POLITECHNIKA POZNAŃSKA
INSTYTUT KONSTRUKCJI BUDOWLANYCH
Zakład Mechaniki Budowli

ĆWICZENIE nr 2

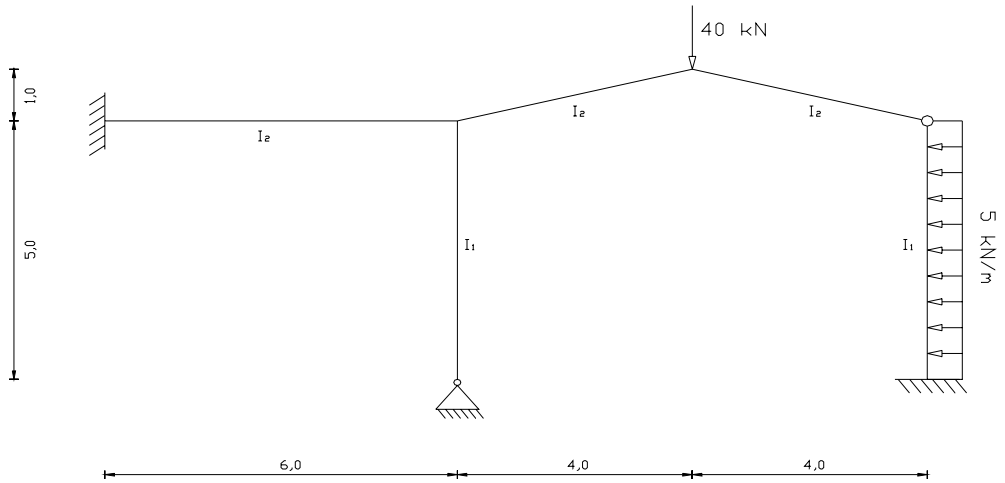
OBLICZANIE RAMY METODĄ PRZEMIESZCZEŃ

Prowadzący: **mgr inż. A. Kaczor**

Wykonał: **Piotr Matysiak**
Grupa 1
Rok akad. 2003/2004

STUDIUM ZAOCZNE

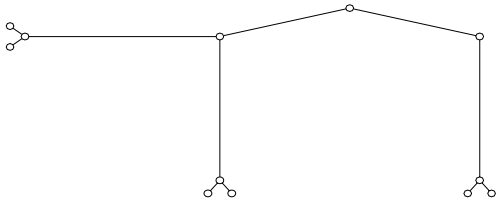
2. *Schemat konstrukcji.*



3. *Stopień geometrycznej niewyznaczalności układu.*

SGN = ∑Δ + ∑φ, gdzie: ∑Δ – łączna liczba niezależnych przesunięć węzłów,
∑φ – łączna liczba obrotów węzłów sztywnych

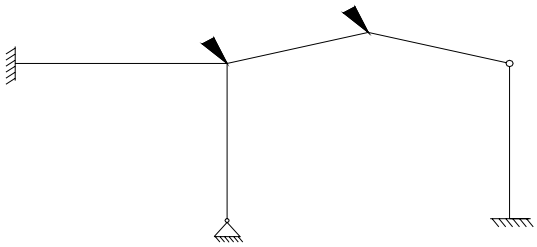
3.1.Określenie liczby przesunięć węzłów – łańcuch kinematyczny:



∑Δ = 2 · w – p, gdzie: w – liczba węzłów; w = 6
p – liczba prętów; p = 11

∑Δ = 2 · 6 – 11 = 1

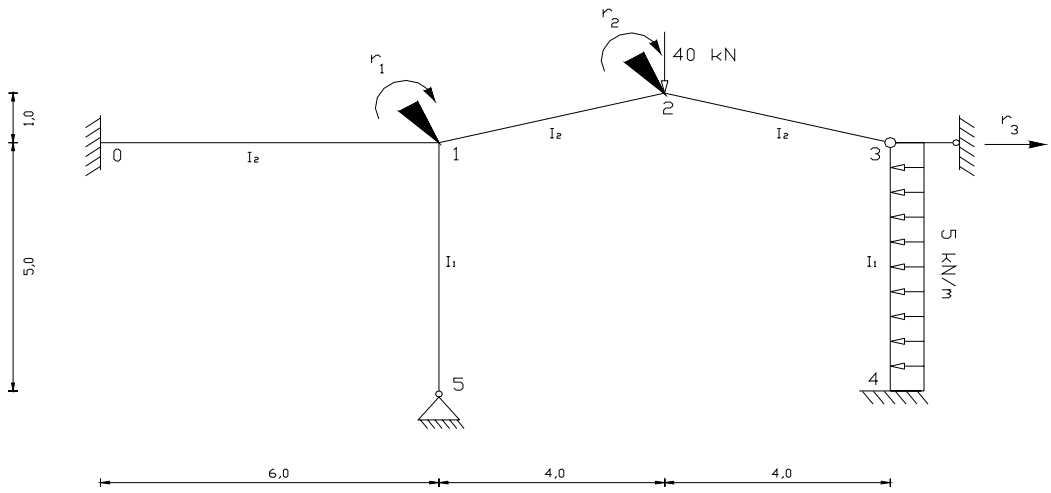
3.2.Określenie liczby obrotów węzłów sztywnych:



∑φ = 2

SGN = 1 + 2 = 3

3.3.Układ podstawowy:



3.4.Układ równań kanonicznych:

$r_{11} \cdot z_1 + r_{12} \cdot z_2 + r_{13} \cdot z_3 + r_{1P} = 0$
$r_{21} \cdot z_1 + r_{22} \cdot z_2 + r_{23} \cdot z_3 + r_{2P} = 0$
$r_{31} \cdot z_1 + r_{32} \cdot z_2 + r_{33} \cdot z_3 + r_{3P} = 0$

Niewiadome: $\varphi_1, \varphi_2, \Delta_3$
Oznaczamy: $z_1 = \varphi_1$
$z_2 = \varphi_2$
$z_3 = \Delta_3$

$r_{11} \cdot \varphi_1 + r_{12} \cdot \varphi_2 + r_{13} \cdot \Delta_3 + r_{1P} = 0$
$r_{21} \cdot \varphi_1 + r_{22} \cdot \varphi_2 + r_{23} \cdot \Delta_3 + r_{2P} = 0$
$r_{31} \cdot \varphi_1 + r_{32} \cdot \varphi_2 + r_{33} \cdot \Delta_3 + r_{3P} = 0$

3.5.Współczynnik porównawczy sztywności:

I_1 – dwuteownik zwykły 220 ($I_x = 3060$ cm^4; W = 278 cm^3);
I_2 – dwuteownik zwykły 260 ($I_x = 5740$ cm^4; W = 442 cm^3);
E – współczynnik sprężystości podłużnej E = 205 GPa

