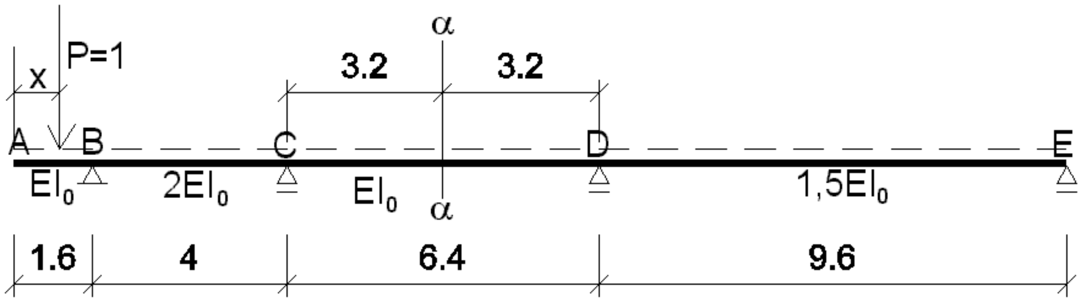
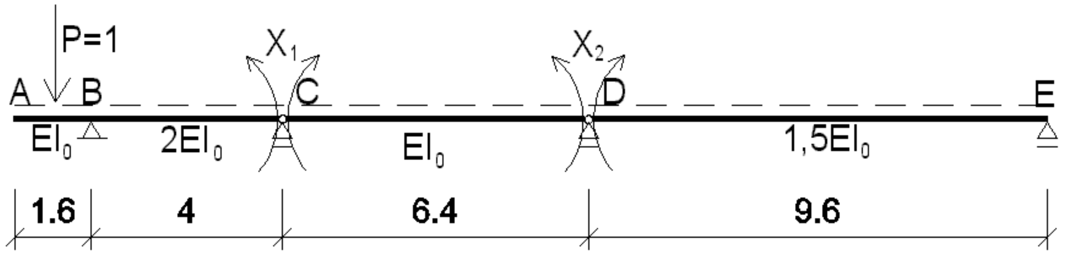
Narysować linie wpływu: R_B, R_C, R_D, R_E, M_α, T_α.



Stopień statycznej niewyznaczalności układu: **SSN=2**

Układ podstawowy:



$$LwR_B = LwR_B^{\ 0} + R_B^{(X_1=1)} \cdot LwX_1 + R_B^{(X_2=1)} \cdot LwX_2$$
$$LwR_C = LwR_C^{\ 0} + R_C^{(X_1=1)} \cdot LwX_1 + R_C^{(X_2=1)} \cdot LwX_2$$
$$LwR_D = LwR_D^{\ 0} + R_D^{(X_1=1)} \cdot LwX_1 + R_D^{(X_2=1)} \cdot LwX_2$$
$$LwR_E = LwR_E^{\ 0} + R_E^{(X_1=1)} \cdot LwX_1 + R_E^{(X_2=1)} \cdot LwX_2$$
$$LwM_\alpha = LwM_\alpha^{\ 0} + M_\alpha^{(X_1=1)} \cdot LwX_1 + M_\alpha^{(X_2=1)} \cdot LwX_2$$
$$LwT_\alpha = LwT_\alpha^{\ 0} + T_\alpha^{(X_1=1)} \cdot LwX_1 + T_\alpha^{(X_2=1)} \cdot LwX_2$$

Wyznaczenie *LwX_1, LwX_2*.

Układ równań kanonicznych:

$$\begin{cases} \delta_{11} \cdot X_1(x) + \delta_{12} \cdot X_2(x) + \delta_{1P}(x) = 0 \\ \delta_{21} \cdot X_1(x) + \delta_{22} \cdot X_2(x) + \delta_{2P}(x) = 0 \end{cases} \qquad \begin{array}{l} LwX_1 = X_1(x) \\ LwX_2 = X_2(x) \end{array}$$

Współczynniki δ_{ik}, δ_{iP} układu równań kanonicznych:

$$\delta_{ik} = \sum \int_s \frac{M_i M_k}{EI} ds$$

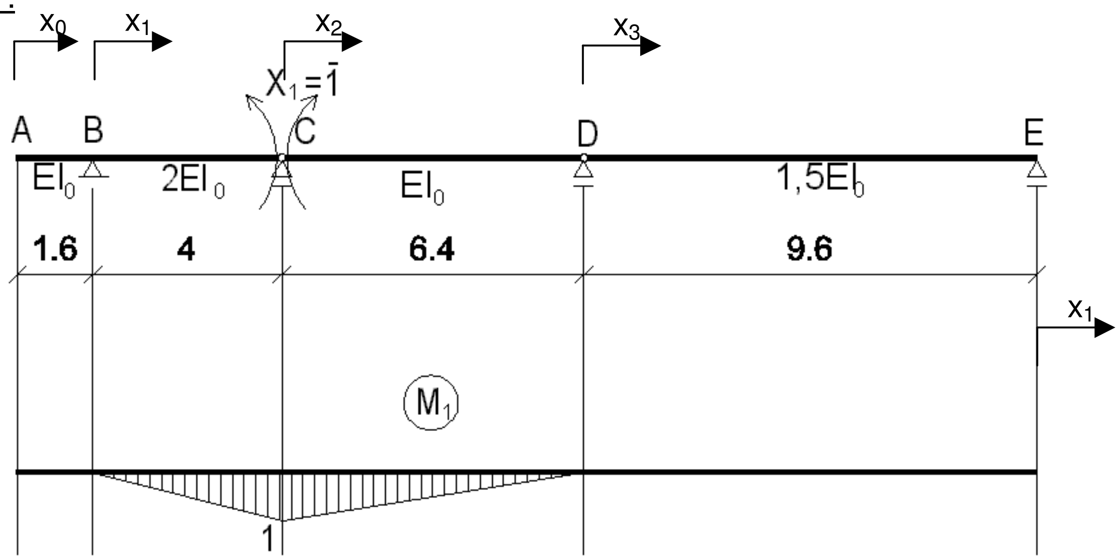
- przemieszczenie pktu przyłożenia siły X_i, po kierunku tej siły, wywołane działaniem siły $X_k=1,0$; $\delta_{ik} = \delta_{ki}$
- w tym przypadku współczynnik np. δ_{12} jest to wzajemny kąt obrotu przekrojów z prawej i lewej strony przegubu w pkcie C, wywołany działaniem siły $X_2=1,0$.

