}{l}(\varphi_2 - \Psi_{12}) = \frac{3El_0}{2}(0 - 0) = 0$$

$$M_{23} = \frac{2El_0}{l}(2\varphi_2 + \varphi_3 - 3\Psi_{23}) = \frac{2El_0}{6}\left(2 \cdot 0 + 0 - 3 \cdot \frac{1}{12}\right) = 0.0833 El_0$$

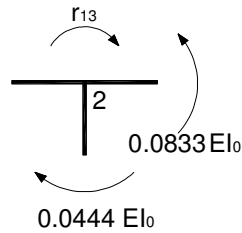
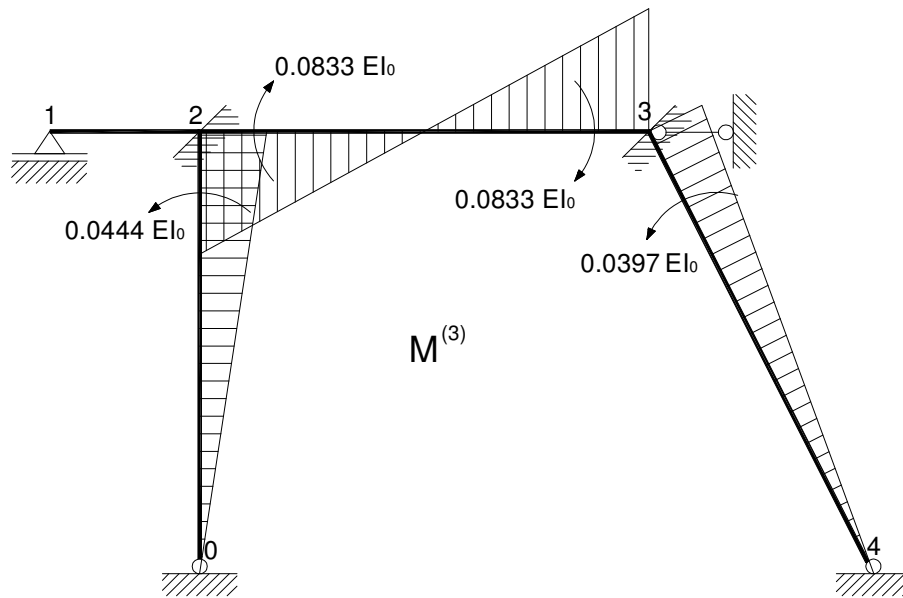
$$M_{32} = \frac{2El_0}{l}(\varphi_2 + 2\varphi_3 - 3\Psi_{23}) = \frac{2El_0}{6}\left(0 + 2 \cdot 0 - 3 \cdot \frac{1}{12}\right) = 0.0833 El_0$$

$$M_{34} = \frac{3El_0}{l}(\varphi_3 - \Psi_{34}) = \frac{3 \cdot 0.5331 \cdot El_0}{6.7082}\left(0 - \frac{1}{6}\right) = -0.0397 El_0$$

$$M_{43} = 0$$

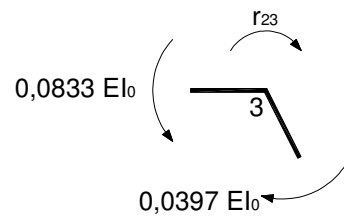
$$M_{02} = 0$$

$$M_{20} = \frac{3El_0}{l}(\varphi_2 - \Psi_{02}) = \frac{3 \cdot 0.5331 \cdot El_0}{6}\left(0 - \frac{1}{6}\right) = -0.0444 El_0$$