$EI_1 = 205 \cdot 10^9 \cdot 3060 \cdot 10^{-8} = 6273000$ Nm^2 = 6273 kNm^2
$EI_2 = 205 \cdot 10^9 \cdot 5740 \cdot 10^{-8} = 11767000$ Nm^2 = 11767 kNm^2

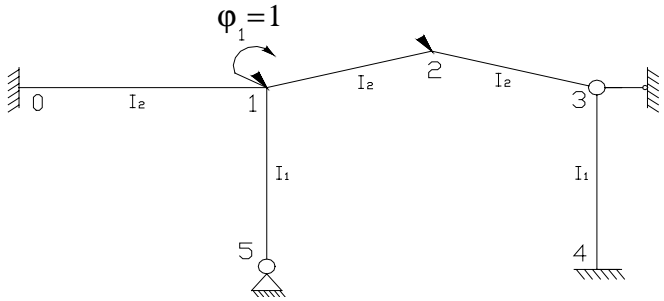
Przyjęto współczynnik porównawczy sztywności $EI_0 = EI_2$

$EI_2 = EI_0 = 11767$ kNm^2, $EI_1 = 0{,}5331\ EI_0$

3.6.Łańcuch kinematyczny:

→ 015	$0 \cdot \Psi_{01} - 5 \cdot \Psi_{15} = 0$	→ $\boldsymbol{\Psi_{15} = 0}$
↑ 015	$6 \cdot \Psi_{01} - 0 \cdot \Psi_{15} = 0$	→ $\boldsymbol{\Psi_{01} = 0}$
→ 43	$5 \cdot \Psi_{34} = \Delta$	→ $\boldsymbol{\Psi 34 = \Delta/5}$
↑ 51234	$0 \cdot \Psi_{15} + 4 \cdot \Psi_{12} + 4 \cdot \Psi_{23} + 0 \cdot \Psi_{34} = 0$	→ $\boldsymbol{\Psi_{12} = -\Psi_{23}}$
→ 5123	$5 \cdot \Psi_{15} + 1 \cdot \Psi_{12} - 1 \cdot \Psi_{23} = \Delta$	
	$5 \cdot 0 - \Psi_{23} - \Psi_{23} = \Delta$	→ $\boldsymbol{\Psi_{23} = -\Delta/2}$
		→ $\boldsymbol{\Psi_{12} = \Delta/2}$

4. *Stan* $\varphi_1 = 1$.



$$M_{01} = \frac{2EI_2}{l}\cdot(2\cdot\varphi_0 + \varphi_1 - 3\cdot\psi_{01}) = \frac{2EI_0}{6}\cdot(2\cdot 0 + 1 - 3\cdot 0) = \frac{1}{3}EI_0$$

$$M_{10} = \frac{2EI_2}{l}\cdot(2\cdot\varphi_1 + \varphi_0 - 3\cdot\psi_{01}) = \frac{2EI_0}{6}\cdot(2\cdot 1 + 0 - 3\cdot 0) = \frac{2}{3}EI_0$$

$$M_{15} = \frac{3EI_1}{l}\cdot(\varphi_1 - \psi_{15}) = \frac{3\cdot 0{,}5331EI_0}{5}\cdot(1-0) = 0{,}31986EI_0$$

$$M_{51} = 0$$

$$M_{12} = \frac{2EI_2}{l}\cdot(2\cdot\varphi_1 + \varphi_2 - 3\cdot\psi_{12}) = \frac{2EI_0}{\sqrt{17}}\cdot(2\cdot 1 + 0 - 3\cdot 0) = 0{,}97014EI_0$$

$$M_{21} = \frac{2EI_2}{l}\cdot(2\cdot\varphi_2 + \varphi_1 - 3\cdot\psi_{12}) = \frac{2EI_0}{\sqrt{17}}\cdot(2\cdot 0 + 1 - 3\cdot 0) = 0{,}48507EI_0$$

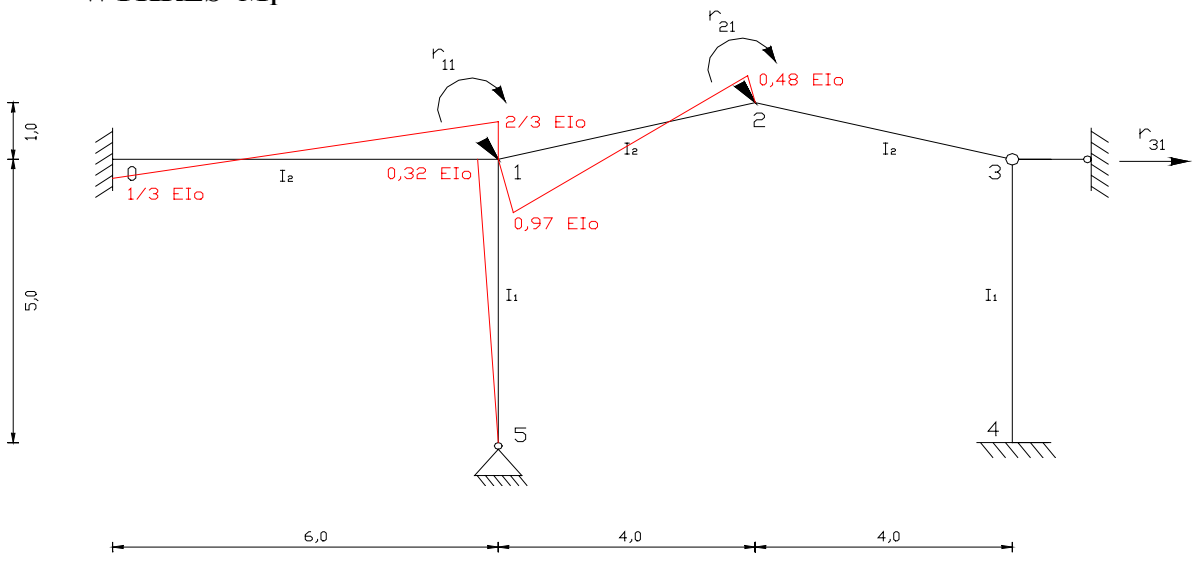
$$M_{23} = \frac{3EI_2}{l}\cdot(\varphi_2 - \psi_{23}) = \frac{3\cdot EI_0}{\sqrt{17}}\cdot(0-0) = 0$$