$$\delta_{iP} = \sum \int_s \frac{M_i M_P}{EI} ds$$

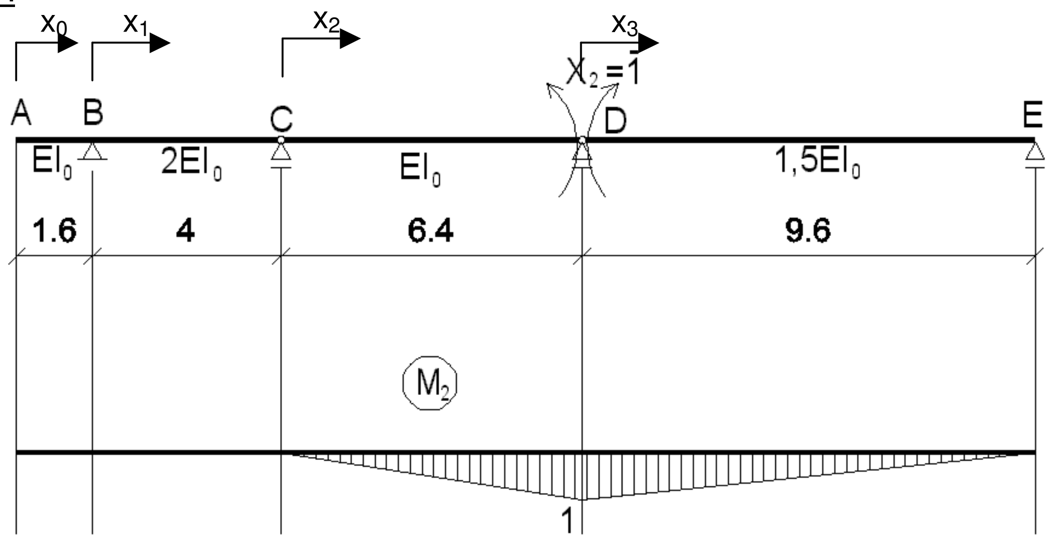
- przemieszczenie pktu przyłożenia siły X_i, po kierunku tej siły, wywołane działaniem obciążenia zewnętrznego;
- w tym przypadku obciążeniem zewnętrznym jest „ruchoma" siła $P=1,0$
- wykorzystując twierdzenie o wzajemności przemieszczeń (tw. Maxwella): $\delta_{iP} = \delta_{Pi}$,

  δ_{Pi} jest to przemieszczenie pktu przyłożenia siły $P=1,0$ wywołane działaniem siły $X_i=1$, a więc **linia ugięcia belki** (w układzie podstawowym) wywołana działaniem siły $X_i=1,0$.

Stan X_1=1:



Stan X_2=1:



Wyznaczenie współczynników δ_{ik} :

$$\delta_{11} = \frac{1}{2EI_0}\left[\frac{1}{2}\cdot 1\cdot 4\cdot \frac{2}{3}\cdot 1\right] + \frac{1}{EI_0}\left[\frac{1}{2}\cdot 1\cdot 6{,}4\cdot \frac{2}{3}\cdot 1\right] = \frac{1}{EI_0}(2{,}8)$$

$$\underline{EI_0 \cdot \delta_{11} = 2{,}8}$$

$$\delta_{12} = \frac{1}{EI_0}\left[\frac{1}{2}\cdot 1\cdot 6{,}4\cdot \frac{1}{3}\cdot 1\right] = \frac{1}{EI_0}\left(\frac{6{,}4}{6}\right)$$

$$\underline{EI_0 \cdot \delta_{12} = 1{,}0667}$$

$$\delta_{22} = \frac{1}{EI_0}\left[\frac{1}{2}\cdot 1\cdot 6{,}4\cdot \frac{2}{3}\cdot 1\right] + \frac{1}{1{,}5EI_0}\left[\frac{1}{2}\cdot 1\cdot 9{,}6\cdot \frac{2}{3}\cdot 1\right] = \frac{1}{EI_0}(4{,}2667)$$

$$\underline{EI_0 \cdot \delta_{22} = 4{,}2667}$$

Wyznaczenie współczynników δ_{Pi} jako linii ugięcia belki wywołanej siłą $X_i=1,0$

δ_{P1}

$x_1 \in <B;C> \quad (0 \le x_1 \le 4)$

$$M(x_1)=\frac{x_1}{4}$$

$$2EI_0\frac{d^2y}{dx^2}=-\frac{x_1}{4}$$

$$2EI_0\frac{dy}{dx}=-\frac{1}{2}\cdot\frac{x_1^2}{4}+C=-\frac{1}{8}x_1^2+C$$

$$2EI_0 y=-\frac{1}{3}\cdot\frac{x_1^3}{8}+C\cdot x_1+D=-\frac{1}{24}x_1^3+C\cdot x_1+D$$

$$EI_0 y=-\frac{1}{48}x_1^3+\frac{1}{2}C\cdot x_1+\frac{1}{2}D$$

warunki brzegowe:

1) $x_1 = 0 \quad y = 0$
2) $x_1 = 4 \quad y = 0$

z 1) $0=-\frac{1}{48}\cdot 0^3+\frac{1}{2}C\cdot 0+\frac{1}{2}D$

$D=0$

z 2) $0=-\frac{1}{48}\cdot 4^3+\frac{1}{2}C\cdot 4+\frac{1}{2}\cdot 0$

$C=\frac{2}{3}$

$$\underline{\delta_{P1}=\frac{1}{EI_0}\left(-\frac{1}{48}x_1^3+\frac{1}{3}x_1\right)}$$

$x_0 \in <A;C> \quad (0 \le x_0 \le 1{,}6)$

$$M(x_0)=0$$

$$EI_0\frac{d^2y}{dx^2}=0$$

$$EI_0\frac{dy}{dx}=C$$

$$EI_0 y=C\cdot x_0+D$$

warunki brzegowe:

1) $\varphi_B^L=\varphi_B^P$
2) $x_0 = 1{,}6 \quad y = 0$

z 1) $2EI_0\varphi_B^P=-\frac{1}{8}\cdot 0^2+\frac{2}{3}=\frac{2}{3}$

$EI_0\varphi_B^P=\frac{1}{3}$

$EI_0\varphi_B^L=C \Rightarrow C=\frac{1}{3}$

z 2) $0=\frac{1}{3}\cdot 1{,}6+D \Rightarrow D=-\frac{1{,}6}{3}$

$$\underline{\delta_{P1}=\frac{1}{EI_0}\left(\frac{1}{3}x_0-\frac{1{,}6}{3}\right)}$$

$x_3 \in <D;E> \quad (0 \le x_3 \le 9{,}6)$

$$M(x_0)=0$$

$$EI_0\frac{d^2y}{dx^2}=0$$

$$EI_0\frac{dy}{dx}=C$$

$$EI_0 y=C\cdot x_3+D$$

warunki brzegowe:

1) $x_3 = 0 \quad y = 0$
2) $x_3 = 9{,}6 \quad y = 0$

z 1) D=0
z 2) C=0

$$\underline{\delta_{P1}=0}$$

δ_{P2}

$x_1 \in <A;C>$ $\quad (-1{,}6 \le x_1 \le 4)$ $\quad \delta_{P2} = 0$

$x_2 \in <C;D>$ $\quad (0 \le x_2 \le 6{,}4)$

$$M(x_2) = \frac{x_2}{6{,}4}$$

$$EI_0 \frac{d^2 y}{dx^2} = -\frac{x_2}{6{,}4}$$

$$EI_0 \frac{dy}{dx} = -\frac{1}{2} \cdot \frac{x_2^2}{6{,}4} + C = -\frac{1}{12{,}8} x_2^2 + C$$

$$EI_0 y = -\frac{1}{3} \cdot \frac{x_2^3}{12{,}8} + C \cdot x_2 + D = -\frac{1}{38{,}4} x_2^3 + C \cdot x_2 + D$$

warunki brzegowe:

1) $x_2 = 0$ $\quad y = 0$

2) $x_2 = 6{,}4$ $\quad y = 0$

z 1) $\quad 0 = \frac{1}{38{,}4} \cdot 0^3 - \frac{1}{2} \cdot 0^2 + C \cdot 0 + D$

$D = 0$

z 2) $\quad 0 = -\frac{1}{38{,}4} \cdot 6{,}4^3 + C \cdot 6{,}4 + 0$

$C = \frac{16}{15}$

$$\delta_{P2} = \frac{1}{EI_0}\left(-\frac{1}{38{,}4} x_2^3 + \frac{16}{15} x_2\right)$$

$x_3 \in <D;E>$ $\quad (0 \le x_3 \le 9{,}6)$

$$M(x_3) = 1 - \frac{x_3}{9{,}6}$$

$$1{,}5EI_0 \frac{d^2 y}{dx^2} = -\left(1 - \frac{x_3}{9{,}6}\right) = -1 + \frac{x_3}{9{,}6}$$

$$1{,}5EI_0 \frac{dy}{dx} = -x_3 + \frac{1}{2} \cdot \frac{x_3^2}{9{,}6} + C = -x_3 + \frac{1}{19{,}2} x_3^2 + C$$

$$1{,}5EI_0 y = -\frac{1}{2} x_3^2 + \frac{1}{3} \cdot \frac{x_3^3}{19{,}2} + C \cdot x_3 + D =$$

$$= \frac{1}{57{,}6} x_3^3 - \frac{1}{2} x_3^2 + C \cdot x_3 + D$$

$$EI_0 y = \frac{1}{86{,}4} x_3^3 - \frac{1}{3} x_3^2 + \frac{2}{3} C \cdot x_3 + \frac{2}{3} D$$

warunki brzegowe:

1) $x_3 = 0$ $\quad y = 0$

2) $x_3 = 9{,}6$ $\quad y = 0$

z 1) $\quad 0 = \frac{1}{84{,}6} \cdot 0^3 - \frac{1}{3} \cdot 0^2 + \frac{2}{3} C \cdot 0 + \frac{2}{3} D$

$D = 0$

z 2)

$$0 = \frac{1}{84{,}6} \cdot 9{,}6^3 - \frac{1}{3} \cdot 9{,}6^2 + \frac{2}{3} C \cdot 9{,}6 + \frac{2}{3} \cdot 0$$

$C = 3{,}2$

$$\delta_{P2} = \frac{1}{EI_0}\left(\frac{1}{86{,}4} x_3^3 - \frac{1}{3} x_3^2 + \frac{32}{15} x_3\right)$$

$$\begin{cases} \frac{2{,}8}{\mathrm{EI}_0} X_1(x) + \frac{1{,}0667}{\mathrm{EI}_0} X_2(x) + \delta_{1P}(x) = 0 / \cdot EI_0 \\ \frac{1{,}0667}{\mathrm{EI}_0} X_1(x) + \frac{4{,}2667}{\mathrm{EI}_0} X_2(x) + \delta_{2P}(x) = 0 / \cdot EI_0 \end{cases}$$

$$\begin{cases} 2{,}8 \cdot X_1(x) + 1{,}0667 \cdot X_2(x) = -\delta_{1P}(x) \cdot EI_0 \\ 1{,}0667 \cdot X_1(x) + 4{,}2667 \cdot X_2(x) = -\delta_{2P}(x) \cdot EI_0 \end{cases}$$

$$W = \begin{vmatrix} 2{,}8 & 1{,}0667 \\ 1{,}0667 & 4{,}2667 \end{vmatrix} = 10{,}8089$$

$$W=\begin{vmatrix}-\delta_{P1}(x) & 1{,}0667\\ -\delta_{P2}(x) & 4{,}2667\end{vmatrix}=-4{,}2667\cdot\delta_{P1}(x)\cdot EI_0+1{,}0667\cdot\delta_{P2}(x)\cdot EI_0$$

$$W=\begin{vmatrix}2{,}8 & -\delta_{P1}(x)\\ 1{,}0667 & -\delta_{P2}(x)\end{vmatrix}=-2{,}8\cdot\delta_{P2}(x)\cdot EI_0+1{,}0667\cdot\delta_{P1}(x)\cdot EI_0$$

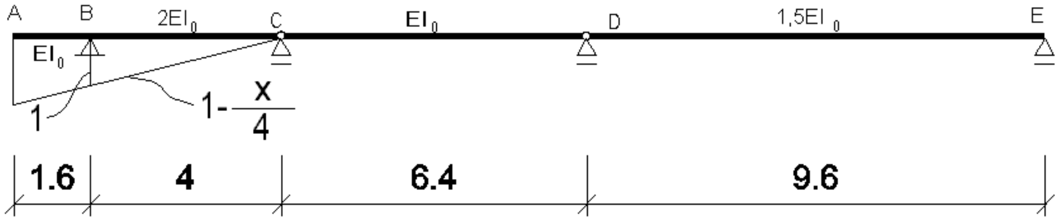
$$X_1(x)=\frac{W_1}{W}=\frac{1}{10{,}8089}\cdot\left(-4{,}2667\cdot\delta_{P1}(x)\cdot EI_0+1{,}0667\cdot\delta_{P2}(x)\cdot EI_0\right)$$

$$X_2(x)=\frac{W_2}{W}=\frac{1}{10{,}8089}\cdot\left(-2{,}8\cdot\delta_{P2}(x)\cdot EI_0+1{,}0667\cdot\delta_{P1}(x)\cdot EI_0\right)$$