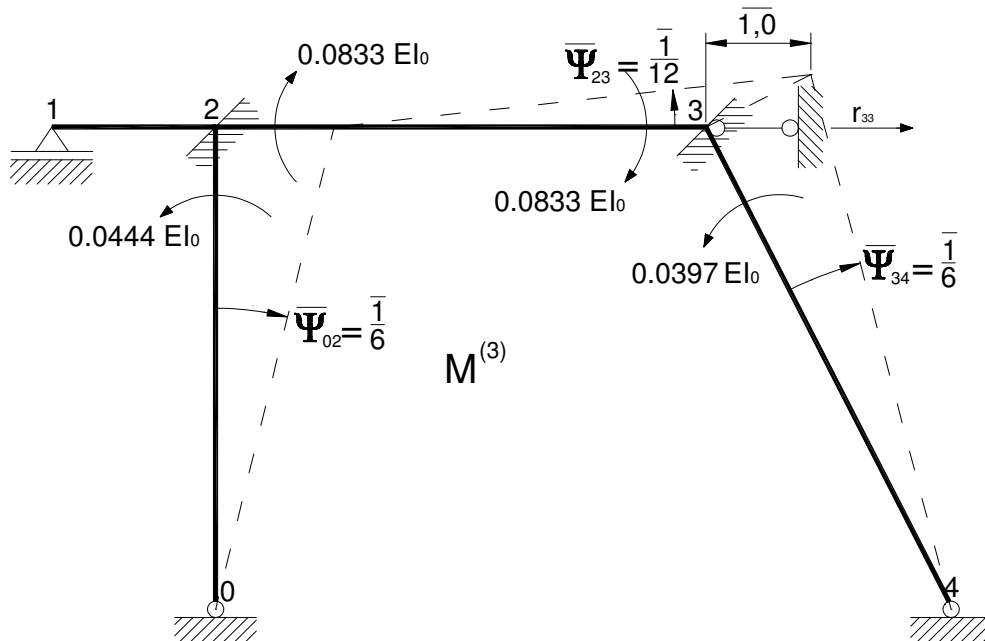
$$r_{13} = -0.0444 + 0.0833$$

$$r_{13} = 0.0389 EI_0$$



$$r_{23} = 0.0833 - 0.0397$$

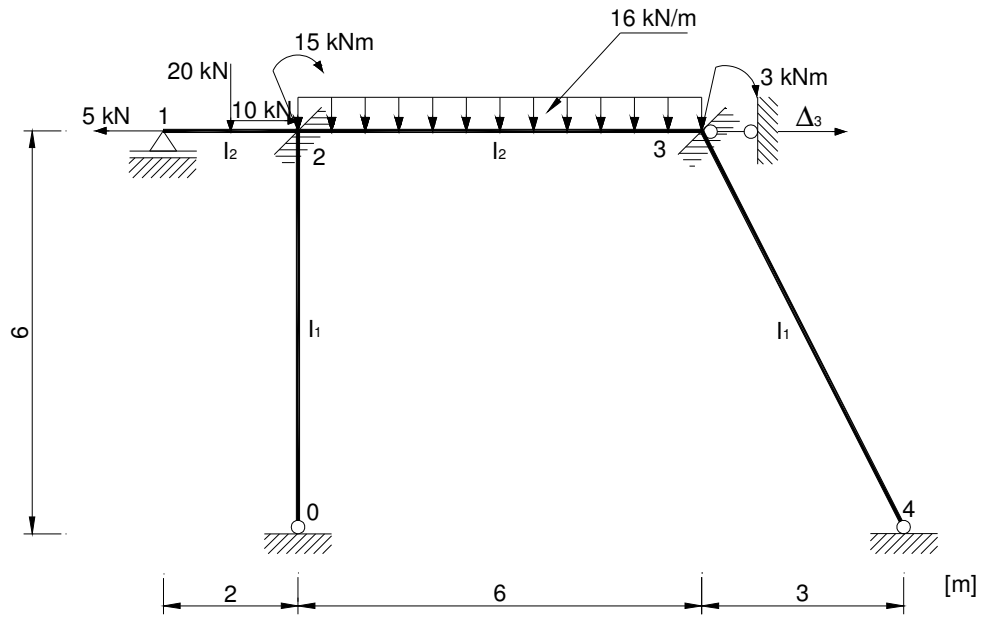
$$r_{23} = 0.0436 EI_0$$



R.P.W.

$$\bar{1},0 \cdot r_{33} - 0.0444 EI_0 \cdot \frac{\bar{1},0}{6} - (0.0833 + 0.0833) EI_0 \cdot \frac{\bar{1},0}{12} - 0.0397 EI_0 \cdot \frac{\bar{1},0}{6} = 0 \quad \rightarrow \quad r_{33} = 0.0279 EI_0$$

Stan „P”



$$M_{12} = 0$$

$$M_{21} = \frac{3}{16}Pl = \frac{3}{16} \cdot 20 \cdot 2 = 7.50 \text{ kNm}$$

$$M_{23} = -\frac{ql^2}{12} = \frac{16 \cdot 6^2}{12} = -48.00 \text{ kNm}$$

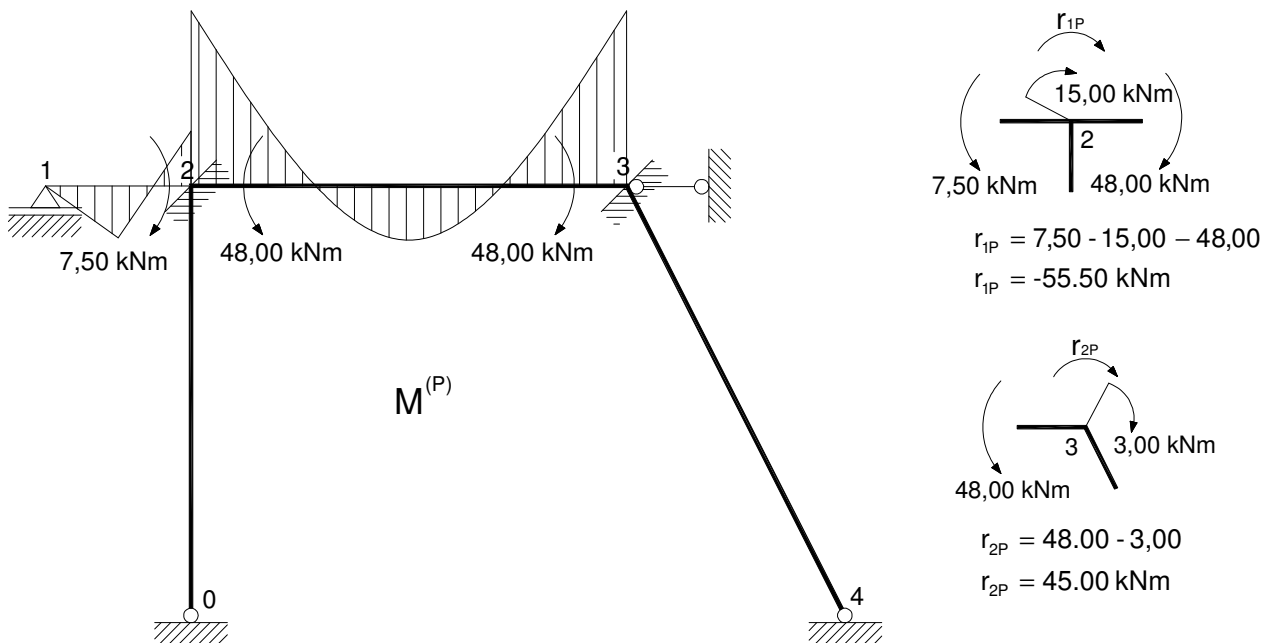
$$M_{32} = \frac{ql^2}{12} = \frac{16 \cdot 6^2}{12} = 48.00 \text{ kNm}$$