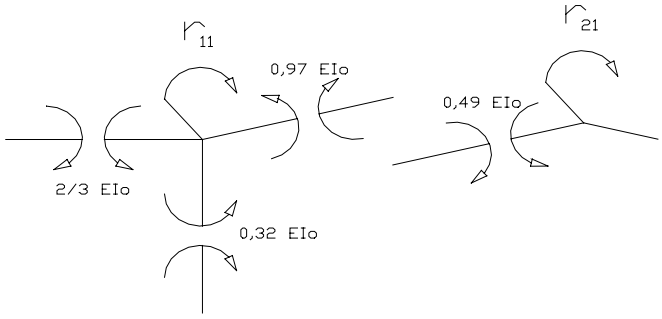
$$M_{32} = 0$$

$$M_{34} = 0$$

$$M_{43} = \frac{3EI_1}{l}\cdot(\varphi_4 - \psi_{34}) = \frac{3\cdot 0{,}5331EI_0}{5}\cdot(0-0) = 0$$

WYKRES M_1





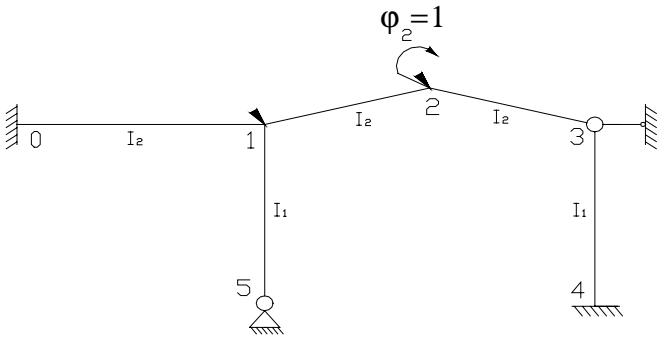
$r_{11} = 0{,}97014EIo + 0{,}31986EIo + 2/3EIo = 1{,}95667EIo$
$r_{21} = 0{,}48507EIo$

$$r_{31} \cdot \bar{1} + (1/3 + 2/3)EI_0 \cdot \psi_{01} + 0{,}31986EI_0 \cdot \psi_{15} + (0{,}97014 + 0{,}48507)EI_0 \cdot \psi_{12} = 0$$

$$r_{31} \cdot \bar{1} + (1/3 + 2/3)EI_0 \cdot 0 + 0{,}31986EI_0 \cdot 0 + (0{,}97014 + 0{,}48507)EI_0 \cdot \frac{1}{2} = 0$$

$$r_{31} = -0{,}727607EI_0$$

5. *Stan $\varphi_2 = 1$.*



$$M_{01} = \frac{2EI_2}{l} \cdot (2 \cdot \varphi_0 + \varphi_1 - 3 \cdot \psi_{01}) = \frac{2EI_0}{6} \cdot (2 \cdot 0 + 0 - 3 \cdot 0) = 0$$

$$M_{10} = \frac{2EI_2}{l} \cdot (2 \cdot \varphi_1 + \varphi_0 - 3 \cdot \psi_{01}) = \frac{2EI_0}{6} \cdot (2 \cdot 0 + 0 - 3 \cdot 0) = 0$$

$$M_{15} = \frac{3EI_1}{l} \cdot (\varphi_1 - \psi_{15}) = \frac{3 \cdot 0{,}5331EI_0}{5} \cdot (0 - 0) = 0$$

$$M_{51} = 0$$

$$M_{12} = \frac{2EI_2}{l} \cdot (2 \cdot \varphi_1 + \varphi_2 - 3 \cdot \psi_{12}) = \frac{2EI_0}{\sqrt{17}} \cdot (2 \cdot 0 + 1 - 3 \cdot 0) = 0{,}48507EI_0$$

$$M_{21} = \frac{2EI_2}{l} \cdot (2 \cdot \varphi_2 + \varphi_1 - 3 \cdot \psi_{12}) = \frac{2EI_0}{\sqrt{17}} \cdot (2 \cdot 1 + 0 - 3 \cdot 0) = 0{,}97014EI_0$$

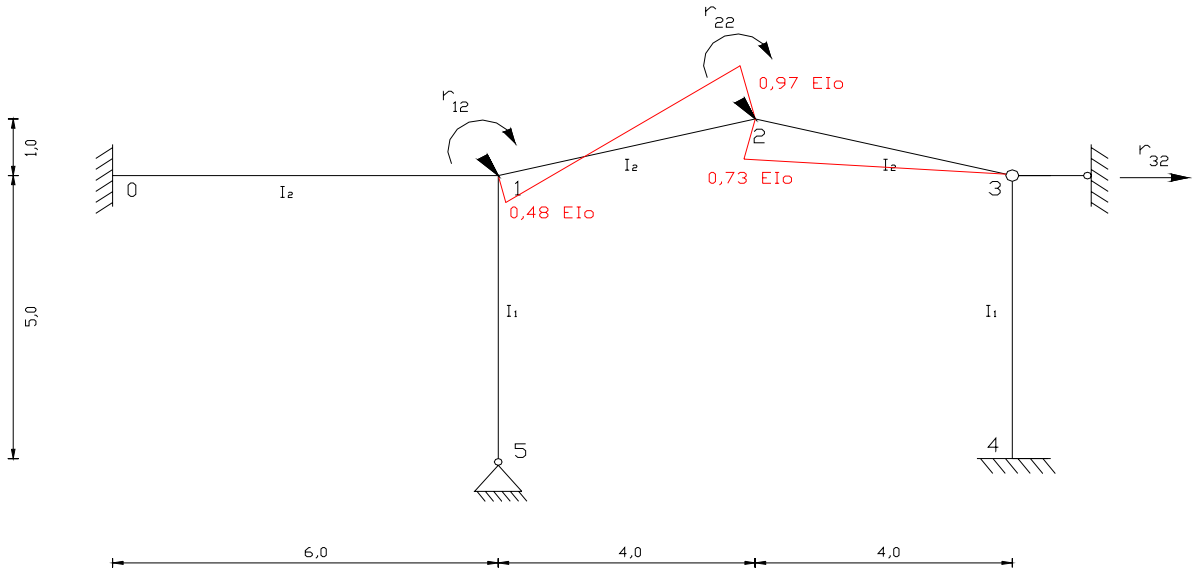
$$M_{23} = \frac{3EI_2}{l} \cdot (\varphi_2 - \psi_{23}) = \frac{3 \cdot EI_0}{\sqrt{17}} \cdot (1 - 0) = 0{,}727607EI_0$$

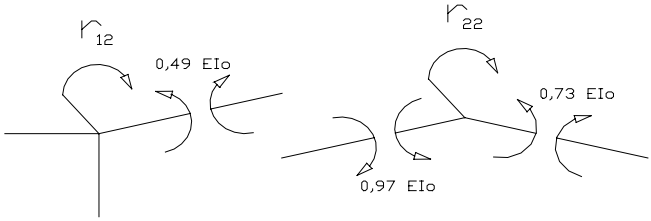
$$M_{32} = 0$$

$$M_{34} = 0$$

$$M_{43} = \frac{3EI_1}{l} \cdot (\varphi_4 - \psi_{34}) = \frac{3 \cdot 0{,}5331EI_0}{5} \cdot (0 - 0) = 0$$