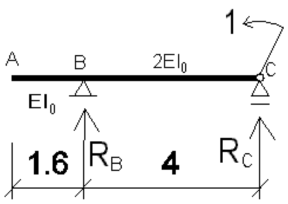
$$X_1(x)=-0{,}3947\cdot\delta_{P1}(x)\cdot EI_0+0{,}0987\cdot\delta_{P2}(x)\cdot EI_0$$
$$X_2(x)=-0{,}2590\cdot\delta_{P2}(x)\cdot EI_0+0{,}0987\cdot\delta_{P1}(x)\cdot EI_0$$

Wyznaczenie linii wpływu w układzie podstawowym oraz wartości reakcji i sił przekrojowych w stanach jednostkowych:

$LwR_B^{\,0}$:



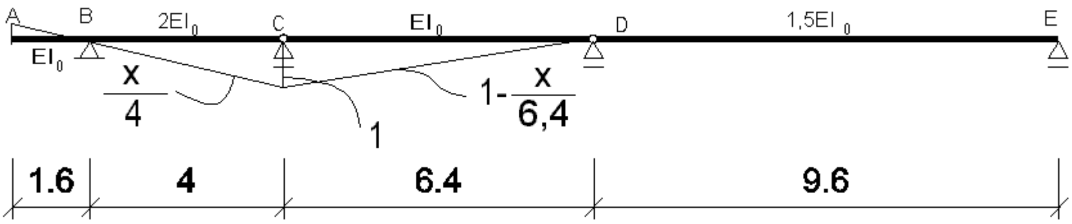
Stan X_1=1



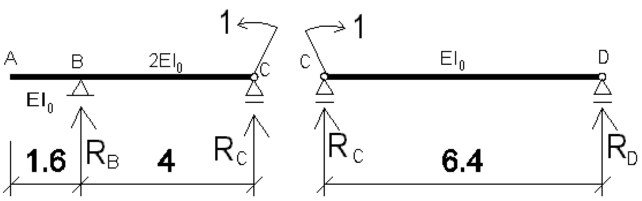
$$R_B^{(X_1=1)}=\frac{1}{4}$$

Stan X_2=1 $\quad R_B^{(X_2=1)}=0$

$LwR_C^{\,0}$:



Stan X_1=1



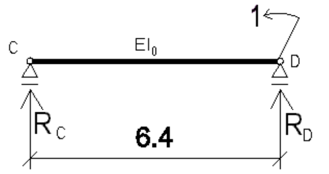
$$R_B=\frac{1}{4}\qquad R_D=\frac{1}{6{,}4}$$

$$R_C^{\,BC}=-R_B=-\frac{1}{4}$$

$$R_C^{\,CD}=-R_D=-\frac{1}{6{,}4}$$

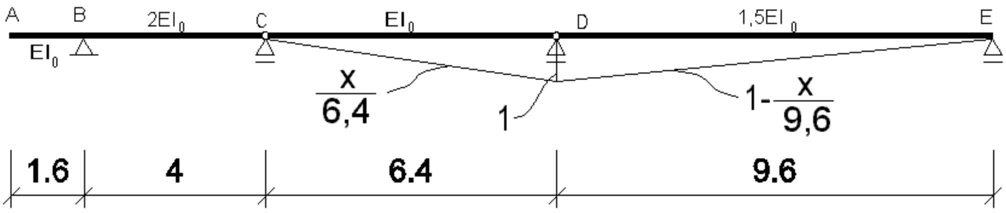
$$R_C^{(X_1=1)}=R_C^{\,BC}+R_C^{\,CD}=-\frac{1}{4}-\frac{1}{6{,}4}=-\frac{13}{32}$$

Stan $X_2=1$

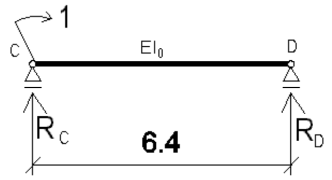


$$R_C^{(X_2=1)} = \frac{1}{6,4}$$

LwR_D^0:

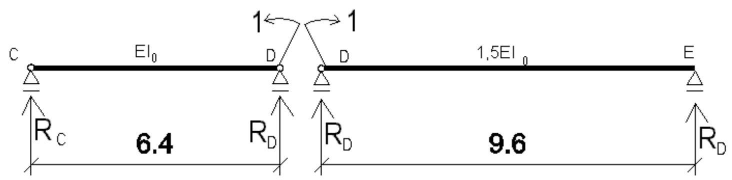


Stan $X_2=1$



$$R_D^{(X_1=1)} = \frac{1}{6,4}$$

Stan $X_2=1$

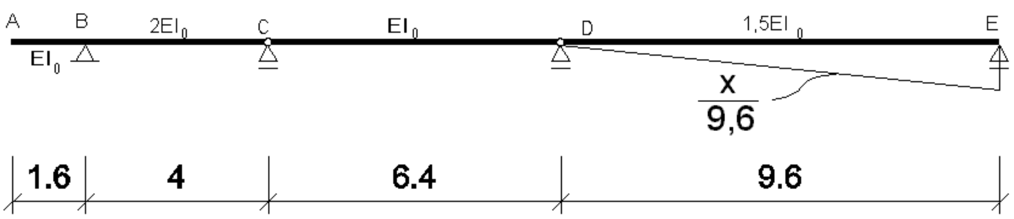


$$R_C = \frac{1}{6,4} \qquad R_E = \frac{1}{9,6}$$

$$R_D^{CD} = -R_C = -\frac{1}{6,4} \qquad R_D^{DE} = -R_E = -\frac{1}{9,6}$$

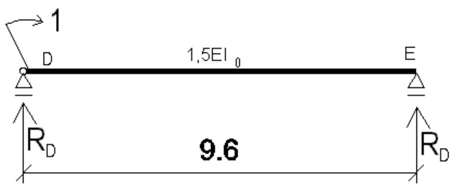
$$R_D^{(X_2=1)} = R_D^{CD} + R_D^{DE} = -\frac{1}{6,4} - \frac{1}{9,6} = -\frac{1}{3,84}$$

LwR_E^0:



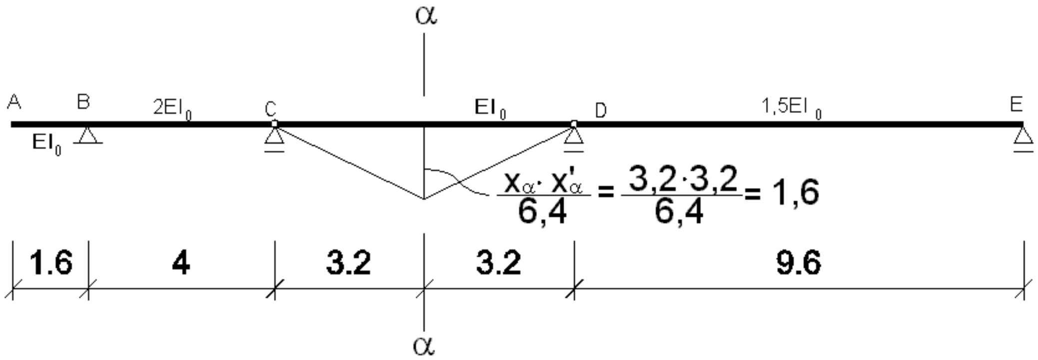
Stan $X_1=1$ $\qquad R_E^{(X_1=1)} = 0$

Stan $X_2=1$

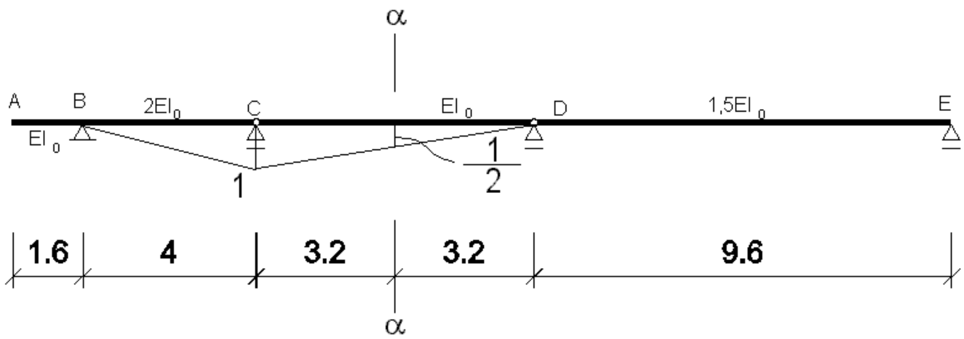


$$R_B^{(X_2=1)} = \frac{1}{9,6}$$

LwM_{α}^{0}:

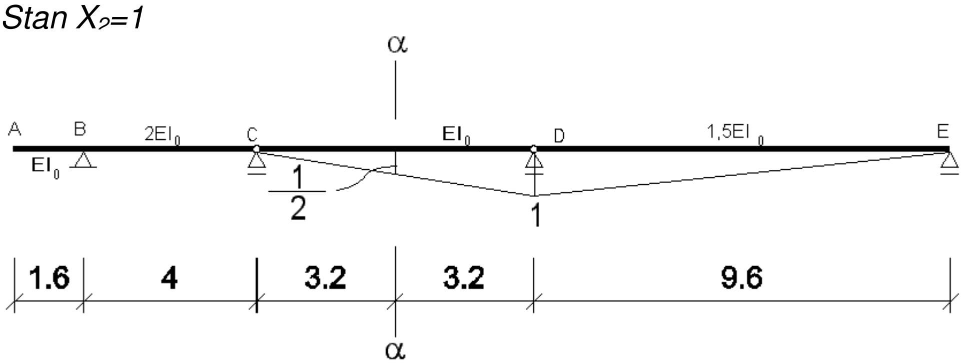


Stan X₁=1



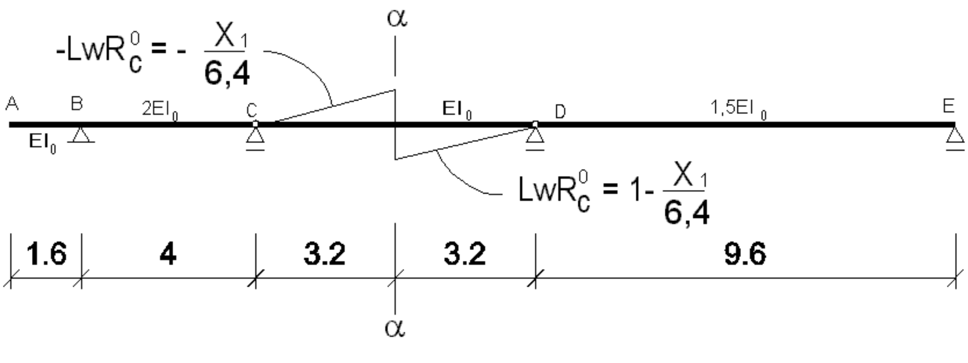
$$M_{\alpha}^{(X_1=1)} = \frac{1}{2}$$

Stan X₂=1



$$M_{\alpha}^{(X_2=1)} = \frac{1}{2}$$

LwT_{α}^{0}:

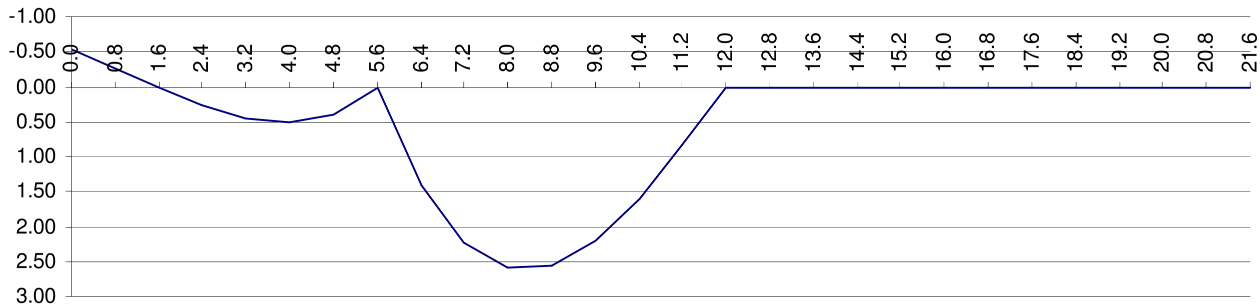


Stan X_1=1 $$T_{\alpha}^{(X_1=1)} = -\frac{1}{6,4}$$

Stan X_2=1 $$T_{\alpha}^{(X_2=1)} = \frac{1}{6,4}$$

x	x_{lok}	$EI_0\delta_{P1}(x)$	$EI_0\delta_{P2}(x)$	LwX_1	LwX_2	LwM_α^0	LwT_α^0	LwR_B^0	LwR_C^0	LwR_D^0	LwR_E^0	LwM_α	LwT_α	LwR_B	LwR_C	LwR_D	LwR_E
0.0	0.0	-0.533	0.000	0.211	-0.053	0.000	0.000	1.400	-0.400	0.000	0.000	0.079	-0.041	1.453	-0.494	0.047	-0.005
0.8	0.8	-0.267	0.000	0.105	-0.026	0.000	0.000	1.200	-0.200	0.000	0.000	0.039	-0.021	1.226	-0.247	0.023	-0.003
1.6	0.0	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
2.4	0.8	0.256	0.000	-0.101	0.025	0.000	0.000	0.800	0.200	0.000	0.000	-0.038	0.020	0.775	0.245	-0.022	0.003
3.2	1.6	0.448	0.000	-0.177	0.044	0.000	0.000	0.600	0.400	0.000	0.000	-0.066	0.035	0.556	0.479	-0.039	0.005
4.0	2.4	0.512	0.000	-0.202	0.051	0.000	0.000	0.400	0.600	0.000	0.000	-0.076	0.039	0.349	0.690	-0.045	0.005
4.8	3.2	0.384	0.000	-0.152	0.038	0.000	0.000	0.200	0.800	0.000	0.000	-0.057	0.030	0.162	0.868	-0.034	0.004
5.6	0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000
6.4	0.8	1.400	0.840	-0.470	-0.079	0.400	-0.125	0.000	0.875	0.125	0.000	0.125	-0.064	-0.117	1.053	0.072	-0.008
7.2	1.6	2.240	1.600	-0.726	-0.193	0.800	-0.250	0.000	0.750	0.250	0.000	0.340	-0.167	-0.182	1.015	0.187	-0.020
8.0	2.4	2.600	2.200	-0.809	-0.313	1.200	-0.375	0.000	0.625	0.375	0.000	0.639	-0.298	-0.202	0.905	0.330	-0.033
8.8	3.2	2.560	2.560	-0.758	-0.411	1.600	-0.500	0.000	0.500	0.500	0.000	1.016	-0.446	-0.189	0.744	0.488	-0.043
9.6	4.0	2.200	2.600	-0.612	-0.456	1.200	0.375	0.000	0.375	0.625	0.000	0.666	0.399	-0.153	0.552	0.648	-0.048
10.4	4.8	1.600	2.240	-0.411	-0.422	0.800	0.250	0.000	0.250	0.750	0.000	0.384	0.248	-0.103	0.351	0.796	-0.044
11.2	5.6	0.840	1.400	-0.193	-0.280	0.400	0.125	0.000	0.125	0.875	0.000	0.163	0.112	-0.048	0.160	0.918	-0.029
12.0	0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
12.8	0.8	0.000	1.499	0.148	-0.388	0.000	0.000	0.000	0.000	0.917	0.083	-0.120	-0.084	0.037	-0.121	1.041	0.043
13.6	1.6	0.000	2.607	0.257	-0.675	0.000	0.000	0.000	0.000	0.833	0.167	-0.209	-0.146	0.064	-0.210	1.049	0.096
14.4	2.4	0.000	3.360	0.332	-0.870	0.000	0.000	0.000	0.000	0.750	0.250	-0.269	-0.188	0.083	-0.271	1.028	0.159
15.2	3.2	0.000	3.793	0.374	-0.982	0.000	0.000	0.000	0.000	0.667	0.333	-0.304	-0.212	0.094	-0.306	0.981	0.231
16.0	4.0	0.000	3.941	0.389	-1.021	0.000	0.000	0.000	0.000	0.583	0.417	-0.316	-0.220	0.097	-0.317	0.910	0.310
16.8	4.8	0.000	3.840	0.379	-0.995	0.000	0.000	0.000	0.000	0.500	0.500	-0.308	-0.215	0.095	-0.309	0.818	0.396
17.6	5.6	0.000	3.526	0.348	-0.913	0.000	0.000	0.000	0.000	0.417	0.583	-0.283	-0.197	0.087	-0.284	0.709	0.488
18.4	6.4	0.000	3.034	0.299	-0.786	0.000	0.000	0.000	0.000	0.333	0.667	-0.243	-0.170	0.075	-0.244	0.585	0.585
19.2	7.2	0.000	2.400	0.237	-0.622	0.000	0.000	0.000	0.000	0.250	0.750	-0.192	-0.134	0.059	-0.193	0.449	0.685
20.0	8.0	0.000	1.659	0.164	-0.430	0.000	0.000	0.000	0.000	0.167	0.833	-0.133	-0.093	0.041	-0.134	0.304	0.789
20.8	8.8	0.000	0.847	0.084	-0.220	0.000	0.000	0.000	0.000	0.083	0.917	-0.068	-0.047	0.021	-0.068	0.154	0.894
21.6	9.6	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000	1.000

Lwδ1P



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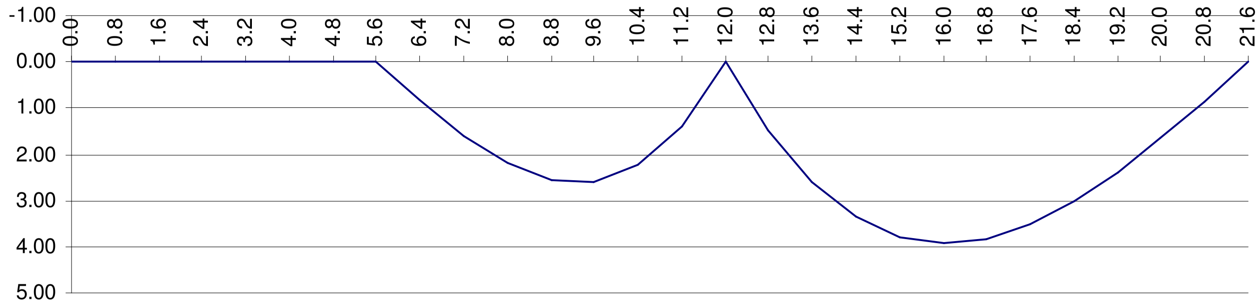

