

Mechanika Budowli

Projekt 2 – Dynamika w ujęciu klasycznym

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Rok akademicki : 2013/2014

Semestr: 4

Grupa: B7

Masy:

$m_1 = 200 \text{ kg}$

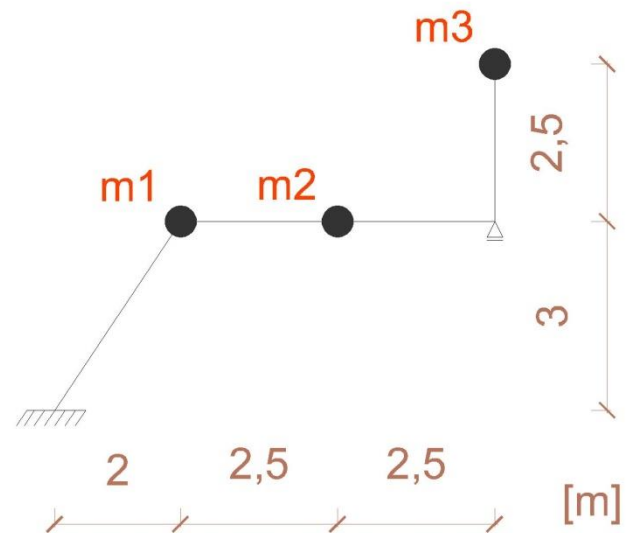
$m_2 = 330 \text{ kg}$

$m_3 = 520 \text{ kg}$

Amplituda siły wymuszającej: $P_0 = 18,0 \text{ kN}$

Częstotliwość siły wymuszającej: $p = 20,5 \text{ Hz}$

Przekrój prętów: **IPE 220**



Drgania swobodne (własne):

$$\begin{cases} q_1 = \delta_{11}B_1 + \delta_{12}B_2 + \delta_{13}B_3 + \delta_{11}B_1' + \delta_{14}B_4 \\ q_2 = \delta_{21}B_1 + \delta_{22}B_2 + \delta_{23}B_3 + \delta_{21}B_1' + \delta_{24}B_4 \\ q_3 = \delta_{31}B_1 + \delta_{32}B_2 + \delta_{33}B_3 + \delta_{31}B_1' + \delta_{34}B_4 \end{cases}$$

$q_1 = A_1 \cos(\omega t)$

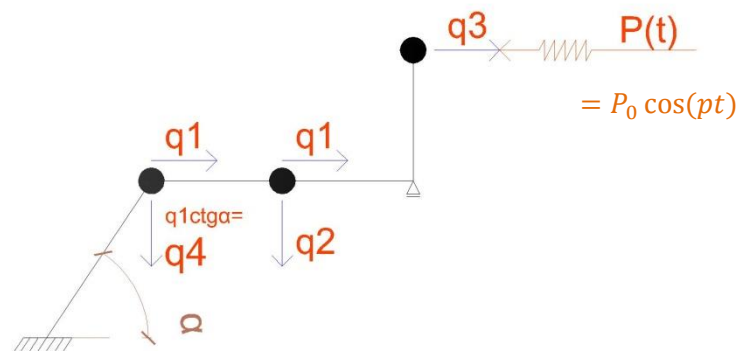
$q_2 = A_2 \cos(\omega t)$

$q_3 = A_3 \cos(\omega t)$

$\ddot{q}_1 = -A_1 \omega^2 \cos(\omega t)$

$\ddot{q}_2 = -A_2 \omega^2 \cos(\omega t)$

$\ddot{q}_3 = -A_3 \omega^2 \cos(\omega t)$



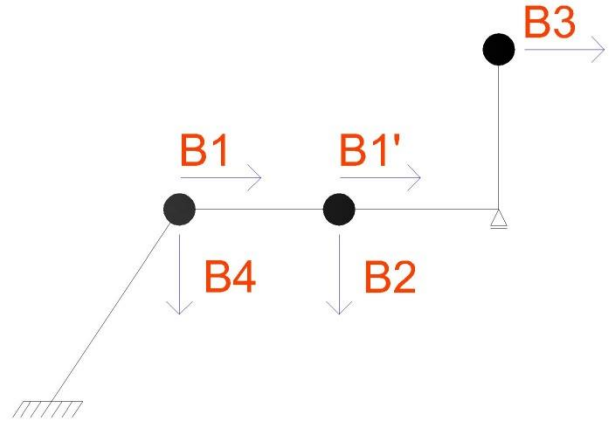
$$B_1 = -\ddot{q}_1 m_1$$

$$B_1' = -\ddot{q}_1 m_2$$

$$B_2 = -\ddot{q}_2 m_2$$

$$B_3 = -\ddot{q}_3 m_3$$

$$B_4 = -\ddot{q}_4 m_1 = -m_1 \operatorname{ctg} \alpha \ddot{q}_1$$



$$\begin{cases} q_1 = \delta_{11}B_1 + \delta_{12}B_2 + \delta_{13}B_3 + \delta_{11}B_1' + \delta_{14}B_4 \\ q_2 = \delta_{21}B_1 + \delta_{22}B_2 + \delta_{23}B_3 + \delta_{21}B_1' + \delta_{24}B_4 \\ q_3 = \delta_{31}B_1 + \delta_{32}B_2 + \delta_{33}B_3 + \delta_{31}B_1' + \delta_{34}B_4 \end{cases}$$

$$\begin{aligned} A_1 \cos(\omega t) &= \delta_{11}(-m_1(-A_1\omega^2 \cos(\omega t))) + \delta_{12}(-m_2(-A_2\omega^2 \cos(\omega t))) \\ &\quad + \delta_{11}(-m_2(-A_1\omega^2 \cos(\omega t))) + \delta_{13}(-m_3(-A_3\omega^2 \cos(\omega t))) \\ &\quad + \delta_{14}(-m_1 \operatorname{ctg} \alpha (-A_1\omega^2 \cos(\omega t))) \end{aligned}$$

