Politechnika Poznańska
Instytut Konstrukcji Budowlanych
Zakład Mechaniki Budowli

Ćwiczenie projektowe nr 2

Dynamika – ujęcie klasyczne

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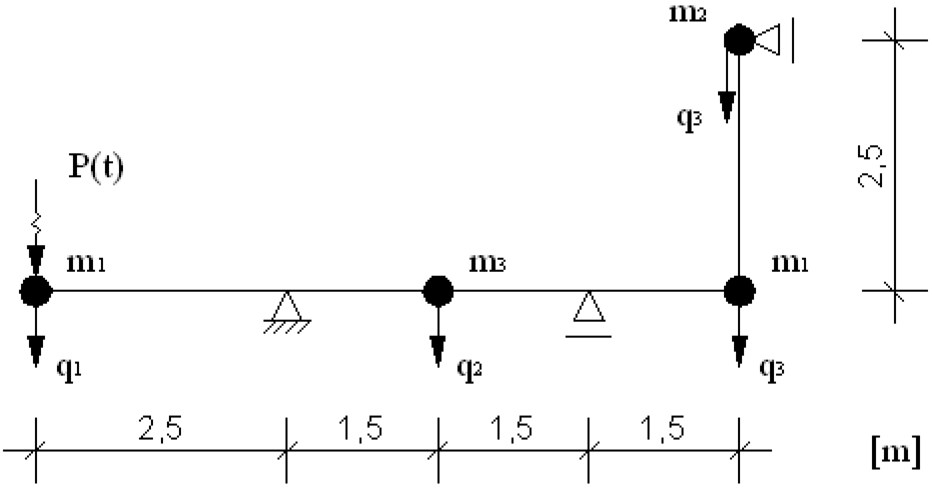
Nr indeksu:	113012, 113010
Semestr:	IV (pierwszy stopień)
Rok:	2013/2014

Ćwiczenie projektowe nr 2 – Dynamika – ujęcie klasyczne

1. Schemat statyczny i stopień dynamicznej swobody konstrukcji:



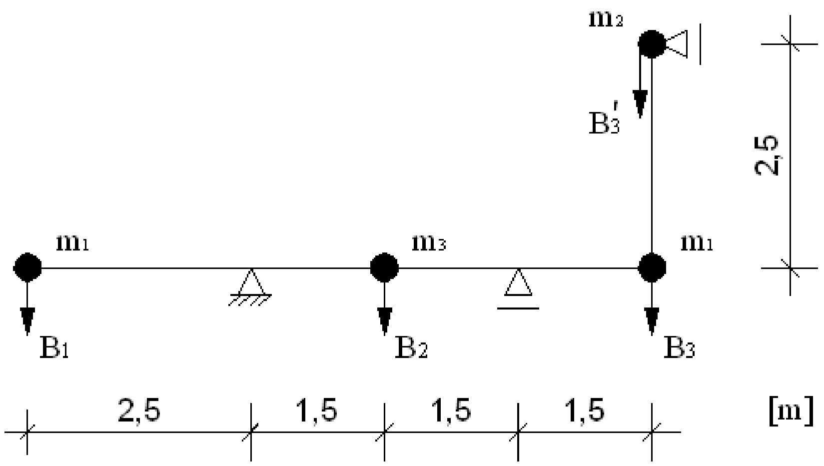
SSD = 3

2. Dane wyjściowe do projektu:

- $E = 205$ GPa
- I200 IPE – $I = 1940\ cm^4$
- $EI = 3977\ kNm^2 = 3977000\ Nm^2$
- $m_1 = 320$ kg
- $m_2 = 430$ kg
- $m_3 = 540$ kg
- $P = P_0 = 62{,}3$ kN = 62300 N
- $p = 28{,}0$ Hz $= 175{,}9292\ \frac{rad}{s}$

I. DRGANIA WŁASNE:

1. Siły działające na układ:



$q_1 = A_1 \cdot \sin(\omega t)$
$q_2 = A_2 \cdot \sin(\omega t)$
$q_3 = A_3 \cdot \sin(\omega t)$

$\ddot{q}_1 = - A_1 \cdot \omega^2 \cdot \sin(\omega t)$
$\ddot{q}_2 = - A_2 \cdot \omega^2 \cdot \sin(\omega t)$
$\ddot{q}_3 = - A_3 \cdot \omega^2 \cdot \sin(\omega t)$