WYKRES M_2





$r_{12} = 0{,}48507EIo$

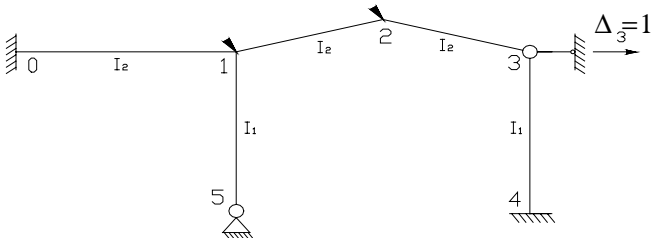
$r_{22} = 0{,}727607EIo + 0{,}97014EIo = 1{,}697749EIo$

$$r_{32} \cdot \bar{1} + (0{,}48507 + 0{,}97014)EI_0 \cdot \psi_{12} + 0{,}727607EI_0 \cdot \psi_{23} = 0$$

$$r_{32} \cdot \bar{1} + (0{,}48507 + 0{,}97014)EI_0 \cdot \frac{\bar{1}}{2} + 0{,}727607EI_0 \cdot \left(-\frac{\bar{1}}{2}\right) = 0$$

$$r_{32} = -0{,}363803EI_0$$

6. *Stan $\Delta_3 = 1$.*



$$M_{01} = \frac{2EI_2}{l} \cdot (2 \cdot \varphi_0 + \varphi_1 - 3 \cdot \psi_{01}) = \frac{2EI_0}{6} \cdot (2 \cdot 0 + 0 - 3 \cdot 0) = 0$$

$$M_{10} = \frac{2EI_2}{l} \cdot (2 \cdot \varphi_1 + \varphi_0 - 3 \cdot \psi_{01}) = \frac{2EI_0}{6} \cdot (2 \cdot 0 + 0 - 3 \cdot 0) = 0$$

$$M_{15} = \frac{3EI_1}{l} \cdot (\varphi_1 - \psi_{15}) = \frac{3 \cdot 0{,}5331EI_0}{5} \cdot (0 - 0) = 0$$

$$M_{51} = 0$$

$$M_{12} = \frac{2EI_2}{l} \cdot (2 \cdot \varphi_1 + \varphi_2 - 3 \cdot \psi_{12}) = \frac{2EI_0}{\sqrt{17}} \cdot \left(2 \cdot 0 + 0 - 3 \cdot \frac{1}{2}\right) = -0{,}727607EI_0$$

$$M_{21} = \frac{2EI_2}{l} \cdot (2 \cdot \varphi_2 + \varphi_1 - 3 \cdot \psi_{12}) = \frac{2EI_0}{\sqrt{17}} \cdot \left(2 \cdot 0 + 0 - 3 \cdot \frac{1}{2}\right) = -0{,}727607EI_0$$

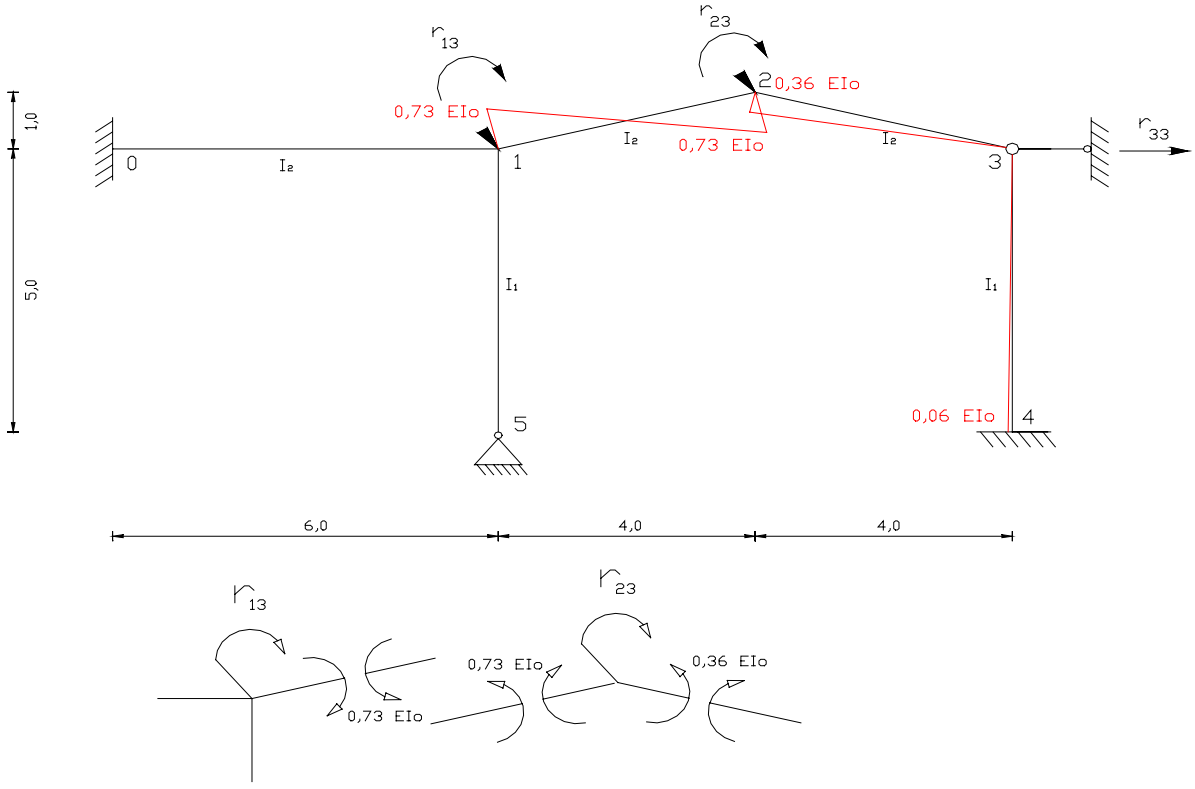
$$M_{23} = \frac{3EI_2}{l} \cdot (\varphi_2 - \psi_{23}) = \frac{3 \cdot EI_0}{\sqrt{17}} \cdot \left(0 + \frac{1}{2}\right) = 0{,}363803EI_0$$

$$M_{32} = 0$$

$$M_{34} = 0$$

$$M_{43} = \frac{3EI_1}{l} \cdot (\varphi_4 - \psi_{34}) = \frac{3 \cdot 0{,}5331EI_0}{5} \cdot \left(0 - \frac{1}{5}\right) = -0{,}063972EI_0$$