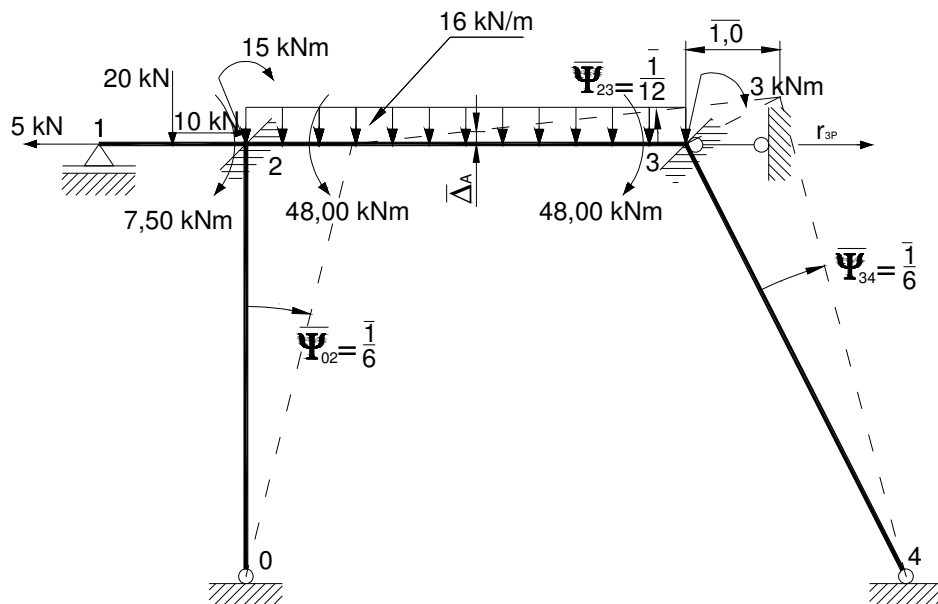
$$M_{34} = 0$$

$$M_{43} = 0$$

$$M_{02} = 0$$

$$M_{20} = 0$$





$$\downarrow 023 \quad 3 \cdot \bar{\Psi}_{23} = \bar{\Delta}_A \Rightarrow \bar{\Delta}_A = 3 \cdot \frac{1,0}{12} \Rightarrow \bar{\Delta}_A = \frac{1,0}{4}$$

R.P.W.

$$1,0 \cdot r_{3P} + (48,0 - 48,0) \cdot \frac{1,0}{12} - 6 \cdot 16 \cdot \bar{\Delta}_A - 5,0 \cdot 1,0 + 10 \cdot 1,0 = 0 \Rightarrow r_{3P} = 19,00 \text{ kN}$$

ROZWIĄZANIE UKŁADU RÓWNAŃ KANONICZNYCH

$$\begin{cases} r_{11} \cdot z_1 + r_{12} \cdot z_2 + r_{13} \cdot z_3 + r_{1P} = 0 \\ r_{21} \cdot z_1 + r_{22} \cdot z_2 + r_{23} \cdot z_3 + r_{2P} = 0 \\ r_{31} \cdot z_1 + r_{32} \cdot z_2 + r_{33} \cdot z_3 + r_{3P} = 0 \end{cases}$$

$$\begin{cases} 2.4332EI_0 \cdot z_1 + 0.3333EI_0 \cdot z_2 + 0.0389EI_0 \cdot z_3 = 55.50 \\ 0.3333EI_0 \cdot z_1 + 0.9051EI_0 \cdot z_2 + 0.0436EI_0 \cdot z_3 = -45.00 \\ 0.0389EI_0 \cdot z_1 + 0.0436EI_0 \cdot z_2 + 0.0279EI_0 \cdot z_3 = -19.00 \end{cases}$$

$$\begin{aligned} z_1 &= \frac{37.9757}{EI_0} \Rightarrow \varphi_2 = \frac{37.9757}{EI_0} \\ z_2 &= \frac{-30.6778}{EI_0} \Rightarrow \varphi_3 = \frac{-30.6778}{EI_0} \\ z_3 &= \frac{-685.6412}{EI_0} \Rightarrow \Delta_3 = \frac{-685.6412}{EI_0} \end{aligned}$$