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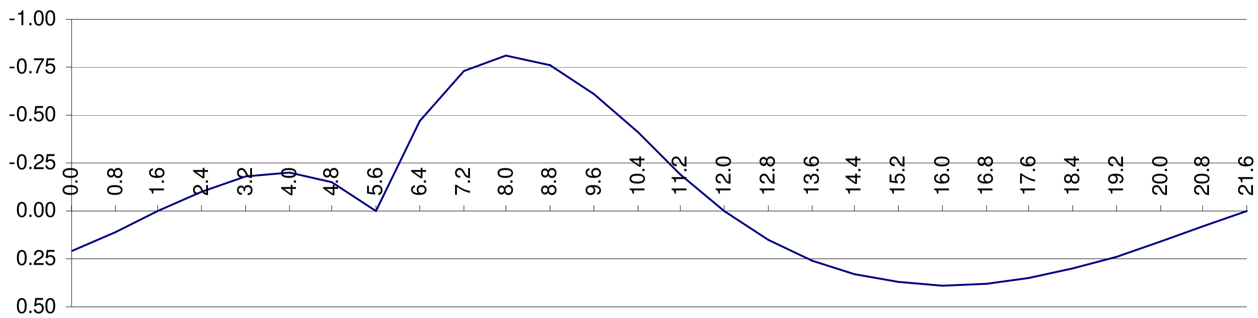

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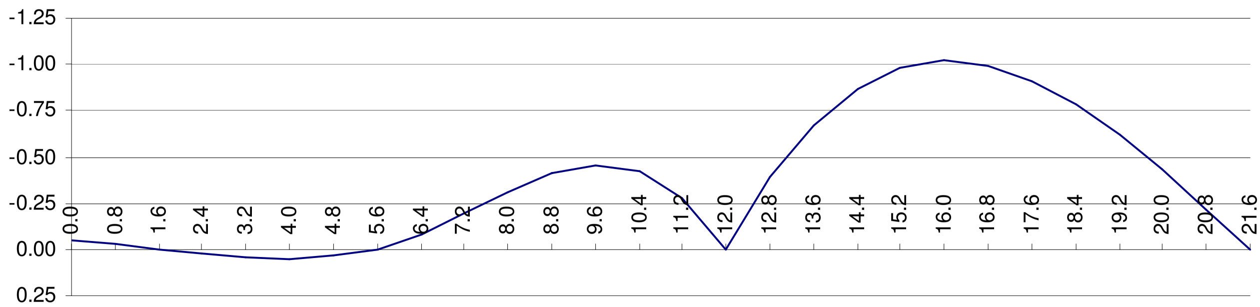