WYKRES M_3



r_{13} = -0,727607EIo

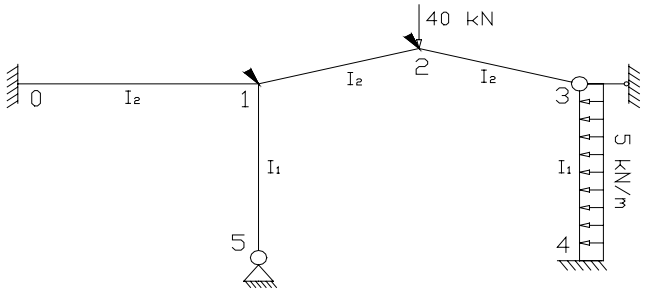
r_{23} = 0,363803EIo – 0,727607EIo = -0,363803

$$r_{33} \cdot \bar{1} + (-0{,}727607 - 0{,}727607)EI_0 \cdot \psi_{12} + 0{,}363803EI_0 \cdot \psi_{23} + (-0{,}063972)EI_0 \cdot \psi_{34} = 0$$

$$r_{33} \cdot \bar{1} + (-0{,}727607 - 0{,}727607)EI_0 \cdot \frac{\bar{1}}{2} + 0{,}363803EI_0 \cdot \left(-\frac{\bar{1}}{2}\right) + (-0{,}063972)EI_0 \cdot \frac{\bar{1}}{5} = 0$$

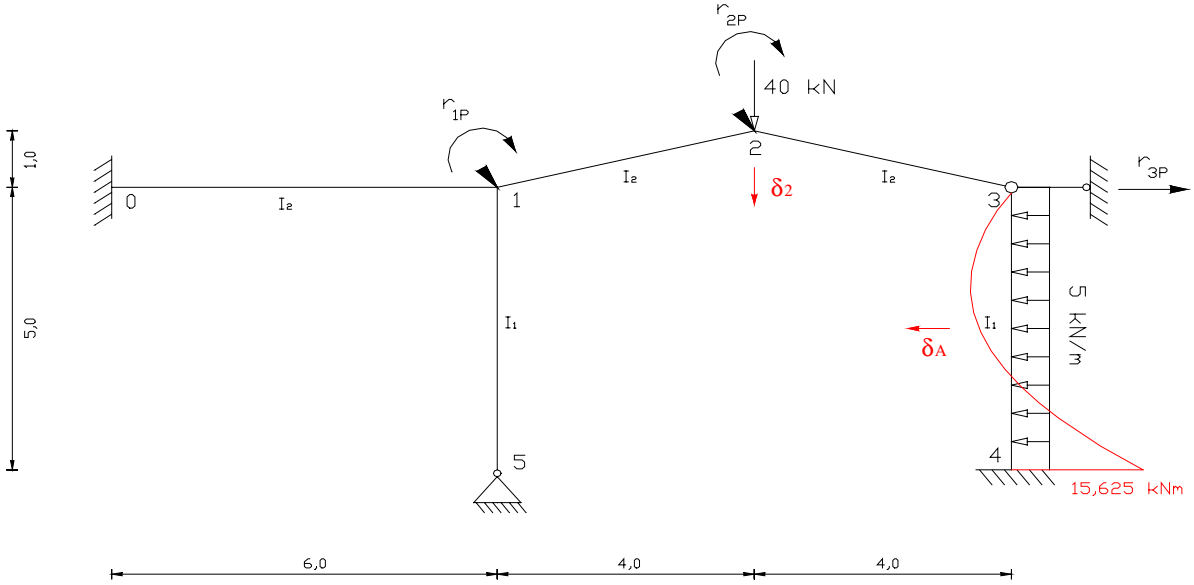
$$r_{33} = 0{,}922303EI_0$$

7. *Stan P.*



$$M_{01} = M_{10} = 0\ kNm$$
$$M_{12} = M_{21} = 0\ kNm$$
$$M_{23} = M_{32} = 0\ kNm$$
$$M_{34} = 0$$
$$M_{43} = \frac{q \cdot l^2}{8} = \frac{5 \cdot 5^2}{8} = 15{,}625\ kNm$$

WYKRES M_P^0



$r_{1P} = 0$ kNm
$r_{2P} = 0$ kNm

↑ 512 $\quad 0 \cdot \Psi_{15} + 4 \cdot \Psi_{12} = -(-\delta_2) \rightarrow \ \delta_2 = 4\Delta/2 = 2\Delta$
→ 4A $\quad 2{,}5 \cdot \Psi_{34} = -\delta_A \ \rightarrow \ \delta_A = 2{,}5\Delta/5 = -0{,}5\Delta$

$$r_{3P} \cdot \bar{1} - (-15{,}625) \cdot \psi_{34} + 5 \cdot 5 \cdot \delta_A + 40 \cdot \delta_2 = 0$$

$$r_{3P} \cdot \bar{1} - (-15{,}625) \cdot \frac{\bar{1}}{5} - 5 \cdot 5 \cdot 0{,}5 \cdot \bar{1} + 40 \cdot 2 \cdot \bar{1} = 0$$

$$r_{3P} = -70{,}625\ kNm$$

8. *Rozwiązanie układu równań kanonicznych.*

$$r_{11} \cdot \varphi_1 + r_{12} \cdot \varphi_2 + r_{13} \cdot \Delta_3 + r_{1P} = 0$$
$$r_{21} \cdot \varphi_1 + r_{22} \cdot \varphi_2 + r_{23} \cdot \Delta_3 + r_{2P} = 0$$
$$r_{31} \cdot \varphi_1 + r_{32} \cdot \varphi_2 + r_{33} \cdot \Delta_3 + r_{3P} = 0$$

$$1{,}95667\ EI_0 \cdot \varphi_1 + 0{,}48507\ EI_0 \cdot \varphi_2 - 0{,}72760\ EI_0 \cdot \Delta_3 + 0 = 0$$
$$0{,}48507\ EI_0 \cdot \varphi_1 + 1{,}69774\ EI_0 \cdot \varphi_2 - 0{,}36380\ EI_0 \cdot \Delta_3 + 0 = 0$$
$$-0{,}72760\ EI_0 \cdot \varphi_1 - 0{,}36380\ EI_0 \cdot \varphi_2 + 0{,}92230\ EI_0 \cdot \Delta_3 - 70{,}625 = 0$$

Obliczenia przeprowadzono w arkuszu kalkulacyjnym Microsoft Excel poprzez odwrócenie macierzy.