$$\begin{aligned} A_2 \cos(\omega t) &= \delta_{21}(-m_1(-A_1\omega^2 \cos(\omega t))) + \delta_{22}(-m_2(-A_2\omega^2 \cos(\omega t))) \\ &\quad + \delta_{21}(-m_2(-A_1\omega^2 \cos(\omega t))) + \delta_{23}(-m_3(-A_3\omega^2 \cos(\omega t))) \\ &\quad + \delta_{24}(-m_1 \operatorname{ctg} \alpha (-A_1\omega^2 \cos(\omega t))) \end{aligned}$$

$$\begin{aligned} A_3 \cos(\omega t) &= \delta_{31}(-m_1(-A_1\omega^2 \cos(\omega t))) + \delta_{32}(-m_2(-A_2\omega^2 \cos(\omega t))) \\ &\quad + \delta_{31}(-m_2(-A_1\omega^2 \cos(\omega t))) + \delta_{33}(-m_3(-A_3\omega^2 \cos(\omega t))) \\ &\quad + \delta_{34}(-m_1 \operatorname{ctg} \alpha (-A_1\omega^2 \cos(\omega t))) \end{aligned}$$

$$\begin{aligned} A_1 \cos(\omega t) &= \delta_{11}(m_1 A_1 \omega^2 \cos(\omega t) + m_2 A_1 \omega^2 \cos(\omega t)) + \delta_{12}(m_2 A_2 \omega^2 \cos(\omega t)) \\ &\quad + \delta_{13}(m_3 A_3 \omega^2 \cos(\omega t)) + \delta_{14}(m_1 \operatorname{ctg} \alpha A_1 \omega^2 \cos(\omega t)) \end{aligned}$$

$$\begin{aligned} A_2 \cos(\omega t) &= \delta_{21}(m_1 A_1 \omega^2 \cos(\omega t) + m_2 A_1 \omega^2 \cos(\omega t)) + \delta_{22}(m_2 A_2 \omega^2 \cos(\omega t)) \\ &\quad + \delta_{23}(m_3 A_3 \omega^2 \cos(\omega t)) + \delta_{24}(m_1 \operatorname{ctg} \alpha A_1 \omega^2 \cos(\omega t)) \end{aligned}$$

$$\begin{aligned} A_3 \cos(\omega t) &= \delta_{31}(m_1 A_1 \omega^2 \cos(\omega t) + m_2 A_1 \omega^2 \cos(\omega t)) + \delta_{32}(m_2 A_2 \omega^2 \cos(\omega t)) \\ &\quad + \delta_{33}(m_3 A_3 \omega^2 \cos(\omega t)) + \delta_{34}(m_1 \operatorname{ctg} \alpha A_1 \omega^2 \cos(\omega t)) \end{aligned}$$

$$A_1 = \delta_{11}A_1\omega^2(m_1 + m_2) + \delta_{12}A_2\omega^2 m_2 + \delta_{13}A_3\omega^2 m_3 + \delta_{14}A_1\omega^2 m_1 \operatorname{ctg} \alpha$$

$$A_2 = \delta_{21}A_1\omega^2(m_1 + m_2) + \delta_{22}A_2\omega^2 m_2 + \delta_{23}A_3\omega^2 m_3 + \delta_{24}A_1\omega^2 m_1 \operatorname{ctg} \alpha$$

$$A_3 = \delta_{31}A_1\omega^2(m_1 + m_2) + \delta_{32}A_2\omega^2 m_2 + \delta_{33}A_3\omega^2 m_3 + \delta_{34}A_1\omega^2 m_1 \operatorname{ctg} \alpha$$

$$A_1(1 - \delta_{11}A_1\omega^2(m_1 + m_2)) - A_2\delta_{12}\omega^2 m_2 - A_3\delta_{13}\omega^2 m_3 - A_1\delta_{14}\omega^2 m_1 \operatorname{ctg} \alpha = 0$$

$$-A_1(\delta_{21}\omega^2(m_1 + m_2) + \delta_{24}\omega^2 m_1 \operatorname{ctg} \alpha) + A_2(1 - \delta_{22}\omega^2 m_2) - A_3\delta_{23}\omega^2 m_3 = 0$$

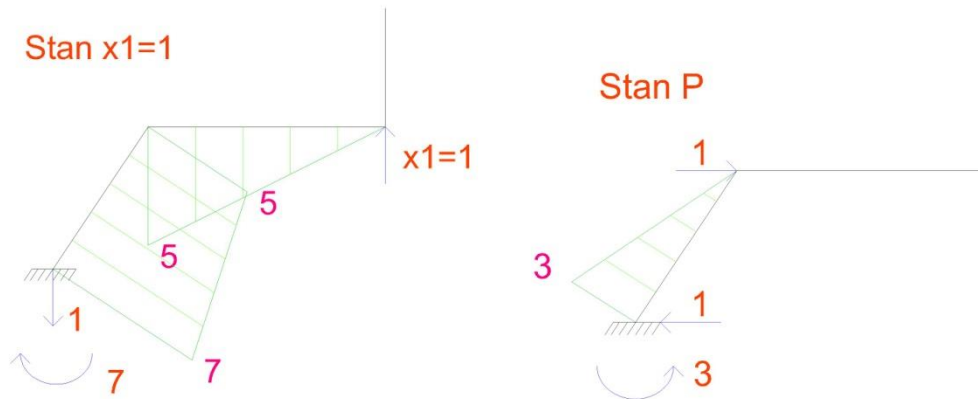
$$-A_1(\delta_{31}\omega^2(m_1 + m_2) + \delta_{34}\omega^2 m_1 \operatorname{ctg} \alpha) - A_2\delta_{32}\omega^2 m_2 + A_3(1 - \delta_{33}\omega^2 m_3) = 0$$

Układ jest statycznie niewyznaczalny (SSN = 1). Zastosowano metodę sił.

$$\delta_{ik} = \sum \int \frac{M_i M_k}{EI} dx$$

Wyznaczenie rozkładu momentów od sił jednostkowych po kierunkach: q_1 , q_2 , q_3 i q_4 .