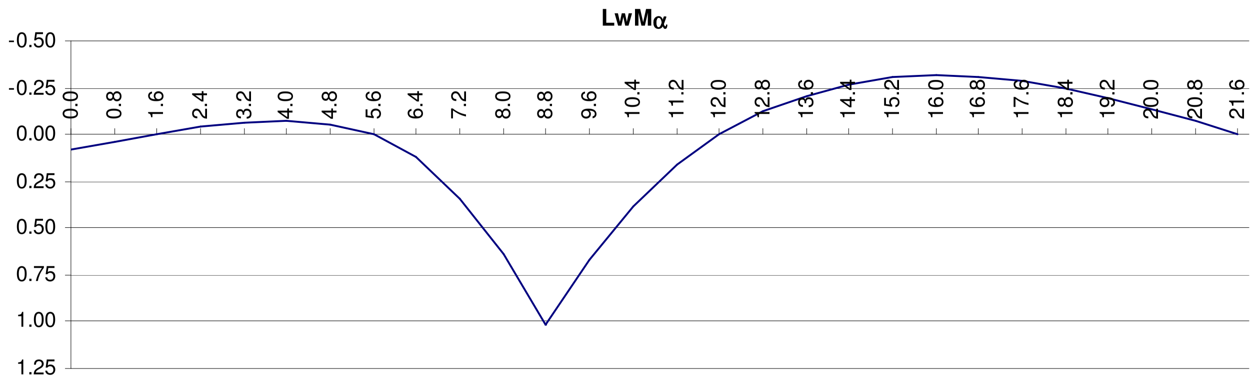
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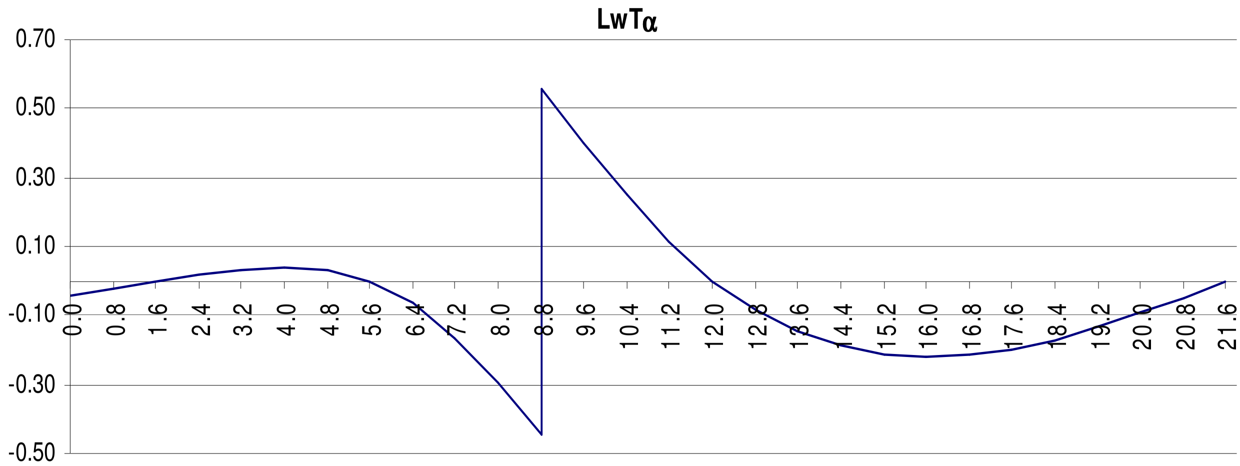
0.00
0.25
0.50
0.75
1.00
1.25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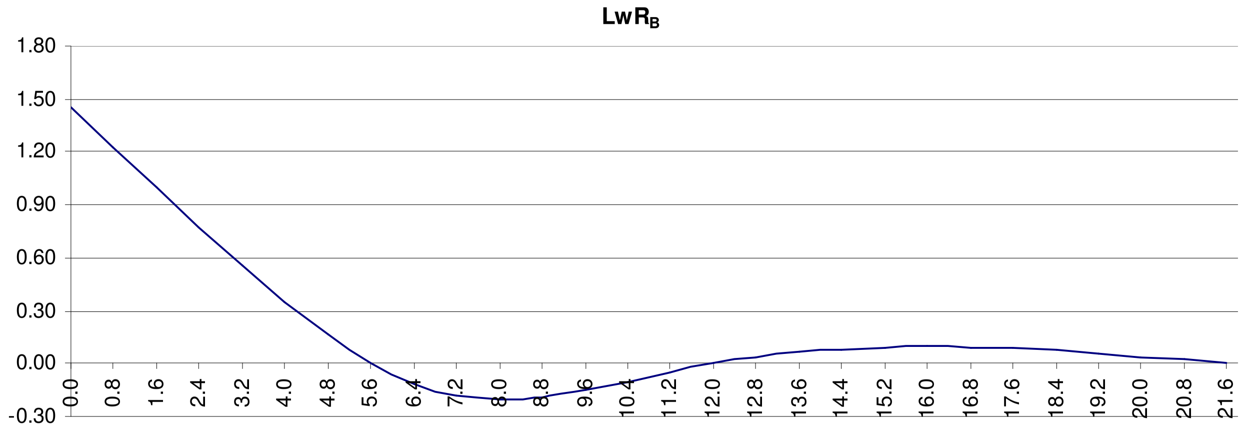
0.30
0.10
-0.10
-0.30
-0.50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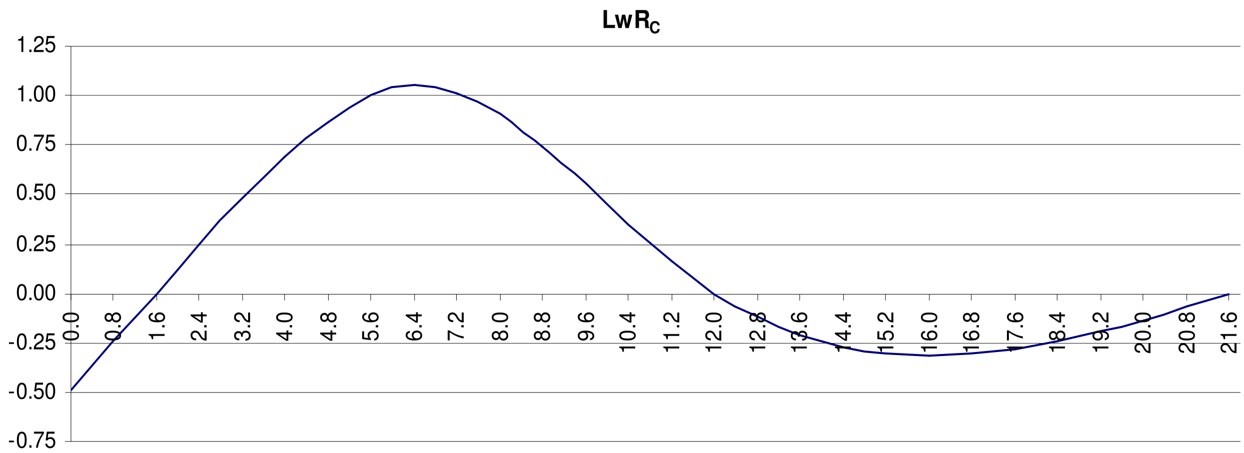
1.20
0.90
0.60
0.30
0.00
-0.30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0.00
-0.25
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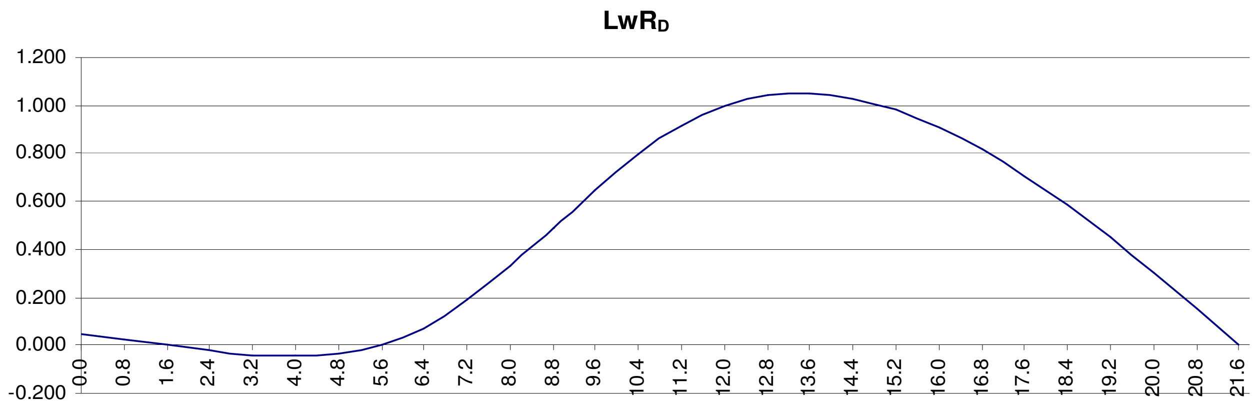
-0.75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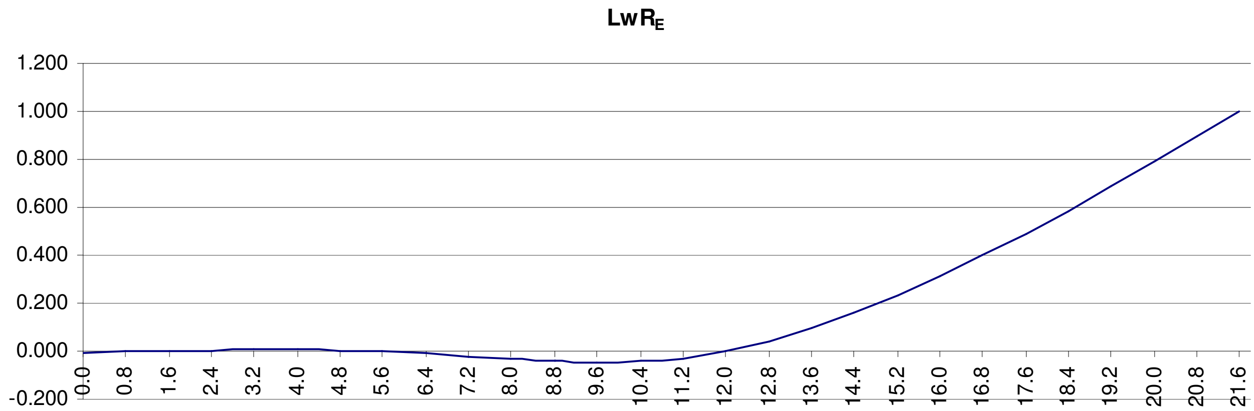
0.800
0.600
0.400
0.200
0.000
-0.200
0.0 0.8 1.6 2.4 3.2 4.0 4.8 5.6 6.4 7.2 8.0 8.8 9.6 10.4 11.2 12.0 12.8 13.6 14.4 15.2 16.0 16.8 17.6 18.4 19.2 20.0 20.8 21.6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0.200
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