WYZNACZENIE WARTOŚCI MOMENTÓW ZGINAJĄCYCH – METODA SUPERPOZYCJI:

$$M_p^{(n)} = M_1 \cdot z_1 + M_2 \cdot z_2 + M_3 \cdot z_3 + M_p$$

$$M_{12} = 0$$

$$M_{21} = 1.50EI_0 \cdot \frac{37.9757}{EI_0} + 0 \cdot \frac{-30.6778}{EI_0} + 0 \cdot \frac{-685.6412}{EI_0} + 7.50 = 64.4635 \text{ kNm}$$

$$M_{23} = 0.6667EI_0 \cdot \frac{37.9757}{EI_0} + 0.3333EI_0 \cdot \frac{-30.6778}{EI_0} + 0.0833EI_0 \cdot \frac{-685.6412}{EI_0} - 48.00 = -90.0456 \text{ kNm}$$

$$M_{32} = 0.3333El_0 \cdot \frac{37.9757}{El_0} + 0.6667El_0 \cdot \frac{-30.6778}{El_0} + 0.0833El_0 \cdot \frac{-685.6412}{El_0} + 48.00 = -16.9301 \text{ kNm}$$

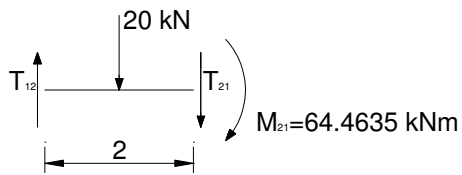
$$M_{34} = 0 \cdot \frac{37.9757}{El_0} + 0.2384El_0 \cdot \frac{-30.6778}{El_0} - 0.0397El_0 \cdot \frac{-685.6412}{El_0} + 0 = 19.9301 \text{ kNm}$$

$$M_{43} = 0$$

$$M_{02} = 0$$

$$M_{20} = 0.2666El_0 \cdot \frac{37.9757}{El_0} + 0 \cdot \frac{-30.6778}{El_0} - 0.0444El_0 \cdot \frac{-685.6412}{El_0} + 0 = 140.5821 \text{ kNm}$$

WYZNACZENIE WARTOŚCI SIŁ TNĄCYCH:



$$\sum M_1 = 0$$

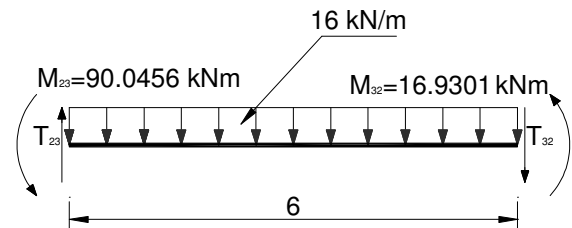
$$20 \cdot 1 + T_{21} \cdot 2 + 64.4635 = 0$$

$$T_{21} = -42.2317 \text{ kN}$$

$$\sum M_2 = 0$$

$$T_{12} \cdot 2 - 20 \cdot 1 + 64.4635 = 0$$

$$T_{12} = -22.2317 \text{ kN}$$



$$\sum M_2 = 0$$

$$16 \cdot 6 \cdot 3 + T_{32} \cdot 6 - 90.0456 - 16.9301 = 0$$

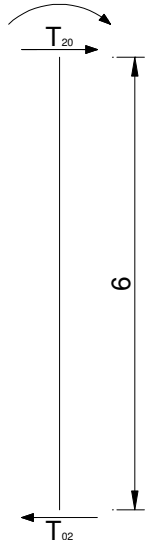
$$T_{32} = -30.1707 \text{ kN}$$

$$\sum M_3 = 0$$

$$T_{23} \cdot 6 - 16 \cdot 6 \cdot 3 - 90.0456 - 16.9301 = 0$$

$$T_{23} = 65.8293 \text{ kN}$$

$$M_{20} = 40.5821 \text{ kNm}$$



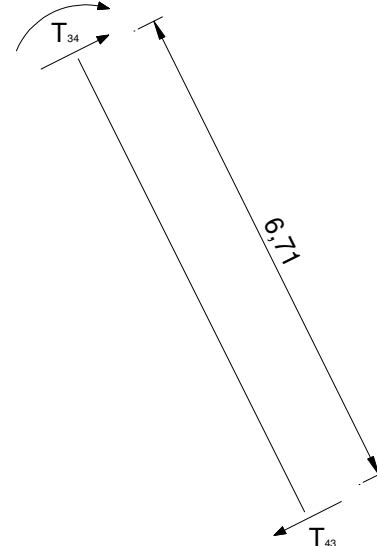
$$\sum M_0 = 0$$

$$T_{20} \cdot 6 + 40.5821 = 0$$

$$T_{20} = -6.764 \text{ kN}$$

$$T_{02} = T_{20} = -6.764 \text{ kN}$$

$$M_{34} = 19.930 \text{ kNm}$$



$$\sum M_3 = 0$$

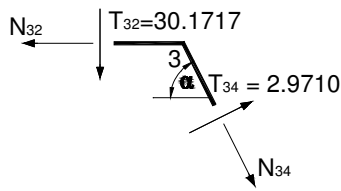
$$T_{43} \cdot 6.7082 + 19.9301 = 0$$

$$T_{43} = -2.9710 \text{ kN}$$

$$T_{34} = T_{43} = -2.9710 \text{ kN}$$

WYZNACZENIE WARTOŚCI SIŁ NORMALNYCH:

$\sin \alpha = 0.8944$; $\cos \alpha = 0.4472$



$\sum Y = 0$

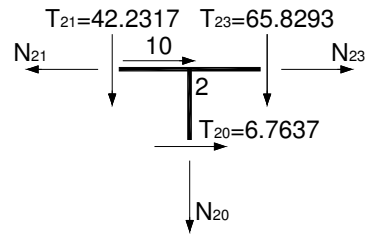
$-N_{34} \cdot \sin \alpha - 30.1707 + 2.9710 \cdot \cos \alpha = 0$