Wyznaczenie M_1^n

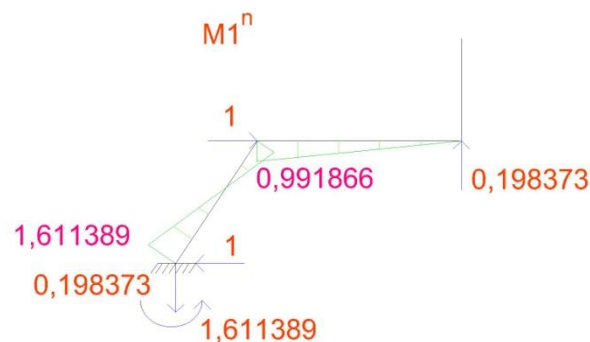


$$\delta_{11} X_1 + \delta_{1P} = 0$$

$$\delta_{11} = \frac{1}{EI} \left[\frac{1}{2} * 5 * 5 * \frac{2}{3} * 5 + \frac{1}{2} * 5 * \sqrt{13} * \left(\frac{2}{3} * 5 + \frac{1}{3} * 7 \right) + \frac{1}{2} * 7 * \sqrt{13} * \left(\frac{2}{3} * 7 + \frac{1}{3} * 5 \right) \right] = \frac{172,668363}{EI}$$

$$\delta_{1P} = \frac{1}{EI} \left[\frac{1}{2} * 3 * \sqrt{13} * \left(\frac{-2}{3} * 7 - \frac{1}{3} * 5 \right) \right] = \frac{-34,252737}{EI}$$

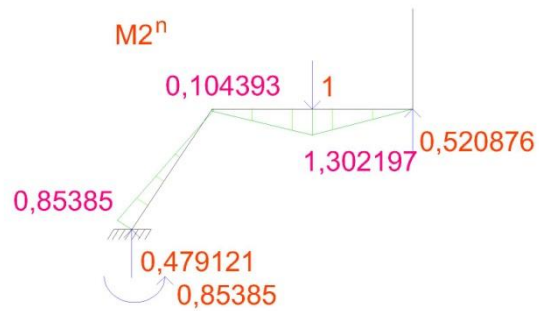
$$X_1 = 0,198373$$



Wyznaczenie M_2^n

$$\delta_{1P} = \frac{1}{EI} \left[\frac{-1}{2} * 4,5 * \sqrt{13} * \left(\frac{2}{3} * 7 + \frac{1}{3} * 5 \right) - \frac{1}{2} * 2,5 * \sqrt{13} * \left(\frac{2}{3} * 5 + \frac{1}{3} * 7 \right) - \frac{1}{2} * 2,5 * 2,5 * \left(\frac{2}{3} * 5 + \frac{1}{3} * 2,5 \right) \right] = \frac{-89,939261}{EI}$$

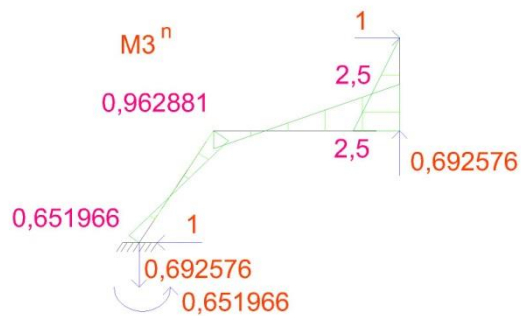
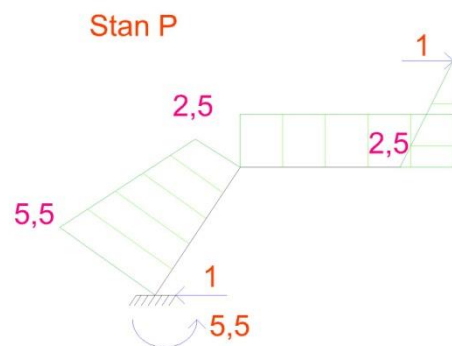
$$X_1 = 0,520876$$



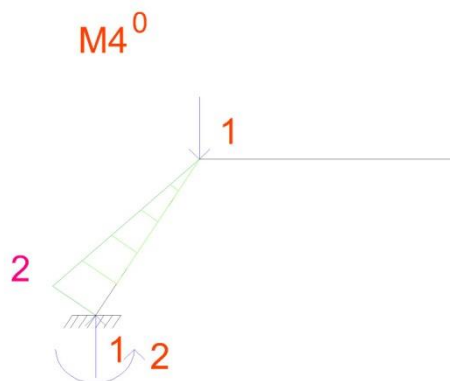
Wyznaczenie M_3^n

$$\delta_{1P} = \frac{1}{EI} \left[\frac{-1}{2} * 5 * 5 * \sqrt{13} * \left(\frac{2}{3} * 7 + \frac{1}{3} * 5 \right) - \frac{1}{2} * 2,5 * \sqrt{13} * \left(\frac{2}{3} * 5 + \frac{1}{3} * 7 \right) - 2,5 * 5 * 2,5 \right] = \frac{-119,586002}{EI}$$

$$X_1 = 0,692576$$



Wyznaczenie M_4^0



Wyznaczenie współczynników δ_{ik} .