$$\varphi_1 = \frac{38{,}430476}{EI_0}$$

$$\varphi_2 = \frac{13{,}026492}{EI_0}$$

$$\Delta_3 = \frac{112{,}030817}{EI_0}$$

9. *Wyznaczenie wartości momentów zginających.*

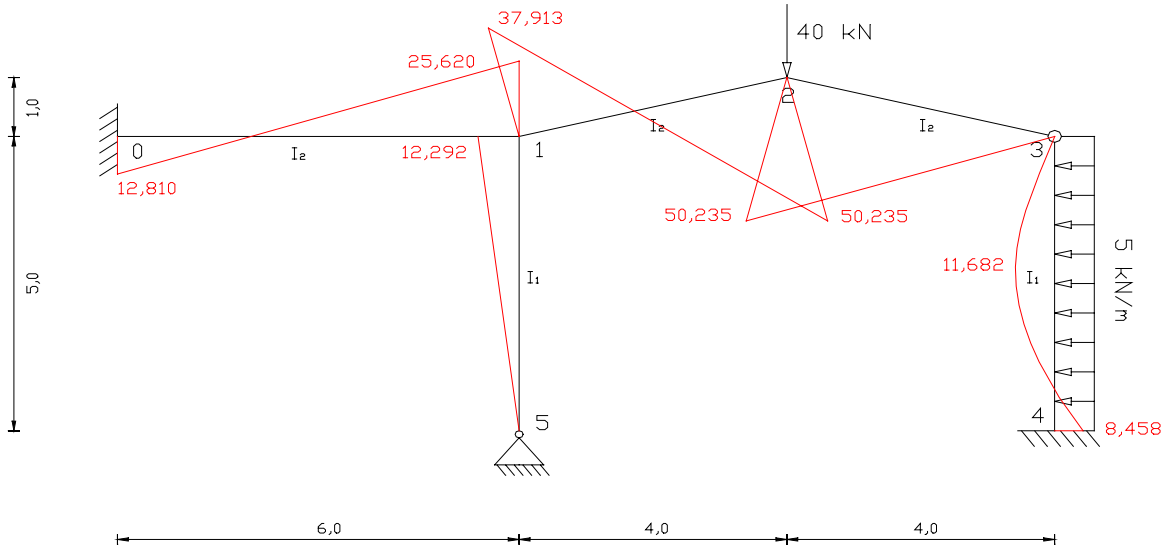
Stosując zasadę superpozycji otrzymamy:

$$M_P^n = M_1 \cdot \varphi_1 + M_2 \cdot \varphi_2 + M_3 \cdot \Delta_3 + M_P^0\ [kNm]$$

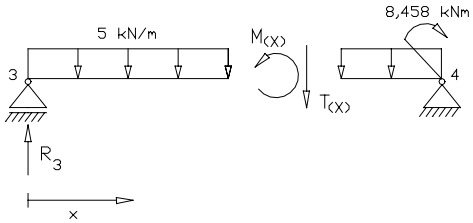
$$M_{01} = \frac{1}{3} EI_0 \cdot \frac{38{,}430}{EI_0} + 0 \cdot \frac{13{,}026}{EI_0} + 0 \cdot \frac{112{,}031}{EI_0} + 0 = 12{,}810\ kNm$$
$$M_{10} = \frac{2}{3} EI_0 \cdot \frac{38{,}430}{EI_0} + 0 \cdot \frac{13{,}026}{EI_0} + 0 \cdot \frac{112{,}031}{EI_0} + 0 = 25{,}620\ kNm$$
$$M_{15} = 0{,}320 EI_0 \cdot \frac{38{,}430}{EI_0} + 0 \cdot \frac{13{,}026}{EI_0} + 0 \cdot \frac{112{,}031}{EI_0} + 0 = 12{,}292\ kNm$$
$$M_{51} = \frac{38{,}430}{EI_0} + 0 \cdot \frac{13{,}026}{EI_0} + 0 \cdot \frac{112{,}031}{EI_0} + 0 = 0\ kNm$$
$$M_{12} = 0{,}970 EI_0 \cdot \frac{38{,}430}{EI_0} + 0{,}485 EI_0 \cdot \frac{13{,}026}{EI_0} - 0{,}728 EI_0 \cdot \frac{112{,}031}{EI_0} + 0 = -37{,}913\ kNm$$
$$M_{21} = 0{,}485 EI_0 \cdot \frac{38{,}430}{EI_0} + 0{,}970 EI_0 \cdot \frac{13{,}026}{EI_0} - 0{,}728 EI_0 \cdot \frac{112{,}031}{EI_0} + 0 = -50{,}235\ kNm$$
$$M_{23} = 0 \cdot \frac{38{,}430}{EI_0} + 0{,}728 EI_0 \cdot \frac{13{,}026}{EI_0} + 0{,}364 EI_0 \cdot \frac{112{,}031}{EI_0} + 0 = 50{,}235\ kNm$$
$$M_{32} = 0\ kNm$$
$$M_{34} = 0\ kNm$$

$$M_{43} = 0 \cdot \frac{38{,}430}{EI_0} + 0 \cdot \frac{13{,}026}{EI_0} - 0{,}064 EI_0 \cdot \frac{112{,}031}{EI_0} + 15{,}625 = 8{,}458\ kNm$$

WYKRES M_P^n [kNm]