$N_{34} = -32.2464 \text{ kN}$

$\sum X = 0$

$-N_{32} + 2.9710 \cdot \sin \alpha + N_{34} \cdot \cos \alpha = 0$

$N_{32} = -11.7637 \text{ kN}$



$\sum Y = 0$

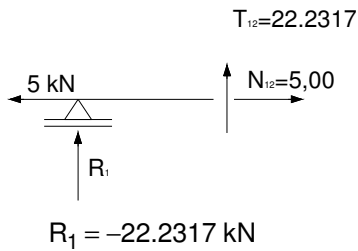
$-N_{20} - 42.2317 - 65.8293 = 0$

$N_{20} = -108.0610 \text{ kN}$

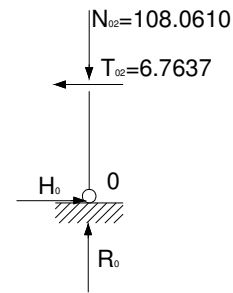
$\sum X = 0$

$-N_{21} + 6.7637 + N_{23} = 0$

$N_{21} = 5.00 \text{ kN}$

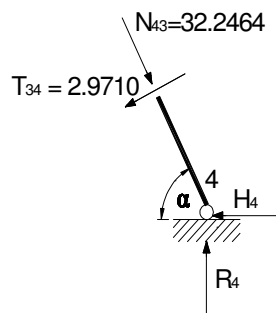


$R_1 = -22.2317 \text{ kN}$



$R_0 = 108.0610 \text{ kN}$

$H_0 = 6.7637 \text{ kN}$