$$\delta_{14} = \sum \int \frac{M_1^n M_4^0}{EI} = \frac{1}{EI} \left[\frac{1}{2} * 1,611389 * \sqrt{13} * \frac{2}{3} * 2 - \frac{1}{2} * 0,991865 * \sqrt{13} * \frac{1}{3} * 2 \right] = \frac{2,681224}{EI}$$

$$\delta_{24} = \sum \int \frac{M_2^n M_4^0}{EI} = \frac{1}{EI} \left[\frac{1}{2} * 2 * \sqrt{13} * \left(\frac{2}{3} * 0,85385 - \frac{1}{3} * 0,104393 \right) \right] = \frac{1,926934}{EI}$$

$$\delta_{34} = \sum \int \frac{M_3^n M_4^0}{EI} = \frac{1}{EI} \left[\frac{1}{2} * 2 * \sqrt{13} * \left(\frac{2}{3} * 0,651966 - \frac{1}{3} * 0,962881 \right) \right] = \frac{0,409893}{EI}$$

$$\delta_{11} = \sum \int \frac{M_1^n M_1^n}{EI} = \frac{1}{EI} \left[\frac{1}{2} * 0,991866 * 5 * \frac{2}{3} * 0,991866 + \frac{1}{2} * 1,611389 * \sqrt{13} * \left(\frac{2}{3} * 1,611389 - \frac{1}{3} * 0,991866 \right) + \frac{1}{2} * 0,991865 * \sqrt{13} * \left(\frac{2}{3} * 0,991866 - \frac{1}{3} * 1,611389 \right) \right] = \frac{4,021836}{EI}$$

$$\delta_{22} = \sum \int \frac{M_2^n M_2^n}{EI} = \frac{1}{EI} \left[\frac{1}{2} * 0,85385 * \sqrt{13} * \left(\frac{2}{3} * 0,85385 - \frac{1}{3} * 0,104393 \right) + \frac{1}{2} * 0,104393 * \sqrt{13} * \left(\frac{2}{3} * 0,104393 - \frac{1}{3} * 0,85385 \right) + \frac{1}{2} * 0,104393 * 2,5 * \left(\frac{2}{3} * 0,104393 + \frac{1}{3} * 1,302197 \right) + \frac{1}{2} * 1,302197 * 2,5 * \left(\frac{2}{3} * 1,302197 + \frac{1}{3} * 0,104393 \right) + \frac{1}{2} * 1,302197 * 2,5 * \frac{2}{3} * 1,302197 \right] = \frac{3,730748}{EI}$$

$$\delta_{12} = \sum \int \frac{M_1^n M_2^n}{EI} = \frac{1}{EI} \left[\frac{1}{2} * 1,611389 * \sqrt{13} * \left(\frac{2}{3} * 0,85385 - \frac{1}{3} * 0,104393 \right) + \frac{1}{2} * 0,991865 * \sqrt{13} * \left(\frac{2}{3} * 0,104393 - \frac{1}{3} * 0,85385 \right) + \frac{1}{2} * 0,991865 * 2,5 * \left(\frac{2}{3} * 0,104393 + \frac{1}{3} * 1,302197 \right) + \frac{1}{2} * 2,5 * 0,495932 * \left(\frac{2}{3} * 1,302197 + \frac{1}{3} * 0,104393 \right) + \frac{1}{2} * 0,495932 * 2,5 * \frac{2}{3} * 1,302197 \right] = \frac{2,890401}{EI}$$

$$\delta_{33} = \sum \int \frac{M_3^n M_3^n}{EI} = \frac{1}{EI} \left[\frac{1}{2} * 0,651966 * \sqrt{13} * \left(\frac{2}{3} * 0,651966 - \frac{1}{3} * 0,962881 \right) + \frac{1}{2} * 0,962881 * \sqrt{13} * \left(\frac{2}{3} * 0,962881 - \frac{1}{3} * 0,651966 \right) + \frac{1}{2} * 0,962881 * 5 * \left(\frac{2}{3} * 0,962881 - \frac{1}{3} * 2,5 \right) + \frac{1}{2} * 5 * 2,5 * \left(\frac{2}{3} * 2,5 - \frac{1}{3} * 0,962881 \right) + \frac{1}{2} * 2,5 * 2,5 * \frac{2}{3} * 2,5 \right] = \frac{14,02889}{EI}$$

$$\delta_{13} = \sum \int \frac{M_1^n M_3^n}{EI} = \frac{1}{EI} \left[\frac{1}{2} * 1,611389 * \sqrt{13} * \left(\frac{2}{3} * 0,651966 - \frac{1}{3} * 0,962881 \right) + \frac{1}{2} * 0,991865 * \sqrt{13} * \left(\frac{2}{3} * 0,962881 - \frac{1}{3} * 0,651966 \right) + \frac{1}{2} * 0,991865 * 5 * \left(\frac{2}{3} * 0,962881 - \frac{1}{3} * 2,5 \right) \right] = \frac{0,614839}{EI}$$

$$\delta_{23} = \sum \int \frac{M_2^n M_3^n}{EI} = \frac{1}{EI} \left[\frac{1}{2} * 0,85385 * \sqrt{13} * \left(\frac{2}{3} * 0,651966 - \frac{1}{3} * 0,962881 \right) + \frac{1}{2} * 0,104393 * \sqrt{13} * \left(\frac{2}{3} * 0,962881 - \frac{1}{3} * 0,768559 \right) + \frac{1}{2} * 1,302197 * 2,5 * \left(-\frac{2}{3} * 0,768559 + \frac{1}{3} * 0,962881 \right) + \frac{1}{2} * 2,5 * 1,302197 * \left(-\frac{2}{3} * 0,768559 - \frac{1}{3} * 2,5 \right) \right] = \frac{-2,196802}{EI}$$