$B_1 = - m_1 \cdot \ddot{q}_1$
$B_2 = - m_3 \cdot \ddot{q}_2$
$B_3 = - m_1 \cdot \ddot{q}_3$
$B_3' = - m_2 \cdot \ddot{q}_3$

$$\begin{cases} q_1 = \delta_{11} \cdot B_1 + \delta_{12} \cdot B_2 + \delta_{13} \cdot (B_3 + B_3') \\ q_2 = \delta_{21} \cdot B_1 + \delta_{22} \cdot B_2 + \delta_{23} \cdot (B_3 + B_3') \\ q_3 = \delta_{31} \cdot B_1 + \delta_{32} \cdot B_2 + \delta_{33} \cdot (B_3 + B_3') \end{cases}$$

$$\begin{cases} A_1 \sin(\omega t) = \delta_{11} \cdot 320{,}0 \cdot A_1 \cdot \omega^2 \sin(\omega t) + \delta_{12} \cdot 540{,}0 \cdot A_2 \cdot \omega^2 \sin(\omega t) + \delta_{13} \cdot (320{,}0 + 430{,}0) \cdot A_3 \cdot \omega^2 \sin(\omega t) \\ A_2 \sin(\omega t) = \delta_{21} \cdot 320{,}0 \cdot A_1 \cdot \omega^2 \sin(\omega t) + \delta_{22} \cdot 540{,}0 \cdot A_2 \cdot \omega^2 \sin(\omega t) + \delta_{23} \cdot (320{,}0 + 430{,}0) \cdot A_3 \cdot \omega^2 \sin(\omega t) \\ A_3 \sin(\omega t) = \delta_{31} \cdot 320{,}0 \cdot A_1 \cdot \omega^2 \sin(\omega t) + \delta_{32} \cdot 540{,}0 \cdot A_2 \cdot \omega^2 \sin(\omega t) + \delta_{33} \cdot (320{,}0 + 430{,}0) \cdot A_3 \cdot \omega^2 \sin(\omega t) \end{cases}$$

$$\begin{cases} A_1 = \delta_{11} \cdot 320{,}0 \cdot A_1 \cdot \omega^2 + \delta_{12} \cdot 540{,}0 \cdot A_2 \cdot \omega^2 + \delta_{13} \cdot (320{,}0 + 430{,}0) \cdot A_3 \cdot \omega^2 \\ A_2 = \delta_{21} \cdot 320{,}0 \cdot A_1 \cdot \omega^2 + \delta_{22} \cdot 540{,}0 \cdot A_2 \cdot \omega^2 + \delta_{23} \cdot (320{,}0 + 430{,}0) \cdot A_3 \cdot \omega^2 \\ A_3 = \delta_{31} \cdot 320{,}0 \cdot A_1 \cdot \omega^2 + \delta_{32} \cdot 540{,}0 \cdot A_2 \cdot \omega^2 + \delta_{33} \cdot (320{,}0 + 430{,}0) \cdot A_3 \cdot \omega^2 \end{cases}$$

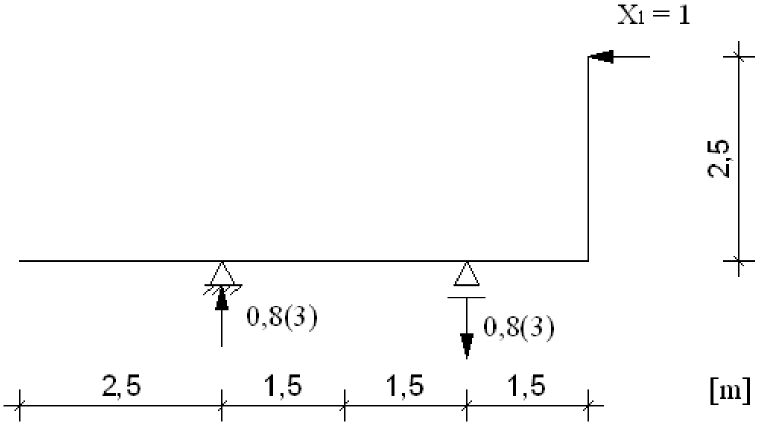
$$\begin{cases} A_1(1 - \delta_{11} \cdot 320{,}0 \cdot \omega^2) - \delta_{12} \cdot 540{,}0 \cdot A_2 \cdot \omega^2 - \delta_{13} \cdot (320{,}0 + 430{,}0) \cdot A_3 \cdot \omega^2 = 0 \\ - \delta_{21} \cdot 320{,}0 \cdot A_1 \cdot \omega^2 + A_2(1 - \delta_{22} \cdot 540{,}0 \cdot \omega^2) + \delta_{23} \cdot (320{,}0 + 430{,}0) \cdot A_3 \cdot \omega^2 = 0 \\ - \delta_{31} \cdot 320{,}0 \cdot A_1 \cdot \omega^2 - \delta_{32} \cdot 540{,}0 \cdot A_2 \cdot \omega^2 + A_3(1 - \delta_{33} \cdot (320{,}0 + 430{,}0) \cdot \omega^2) = 0 \end{cases}$$