Wyznaczenie M_{max}



$\sum M_4 = 0$

$R_3 \cdot 5 - 5 \cdot 5 \cdot 2{,}5 + 8{,}458 = 0$

$R_3 = 10{,}808$ kN

$T_{(X)} = -5x + R_3 = -5x + 10{,}808$

$M_{(X)} = R_3 \cdot x - 5 \cdot x \cdot x/2 = -5/2x^2 + 10{,}808x$

$T_{(Xo)} = 0$

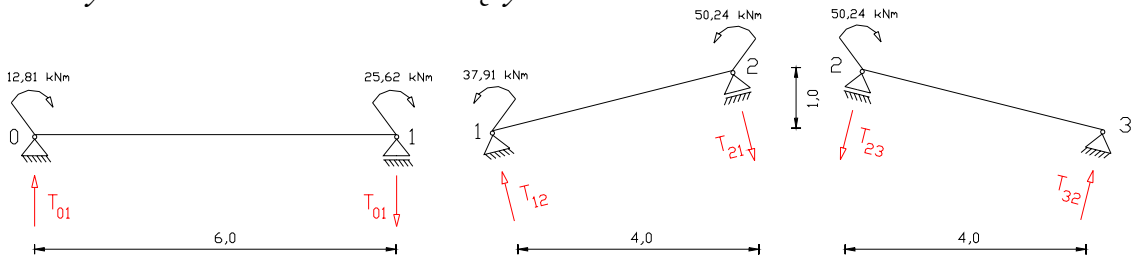
$-5x_o + 10{,}808 = 0$

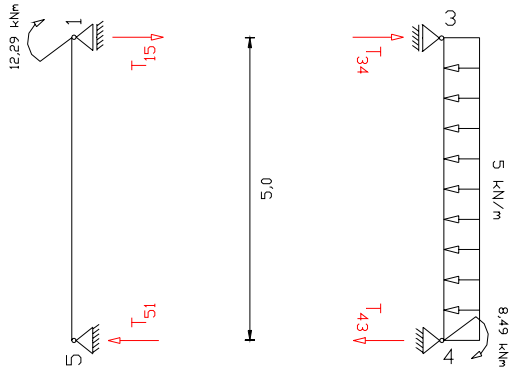
$x_o = 2{,}16$ m

$M_{max} = M_{(Xo)} = -5/2 \cdot 2{,}16^2 + 10{,}808 \cdot 2{,}16$

$M_{max} = 11{,}682$ kNm

10. Wyznaczenie wartości sił tnących.





$$T_{01} = T_{10} = -\frac{12{,}81 + 25{,}62}{6} = -\,6{,}405\ kN$$

$$T_{12} = T_{21} = \frac{37{,}91 + 50{,}24}{\sqrt{17}} = 21{,}379\ kN$$

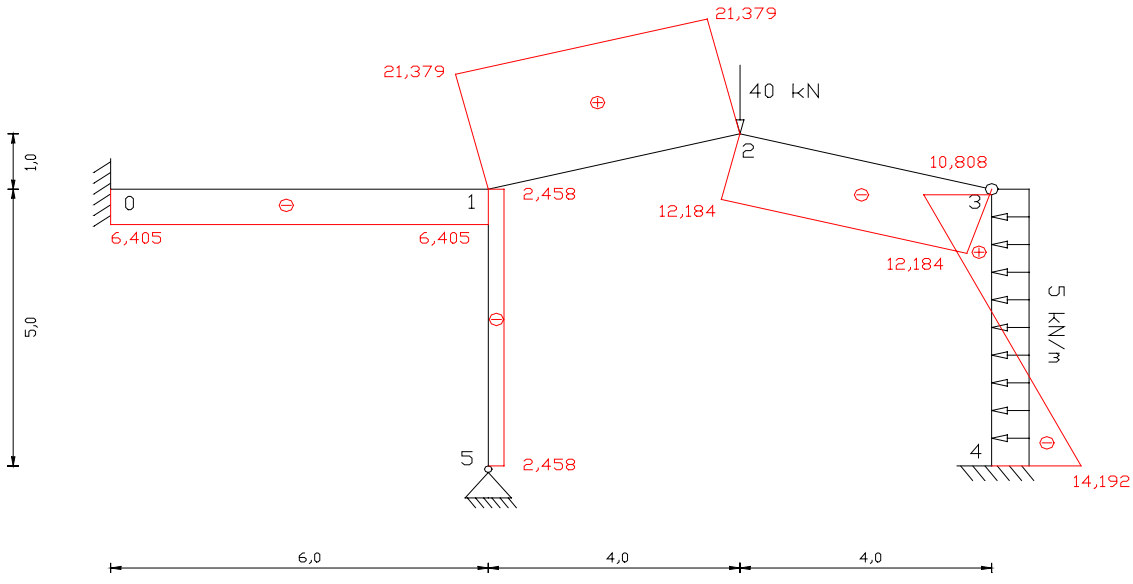
$$T_{23} = T_{32} = -\frac{50{,}24}{\sqrt{17}} = -\,12{,}184\ kN$$

$$T_{15} = T_{51} = -\frac{12{,}29}{5} = -\,2{,}458\ kN$$

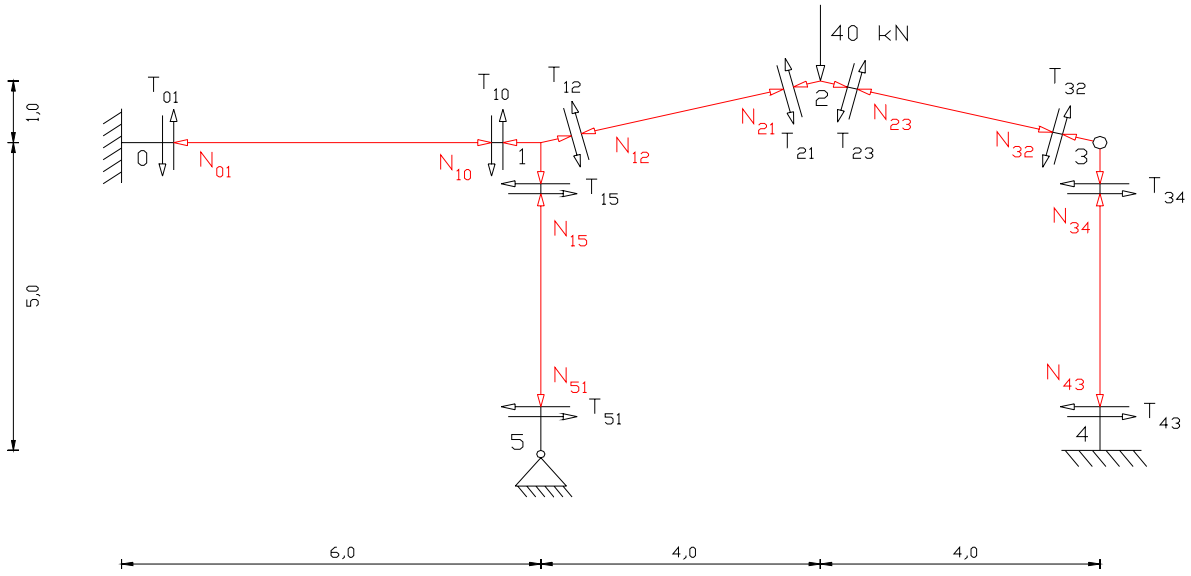
$\sum M_4 = 0 \quad T_{34} \cdot 5 - 5 \cdot 5 \cdot 2{,}5 + 8{,}46 = 0 \quad \rightarrow \quad T_{34} = 10{,}808\ kN$

$\sum M_3 = 0 \quad T_{43} \cdot 5 + 5 \cdot 5 \cdot 2{,}5 + 8{,}46 = 0 \quad \rightarrow \quad T_{43} = -\,14{,}192\ kN$

WYKRES T [kN]