$\sum Y = 0$

$R_4 - 2.9710 \cdot \cos \alpha - 32.2464 \cdot \sin \alpha = 0$

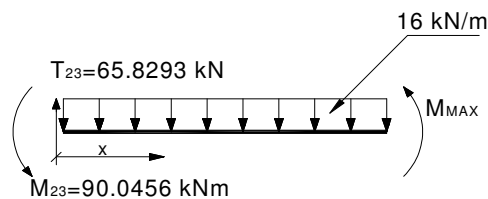
$R_4 = 30.1707 \text{ kN}$

$\sum X = 0$

$-H_4 - 2.9710 \cdot \sin \alpha + 32.2464 \cdot \cos \alpha = 0$

$H_4 = 11.7637 \text{ kN}$

WYZNACZENIE M_{\max}

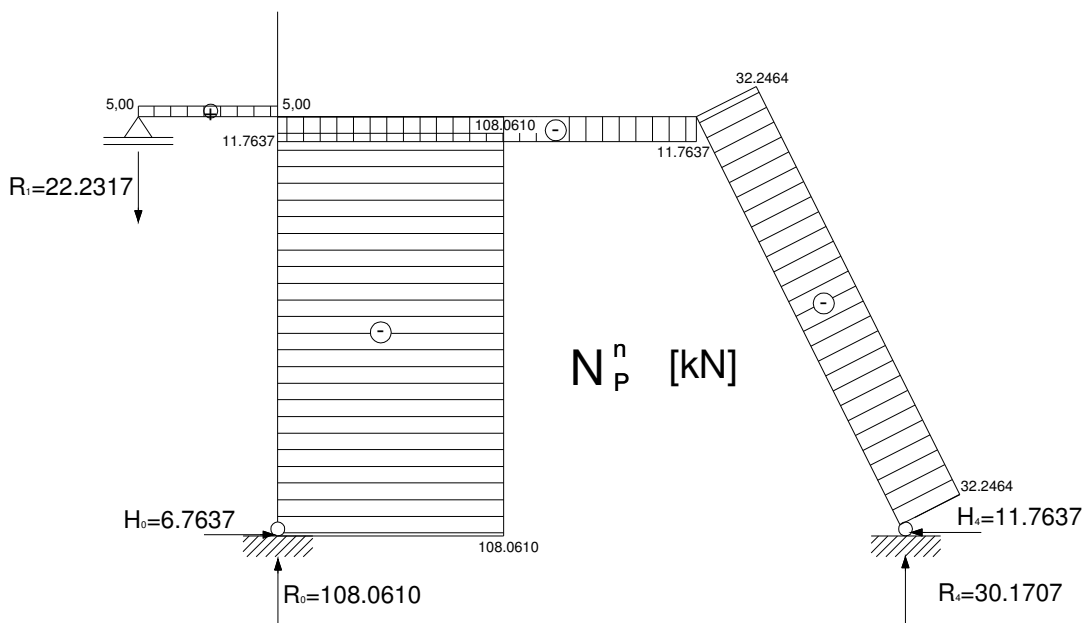
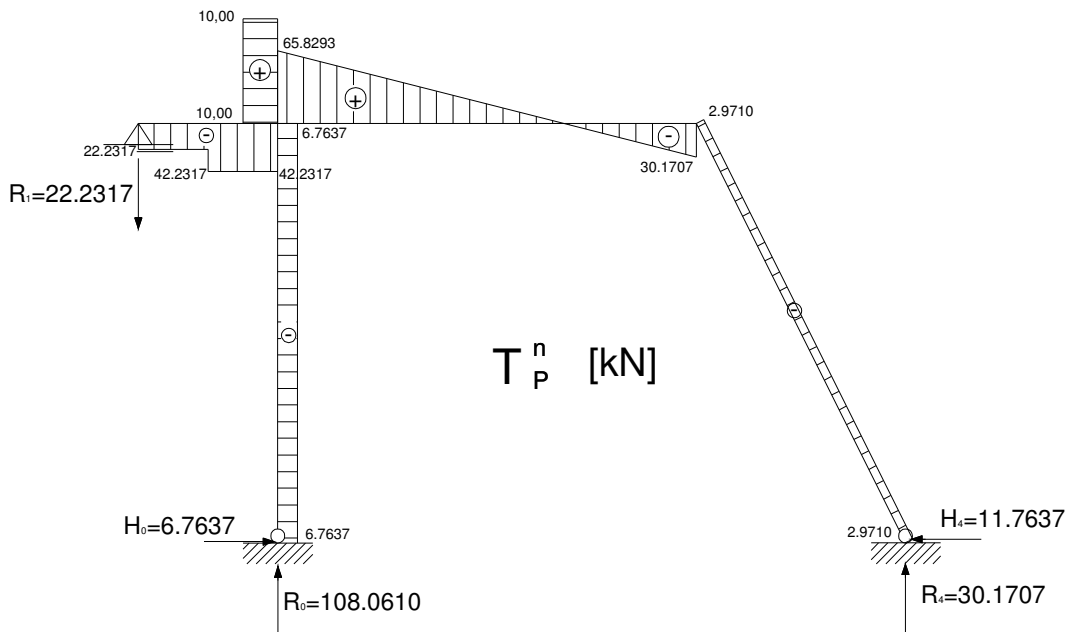
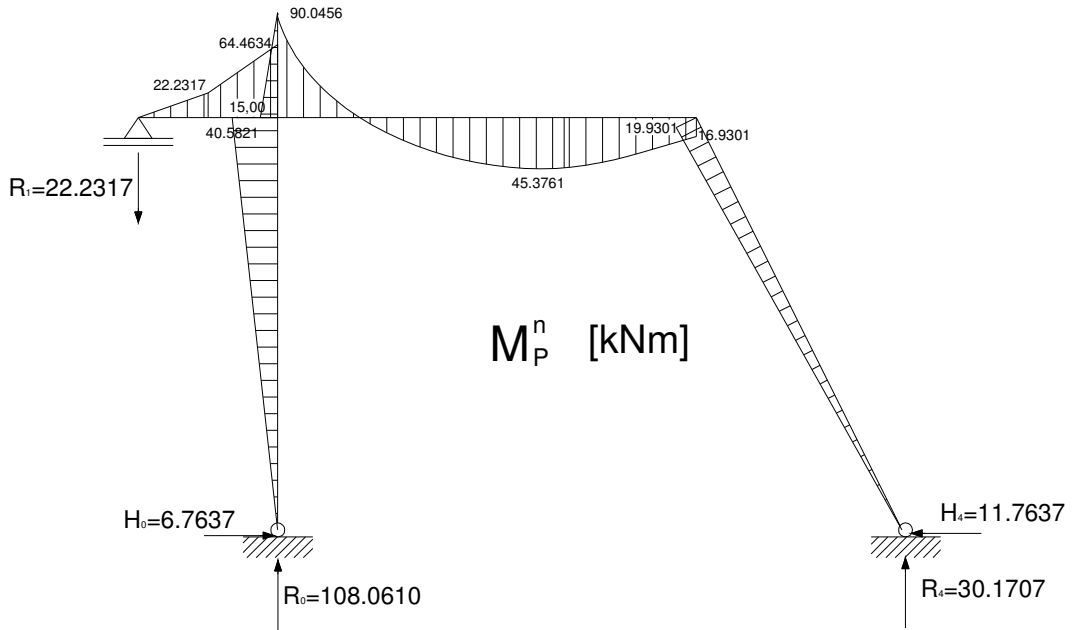


$\sum Y = 0$

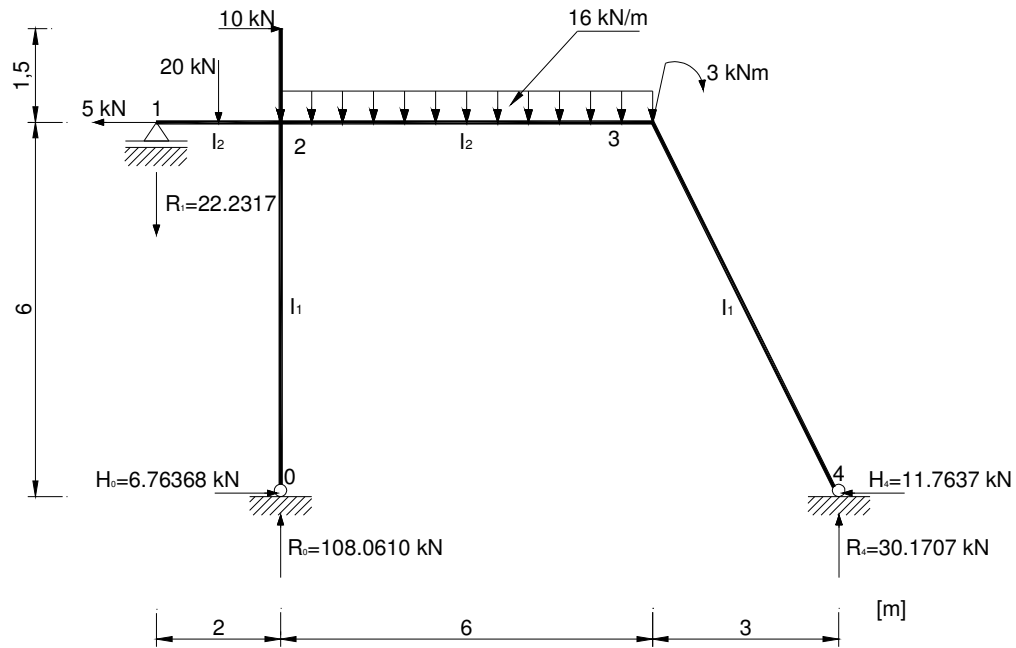
$65.8293 - 16 \cdot x = 0 \Rightarrow x = 45.3761 \text{ m}$