Przyjęto:

$$m_1 = m_0 = 200kg$$

$$m_2 = 1,65 m_0 = 330kg$$

$$m_3 = 2,6 m_0 = 520kg$$

$$\lambda = \frac{m_0 \omega^2}{EI}$$

$$\operatorname{ctg} \alpha = \frac{2}{3}$$

$$A_1 \left(1 - 4,021836 * 2,65\lambda - 2,681224 * \lambda * \frac{2}{3} \right) - A_2 2,890401 * 1,65\lambda - A_3 0,614839 * 2,6\lambda = 0$$

$$-A_1 \left(2,890401 * 2,65\lambda + 1,926934 * \frac{2}{3}\lambda \right) + A_2 (1 - 3,730748 * 1,65\lambda) - A_3 (-2,196802 * 2,6\lambda) = 0$$

$$-A_1 \left(0,614389 * 2,65\lambda + 0,409893 * \frac{2}{3}\lambda \right) - A_2 (-2,196802 * 1,65\lambda) + A_3 (1 - 14,02889 * 2,6\lambda) = 0$$

$$A_1 (1 - 12,445349\lambda) - A_2 4,769162\lambda - A_3 1,598582\lambda = 0$$

$$-A_1 8,944185\lambda + A_2 (1 - 6,155734\lambda) + A_3 5,711684 = 0$$

$$-A_1 1,902585\lambda + A_2 3,624723\lambda + A_3 (1 - 36,475111\lambda) = 0$$

$$\begin{vmatrix} 1 - 12,445349\lambda & -4,769162\lambda & -1,598582\lambda \\ -8,944185\lambda & 1 - 6,155734\lambda & 5,711684 \\ -1,902585\lambda & 3,624723\lambda & 1 - 36,475111\lambda \end{vmatrix} = 0$$

$$1 - 55,076195\lambda + 688,685879\lambda^2 - 858,441382\lambda^3 = 0$$

$$\lambda_3 = 0,714771$$

$$\lambda_2 = 0,060577$$

$$\lambda_1 = 0,026904$$

$$EI = 205 * 10^6 * 3060 * 10^{-8} = 6273 \text{ kNm}^2 = 6273000 \text{ Nm}^2$$

$$\omega_3 = \sqrt{\frac{EI\lambda_3}{m}} = 149,729048 \frac{\text{rad}}{\text{s}}$$

$$\omega_2 = \sqrt{\frac{EI\lambda_2}{m}} = 43,58881 \frac{\text{rad}}{\text{s}}$$

$$\omega_1 = \sqrt{\frac{EI\lambda_1}{m}} = 29,049024 \frac{\text{rad}}{\text{s}}$$

III Postać drgań własnych

$$\lambda_3 = 0,714771$$

$$A_1^{III} = 1,0$$

$$\begin{cases} 1(1 - 12,445349\lambda) - A_2 4,769162\lambda - A_3 1,598582\lambda = 0 \\ -1 * 8,944185\lambda + A_2 (1 - 6,155734\lambda) + A_3 5,711684 = 0 \end{cases}$$

$$\begin{cases} A_2 = -6,254772 \\ A_3 = 13,721193 \end{cases}$$

II Postać drgań własnych

$$\lambda_2 = 0,060577$$

$$A_1^{II} = 1,0$$

$$\begin{cases} 1(1 - 12,445349\lambda) - A_2 4,769162\lambda - A_3 1,598582\lambda = 0 \\ -1 * 8,944185\lambda + A_2(1 - 6,155734\lambda) + A_3 5,711684 = 0 \end{cases}$$

$$\begin{cases} A_2 = 0,83311 \\ A_3 = 0,055952 \end{cases}$$

I Postać drgań własnych

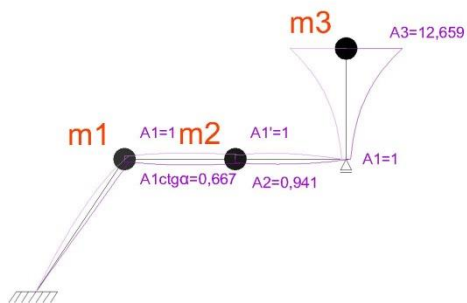
$$\lambda_1 = 0,026904$$

$$A_1^I = 1,0$$

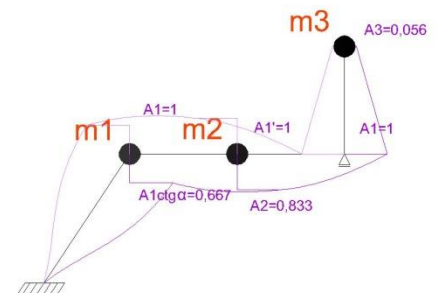
$$\begin{cases} 1(1 - 12,445349\lambda) - A_2 4,769162\lambda - A_3 1,598582\lambda = 0 \\ -1 * 8,944185\lambda + A_2(1 - 6,155734\lambda) + A_3 5,711684 = 0 \end{cases}$$