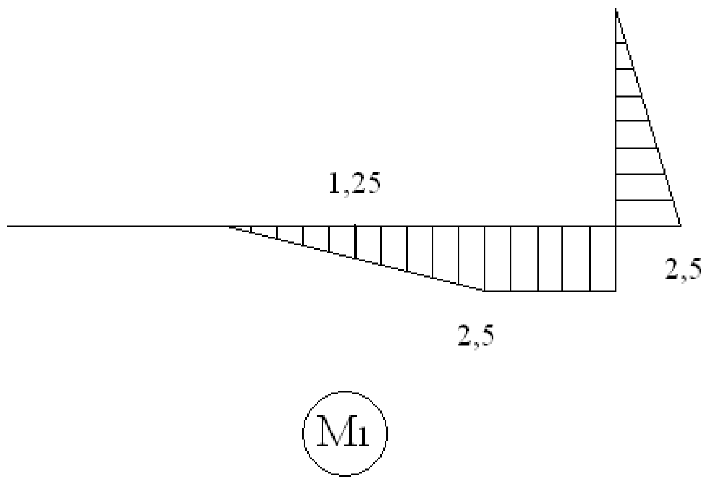
2. Obliczenie współczynników δ_{ik}:

$\delta_{ik} = \sum \int \frac{M_i \cdot M_k}{EI} dx$, M_i, M_k – momenty zginające wywołane jednostkową siłą po kierunku odpowiednio q_i oraz q_k

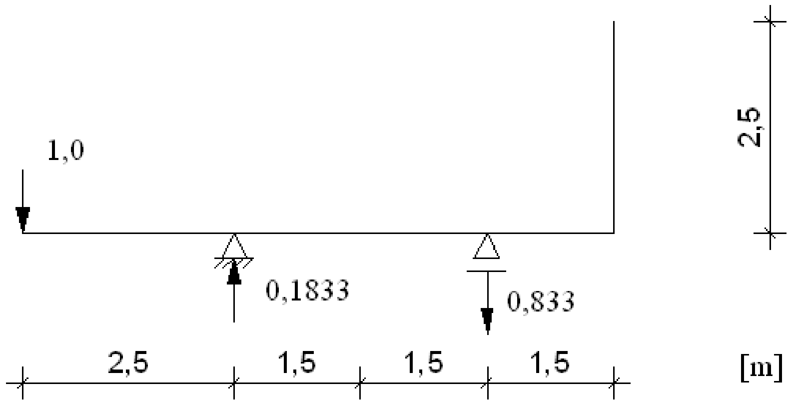
- **Wyznaczenie M_1^n:**

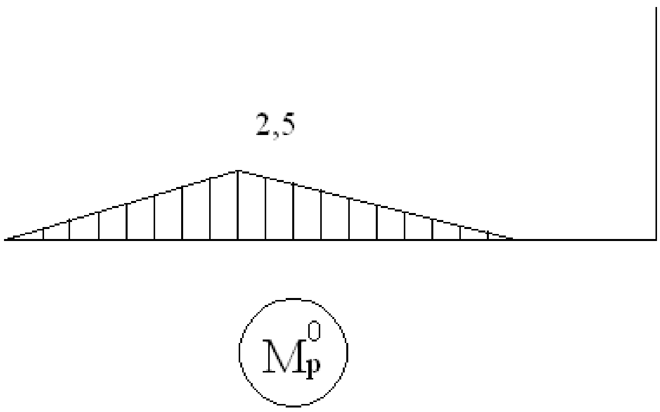
- Stan $X_1 = 1$:





- Stan „P_1" - siła jednostkowa po kierunku q_1:





$$\delta_{11}=\frac{1}{EI}\left[\frac{1}{2}\cdot 2{,}5\cdot 2{,}5\cdot\frac{2}{3}\cdot 2{,}5+\frac{1}{2}\cdot 3{,}0\cdot 2{,}5\cdot\frac{2}{3}\cdot 2{,}5+2{,}5\cdot 1{,}5\cdot 2{,}5\right]$$

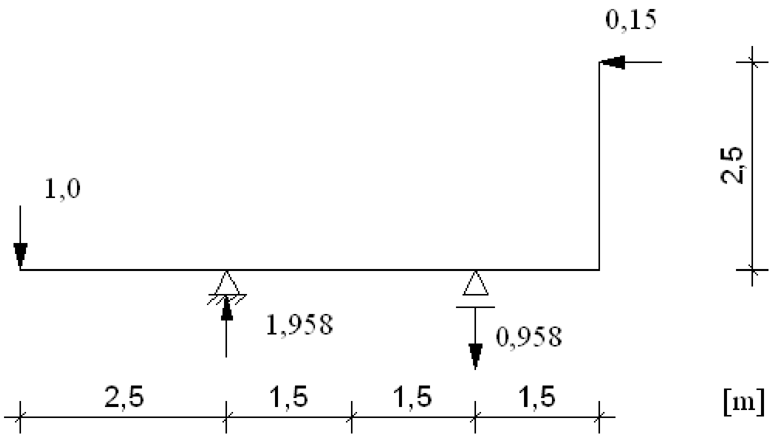
$$\boldsymbol{\delta_{11}=\frac{20{,}8333}{EI}}$$

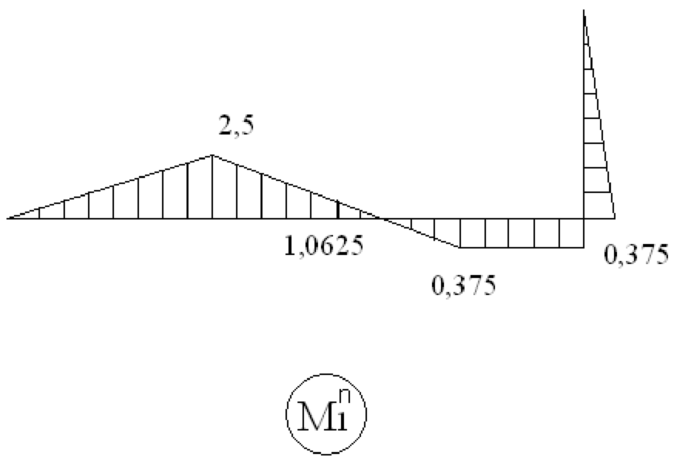
$$\delta_{1P}=\frac{1}{EI}\left[-\frac{1}{2}\cdot 2{,}5\cdot 3{,}0\cdot\frac{1}{3}\cdot 2{,}5\right]$$