$M_{\max} = -90.0456 + 65.8293 \cdot x - 16 \cdot \frac{x^2}{2}$

$M_{\max} = 45.3761 \text{ kNm}$

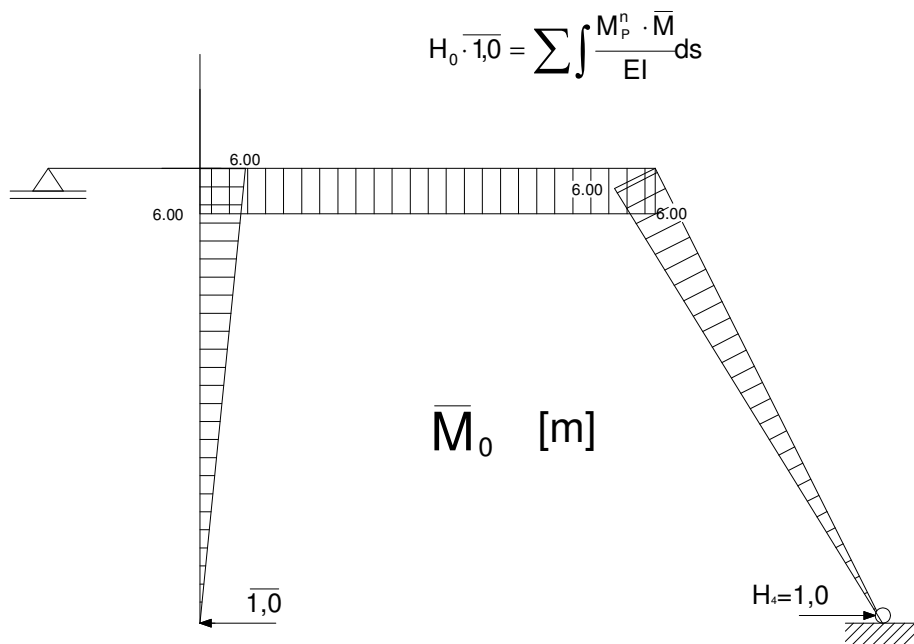


KONTROLA STATYCZNA



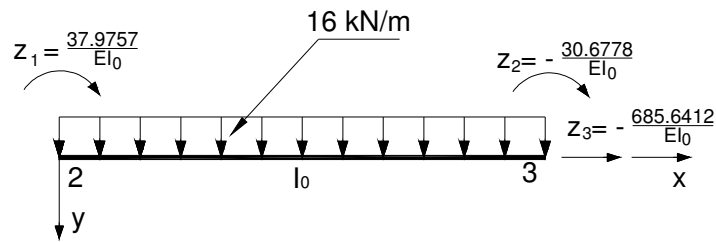
$$\begin{aligned} \sum X = 0 & \quad -5 + 10 + 6.7637 - 11.7637 = 0 \quad \Rightarrow \quad \underline{0 = 0} \\ \sum Y = 0 & \quad -22.2317 - 20 + 108.0610 - 16 \cdot 6 + 30.1707 = 0 \quad \Rightarrow \quad \underline{0 = 0} \\ \sum M_0 = 0 & \quad -5 \cdot 6 - 22.2317 \cdot 2 - 20 \cdot 1 + 10 \cdot 7.5 + 16 \cdot 6 \cdot 3 + 3 - 30.1707 \cdot 9 = 0 \quad \Rightarrow \quad \underline{0 = 0} \end{aligned}$$

KONTROLA KINEMATYCZNA



$$H_0 \cdot \bar{1,0} = \sum \int \frac{M_p^n \cdot \bar{M}}{EI} ds$$

$$\begin{aligned} H_0 \cdot \bar{1,0} &= \frac{1}{EI_0} \left[-\frac{1}{2} \cdot 6 \cdot 90.0456 \cdot 6 + \frac{1}{2} \cdot 6 \cdot 16.9301 \cdot 6 + \frac{2}{3} \cdot 6 \cdot \frac{16 \cdot 6^2}{8} \cdot 6 \right] + \\ &+ \frac{1}{0.5331EI_0} \left[-\frac{1}{2} \cdot 6 \cdot 40.5821 \cdot \frac{2}{3} \cdot 6 + \frac{1}{2} \cdot 6.7082 \cdot 19.9301 \cdot \frac{2}{3} \cdot 6 \right] \\ H_0 \cdot \bar{1,0} &= \frac{411.920613}{EI_0} + \frac{-219.595}{0.5331EI_0} = 0.0350 - 0.0350 = 1.38778E - 17 \approx 0,0 \end{aligned}$$