$$\begin{cases} A_2 = 0,940904 \\ A_3 = 12,658997 \end{cases}$$

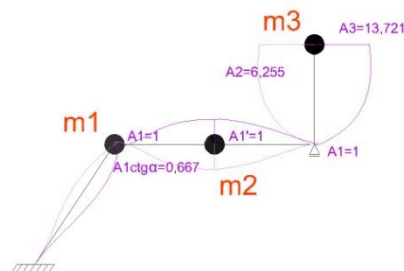
I Postać drgań własnych



II Postać drgań własnych



III Postać drgań własnych

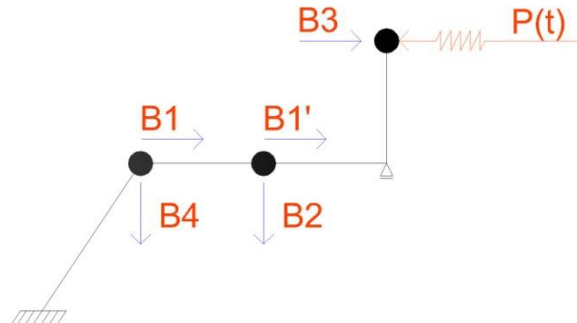


Drgania wymuszone

$$P(t) = P_0 \cos(pt)$$

$$P_0 = 18,0 \text{ kN} = 18000 \text{ N}$$

$$p = 20,5 \text{ Hz} = 128,805299 \text{ rad/s}$$



$$q_1 = A_1 \cos(pt)$$

$$q_2 = A_2 \cos(pt)$$

$$q_3 = A_3 \cos(pt)$$

$$\ddot{q}_1 = -A_1 p^2 \cos(pt)$$

$$\ddot{q}_2 = -A_2 p^2 \cos(pt)$$

$$\ddot{q}_3 = -A_3 p^2 \cos(pt)$$

$$B_1 = -\ddot{q}_1 m_1$$

$$B_1' = -\ddot{q}_1 m_2$$

$$B_2 = -\ddot{q}_2 m_2$$

$$B_3 = -\ddot{q}_3 m_3$$

$$B_4 = -\ddot{q}_4 m_1 = -m_1 \operatorname{ctg} \alpha \ddot{q}_1$$

$$\begin{cases} q_1 = \delta_{11} B_1 + \delta_{12} B_2 + \delta_{13} B_3 + \delta_{11} B_1' + \delta_{14} B_4 - \delta_{13} P(t) \\ q_2 = \delta_{21} B_1 + \delta_{22} B_2 + \delta_{23} B_3 + \delta_{21} B_1' + \delta_{24} B_4 - \delta_{23} P(t) \\ q_3 = \delta_{31} B_1 + \delta_{32} B_2 + \delta_{33} B_3 + \delta_{31} B_1' + \delta_{34} B_4 - \delta_{33} P(t) \end{cases}$$

$$A_1 \cos(pt) =$$

$$\delta_{11}(m_1 A_1 p^2 \cos(pt) + m_2 A_1 p^2 \cos(pt)) + \delta_{12}(m_2 A_2 p^2 \cos(pt)) + \delta_{13}(m_3 A_3 p^2 \cos(pt)) + \delta_{14}(m_1 \operatorname{ctg} \alpha A_1 p^2 - \delta_{13} P_0 \cos(pt))$$

$$A_2 \cos(pt) =$$

$$\delta_{21}(m_1 A_1 p^2 \cos(pt) + m_2 A_1 p^2 \cos(pt)) + \delta_{22}(m_2 A_2 p^2 \cos(pt)) + \delta_{23}(m_3 A_3 p^2 \cos(pt)) + \delta_{24}(m_1 \operatorname{ctg} \alpha A_1 p^2 \cos(pt)) - \delta_{23} P_0 \cos(pt)$$

$$A_3 \cos(pt) = \delta_{31}(m_1 A_1 p^2 \cos(pt) + m_2 A_1 p^2 \cos(pt)) + \delta_{32}(m_2 A_2 p^2 \cos(pt)) + \delta_{33}(m_3 A_3 p^2 \cos(pt)) + \delta_{34}(m_1 \operatorname{ctg} \alpha A_1 p^2 \cos(pt)) - \delta_{33} P_0 \cos(pt)$$

$$A_1(1 - \delta_{11} A_1 p^2 (m_1 + m_2)) - A_2 \delta_{12} p^2 m_2 - A_3 \delta_{13} p^2 m_3 - A_1 \delta_{14} p^2 m_1 \operatorname{ctg} \alpha = -\delta_{13} P_0$$

$$-A_1(\delta_{21} p^2 (m_1 + m_2) + \delta_{24} p^2 m_1 \operatorname{ctg} \alpha) + A_2(1 - \delta_{22} p^2 m_2) - A_3 \delta_{23} p^2 m_3 = -\delta_{23} P_0$$

$$-A_1(\delta_{31} p^2 (m_1 + m_2) + \delta_{34} p^2 m_1 \operatorname{ctg} \alpha) - A_2 \delta_{32} p^2 m_2 + A_3(1 - \delta_{33} p^2 m_3) = -\delta_{33} P_0$$

$$m_1 = 200 \text{ kg}$$

$$m_2 = 330 \text{ kg}$$

$$m_3 = 520 \text{ kg}$$

$$P_0 = 18000 \text{ N}$$

$$p = 128,805299 \text{ rad/s}$$

$$\begin{cases} A_1 = 0,004404 \text{ m} \\ A_2 = -0,009652 \text{ m} \\ A_3 = -0,003401 \text{ m} \end{cases}$$

Siły bezwładności:

$$|B_1| = -\ddot{q}_1 m_1 = m_1 A_1 p^2 \cos(pt)$$

$$|B_2| = -\ddot{q}_2 m_2 = m_2 A_2 p^2 \cos(pt)$$

$$|B_3| = -\ddot{q}_3 m_3 = m_3 A_3 p^2 \cos(pt)$$

$$|B_4| = -\ddot{q}_4 m_1 = -m_1 \operatorname{ctg} \alpha q_1 = m_1 A_1 p^2 \cos(pt) \operatorname{ctg} \alpha$$

$$|B_1'| = -\ddot{q}_1 m_2 = m_2 A_1 p^2 \cos(pt)$$

Maksymalne wartości (amplitudy) sił bezwładności otrzymamy dla $\cos(pt) = 1$ lub $\cos(pt) = -1$.