11. Wyznaczenie wartości sił normalnych.



$\sin\alpha = 0{,}242536$; $\cos\alpha = 0{,}970732$

WĘZEŁ 3

$\sum X = 0$ $\quad -N_{32}\cdot\cos\alpha + T_{32}\cdot\sin\alpha - T_{34} = 0$

$N_{32} = -14{,}178$ kN

$\sum Y = 0$ $\quad -N_{34} + N_{32}\cdot\sin\alpha - T_{32}\cdot\cos\alpha = 0$

$N_{34} = -15{,}266$ kN

PRĘT 2-3

$\sum X = 0$ $\quad N_{32} = N_{23} = -14{,}178$ kN (ponieważ $T_{32} = T_{23}$)

PRĘT 3-4

$\sum Y = 0$ $\quad N_{43} = N_{34} = -15{,}266$ kN

WĘZEŁ 2

$\sum X = 0$ $\quad -N_{21}\cdot\cos\alpha - T_{21}\cdot\sin\alpha + N_{23}\cdot\cos\alpha - T_{23}\cdot\sin\alpha = 0$

$N_{21} = -16{,}476$ kN

PRĘT 1-2

$\sum X = 0$ $\quad N_{12} = N_{21} = -16{,}476$ kN (ponieważ $T_{12} = T_{21}$)

WĘZEŁ 1

$\sum X = 0$ $\quad -N_{10} + N_{12}\cdot\cos\alpha + T_{12}\cdot\sin\alpha - T_{15} = 0$

$N_{10} = -8{,}350$ kN

$\sum Y = 0$ $\quad -N_{15} + N_{12}\cdot\sin\alpha - T_{12}\cdot\cos\alpha + T_{10} = 0$

$N_{15} = -31{,}154$ kN

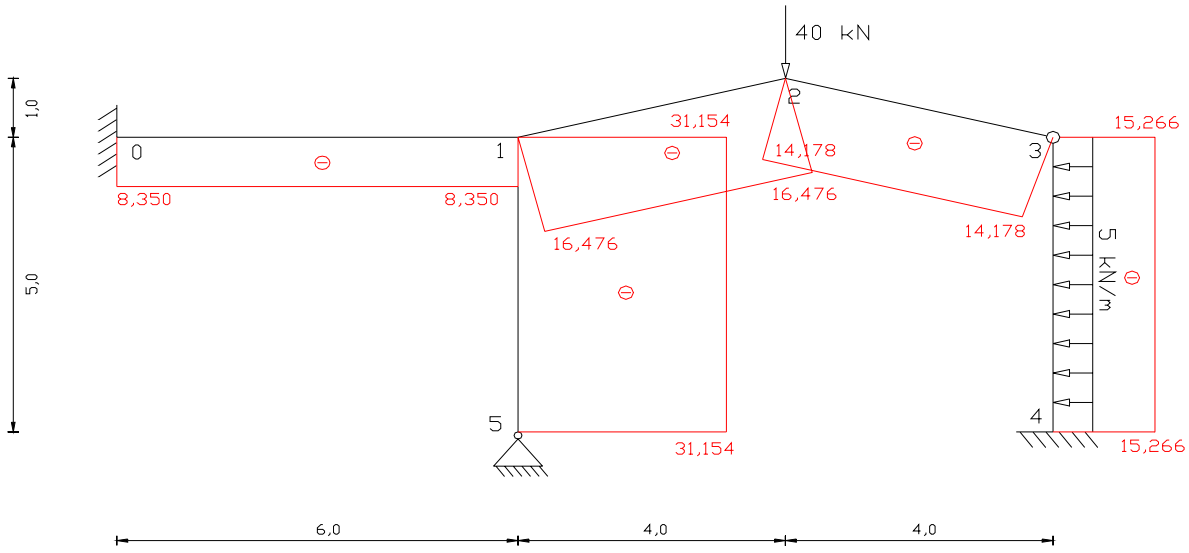
PRĘT 0-1

$\sum X = 0$ $\quad N_{01} = N_{10} = -8{,}350$ kN

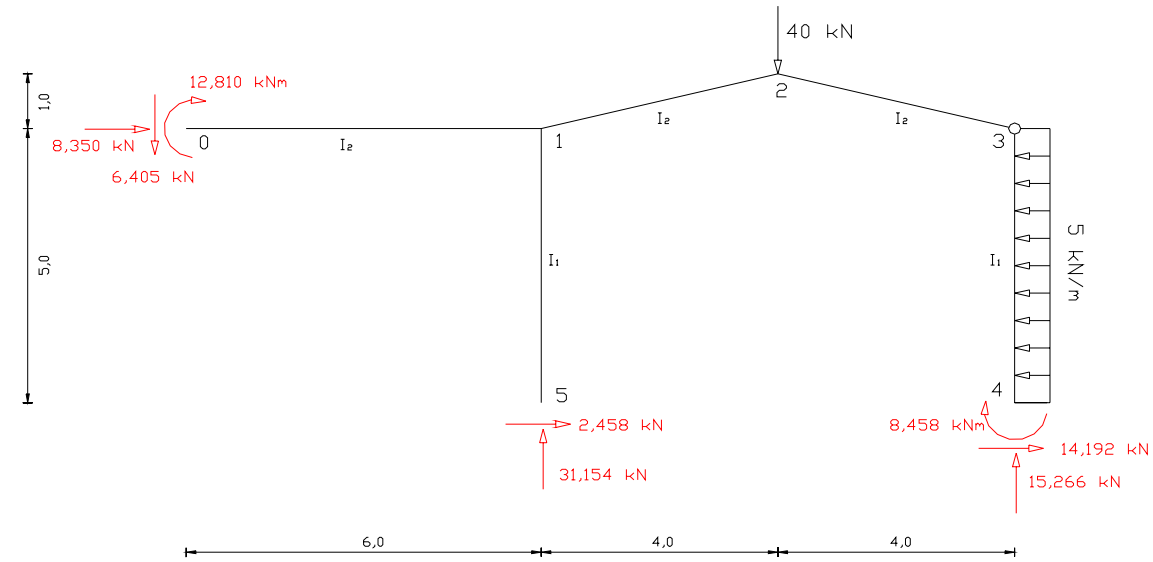
PRĘT 1-5

$\sum Y = 0$ $\quad N_{15} = N_{51} = -31{,}154$ kN

WYKRES N [kN]



12.Kontrola statyczna.



$\sum M_4 = 0$

$8{,}350 \cdot 5 - 6{,}405 \cdot 14 + 12{,}810 + 31{,}154 \cdot 8 - 40 \cdot 4 - 5 \cdot 5 \cdot 2{,}5 + 8{,}458 = 0$

$0{,}0812 \approx 0$

$\sum X = 0$

$8{,}350 + 2{,}458 + 14{,}192 - 5 \cdot 5 = 0$

$0 = 0$

$\sum Y = 0$

$-6{,}405 + 31{,}154 - 40 + 15{,}266 = 0$

$0{,}0153 \approx 0$