RÓWNANIE RÓŻNICZKOWE LINII UGIĘCIARównanie różniczkowe linii ugięcia dla pręta 2-3, $q(x) = 16 \text{ kN/m}$ 

$$EI_0 \frac{d^4 y}{dx^4} = q(x) \Rightarrow EI_0 \frac{d^4 y}{dx^4} = 16$$

$$EI_0 \frac{d^3 y}{dx^3} = -T(x) \Rightarrow EI_0 \frac{d^3 y}{dx^3} = 16x + C$$

$$EI_0 \frac{d^2 y}{dx^2} = -M(x) \Rightarrow EI_0 \frac{d^2 y}{dx^2} = \frac{16}{2} x^2 + Cx + D$$

$$EI_0 \frac{dy}{dx} = \varphi(x) \Rightarrow EI_0 \frac{dy}{dx} = \frac{16}{2 \cdot 3} x^3 + \frac{1}{2} Cx^2 + Dx + E$$

$$EI_0 y = w(x) \Rightarrow EI_0 y = \frac{16}{2 \cdot 3 \cdot 4} x^4 + \frac{1}{2 \cdot 3} Cx^3 + \frac{1}{2} Dx^2 + Ex + F$$

Warunki brzegowe:

$$1. \quad x=0 \quad \varphi_2 = z_1 \quad z_1 = \frac{37.9757}{EI_0}$$

$$2. \quad x=0 \quad y_2 = 0$$

$$3. \quad x=l_0 \quad \varphi_3 = z_2 \quad z_2 = \frac{-30.6778}{EI_0}$$

$$4. \quad x=l_0 \quad y_3 = \Psi_{23} \cdot 6$$

$$\Psi_{23} = -\frac{z_3}{12} \Rightarrow \Psi_{23} = \frac{-57.1368}{EI_0}$$

$$y_3 = \frac{-57.1368}{EI_0} \cdot 6 \Rightarrow y_3 = \frac{-342.8206}{EI_0}$$

$$37.9757 = \frac{16}{2 \cdot 3} \cdot 0^3 + \frac{1}{2} C \cdot 0^2 + D \cdot 0 + E \quad \Rightarrow \quad E = 37.9757$$

$$0 = \frac{16}{2 \cdot 3 \cdot 4} 0^4 + \frac{1}{2 \cdot 3} C \cdot 0^3 + \frac{1}{2} D \cdot 0^2 + E \cdot 0 + F \quad \Rightarrow \quad F = 0$$

$$-30.6778 = \frac{16}{2 \cdot 3} 6^3 + \frac{1}{2} \cdot 6^2 \cdot C + 6 \cdot D + 37.9757 \quad \Rightarrow \quad 18C + 6D = -644.6534$$

$$-342.8206 = \frac{16}{2 \cdot 3 \cdot 4} \cdot 6^4 + \frac{1}{2 \cdot 3} \cdot 6^3 \cdot C + \frac{1}{2} \cdot 6^2 D + 37.9757 \cdot 6 + 0 \quad \Rightarrow \quad 36C + 18D = -749.0333$$

$$18C + 6D = -644.6534 \quad \Rightarrow \quad C = -65.8293$$

$$36C + 18D = -749.0333 \quad \Rightarrow \quad D = 90.04559$$

$$M(0) = -\left(\frac{16}{2} \cdot 0^2 - 65.8293 \cdot 0 + 90.04559\right) = -90.0456 \text{ kNm}$$

- rozciągnięte włókna górne

$$M(6) = -\left(\frac{16}{2} \cdot 6^2 - 65.8293 \cdot 6 + 90.04559\right) = 16.9301 \text{ kNm}$$

- rozciągnięte włókna dolne

Znakowanie wg zasady metody przemieszczeń

 $M_{23} = -90.0456 \text{ kNm}$ - rozciągnięte włókna górne $M_{32} = -16.9301 \text{ kNm}$ - rozciągnięte włókna dolne

$$T(0) = -(16 \cdot 0 - 65.8293) = 65.8293 \text{ kN}$$

$$T_{23} = 65.8293 \text{ kN}$$

$$T(6) = -(16 \cdot 6 - 65.8293) = -30.17072 \text{ kN}$$

$$T_{32} = -30.1707 \text{ kN}$$

SPRAWDZENIE NAPRĘŻEŃ NORMALNYCH WYWOŁANYCH MOMENTEM ZGINAJĄCYM

$$\sigma_{\max} \leq \sigma_{\text{dop}}$$

$$\sigma_{\max} = \frac{M_{\max}}{W} \leq \sigma_{\text{dop}} = 205 \text{ MPa}$$

Pręty grupy 1 I220 ($W_x = 278.18 \text{ cm}^3, I_x = 3060 \text{ cm}^4$)

$$M_{\max} = 40.5821 \text{ kNm}$$

$$\sigma_{\max} = \frac{4058.21}{278.18} = 14.5884 \text{ kN/cm}^2$$

$$\sigma_{\max} = 14.5884 \text{ kN/cm}^2 \leq \sigma_{\text{dop}} = 20,5 \text{ kN/cm}^2$$

Pręty grupy 2 I260 ($W_x = 441.54 \text{ cm}^3, I_x = 5740 \text{ cm}^4$)

$$M_{\max} = 90.0456 \text{ kNm}$$

$$\sigma_{\max} = \frac{9004.56}{441.54} = 20.3935 \text{ kN/cm}^2$$

$$\sigma_{\max} = 20.3935 \text{ kN/cm}^2 \leq \sigma_{\text{dop}} = 20,5 \text{ kN/cm}^2$$