Dla $\cos(pt) = 1,0$:

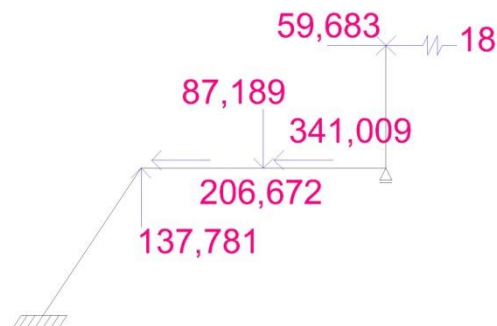
$$|B_1| = -206,672 \text{ kN}$$

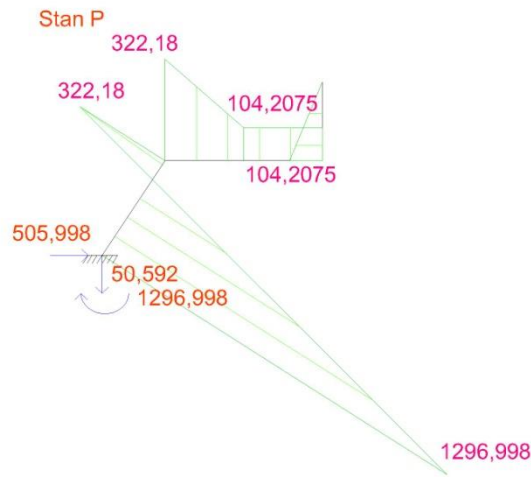
$$|B_2| = 87,189 \text{ kN}$$

$$|B_3| = 59,683 \text{ kN}$$

$$|B_4| = -137,781 \text{ kN}$$

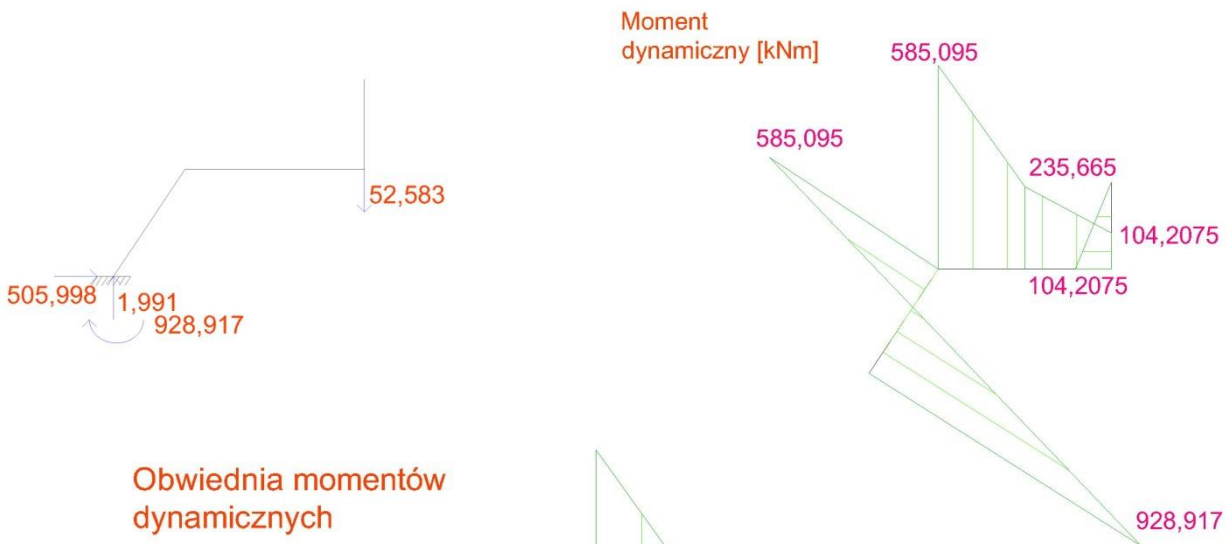
$$|B_1'| = -341,009 \text{ kN}$$



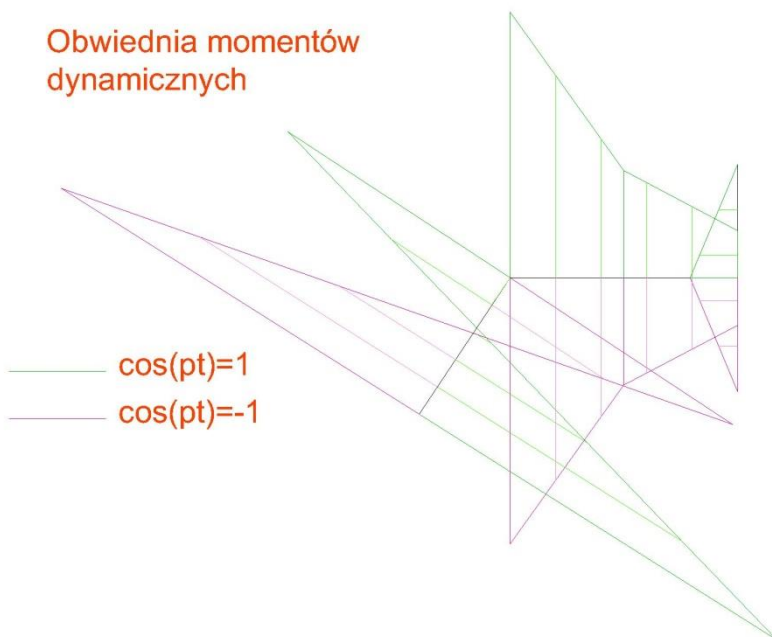


$$\delta_{1P} = \frac{1}{EI} \left[\frac{1}{2} * 322,18 * \sqrt{13} * \left(-\frac{2}{3} * 5 - \frac{1}{3} * 7 \right) + \frac{1}{2} * 1296,998 * \sqrt{13} * \left(\frac{2}{3} * 7 + \frac{1}{3} * 5 \right) + \frac{1}{2} * 104,2075 * 2,5 * \left(-\frac{1}{3} * 5 - \frac{2}{3} * 2,5 \right) + \frac{1}{2} * 322,18 * 2,5 * \left(-\frac{1}{3} * 5 - \frac{2}{3} * 2,5 \right) + 104,2075 * 2,5 * 0,5 * 2,5 \right] = \frac{9079,406546}{EI}$$

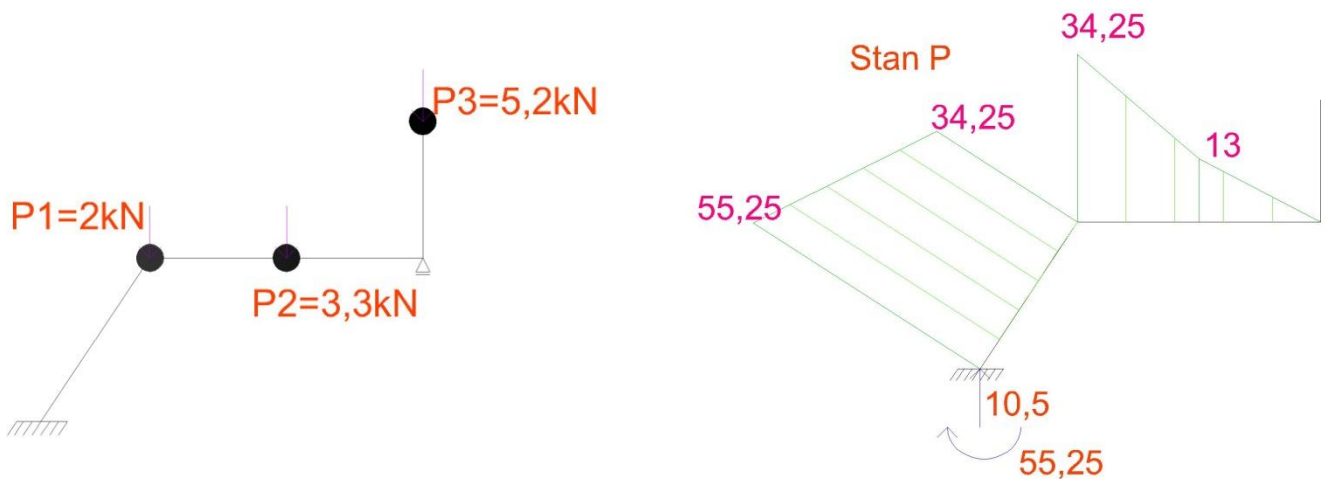
$$X_1 = -52,583 \text{ kN}$$



Obwiednia momentów dynamicznych

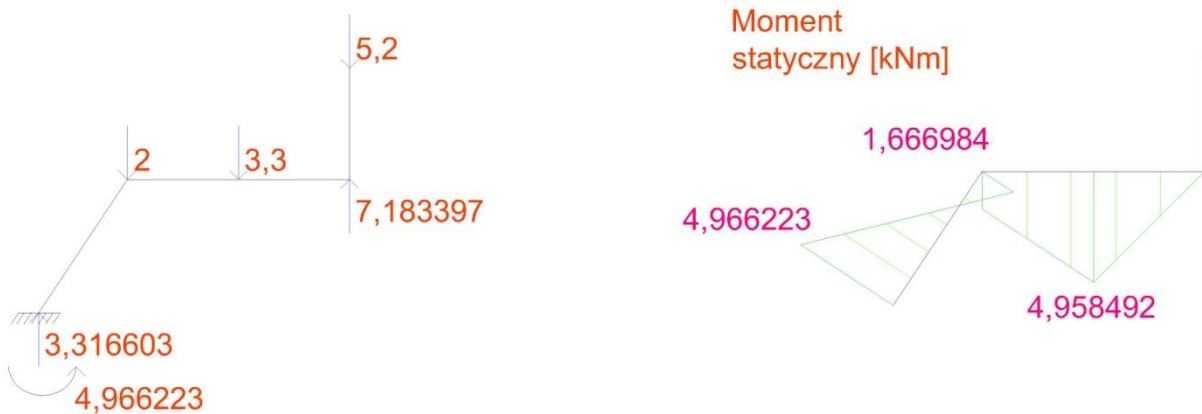


Obciążenie statyczne ciężarami mas



$$\delta_{1P} = \frac{1}{EI} \left[\frac{1}{2} * 55,25 * \sqrt{13} * \left(-\frac{2}{3} * 7 - \frac{1}{3} * 5 \right) + \frac{1}{2} * 34,25 * \sqrt{13} * \left(-\frac{2}{3} * 5 - \frac{1}{3} * 7 \right) - \frac{1}{2} * 34,25 * 2,5 * \left(\frac{2}{3} * 5 + \frac{1}{3} * 2,5 \right) + \frac{1}{2} * 13 * 2,5 * \left(-\frac{2}{3} * 2,5 - \frac{1}{3} * 5 \right) + \frac{1}{2} * 13 * 2,5 * \frac{2}{3} * 2,5 \right] = \frac{-1240,345364}{EI}$$

$$X_1 = 7,183397 \text{ kN}$$



Sprawdzenie maksymalnych naprężeń normalnych.

$$W = 278 \text{ cm}^3$$

$$M_{\max} = 5 * 928,917 + 1,2 * 4,966223 = 4646,809468 \text{ kNm}$$

$$\sigma_{dop} = 215 \text{ MPa}$$

$$\sigma = \frac{M_{\max}}{W}$$

$$\sigma = 16715,142 \text{ MPa}$$

$$\sigma > \sigma_{dop}$$

Warunek nie został spełniony, naprężenia przekroczone 78 – krotnie.