$$\boldsymbol{\delta_{1P}=-\frac{3{,}125}{EI}}$$

$$X_1=-\frac{\delta_{1P}}{\delta_{11}}$$

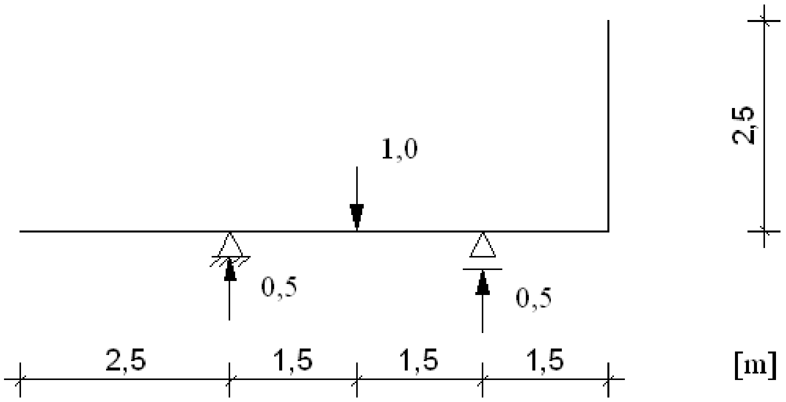
$\mathbf{X_1 = 0{,}15}$

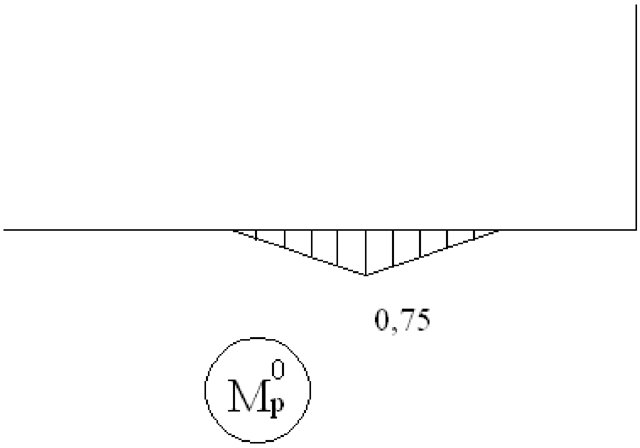




- **Wyznaczenie M_2^n:**

- Stan $X_1 = 1$ – jak poprzednio (str. 3):
- Stan „P” - siła jednostkowa po kierunku q_2:





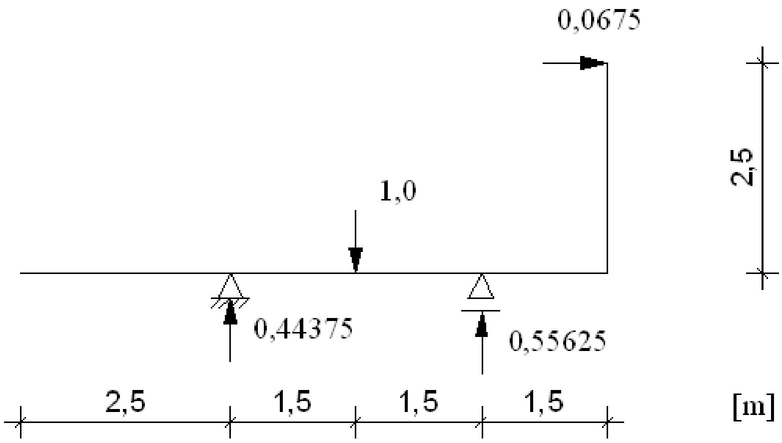
$\boldsymbol{\delta_{11}}$= $\frac{\mathbf{20,8333}}{\mathbf{EI}}$

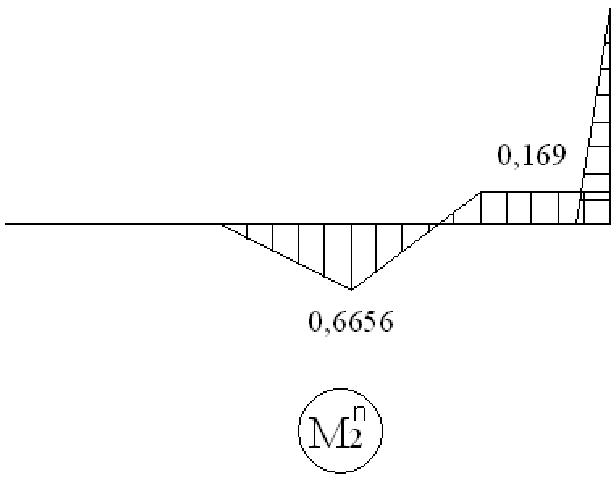
$\delta_{1P}= \frac{1}{EI}\left[\frac{1}{2} \cdot 1,5 \cdot 0,75 \cdot \frac{2}{3} \cdot 1,25 + \frac{1}{2} \cdot 1,5 \cdot 0,75 \cdot (\frac{2}{3} \cdot 1,25 + \frac{1}{3} \cdot 2,5\right]$

$\boldsymbol{\delta_{1P}}$= $\frac{\mathbf{1,40625}}{\mathbf{EI}}$

$X_1 = - \frac{\delta_{1P}}{\delta_{11}}$

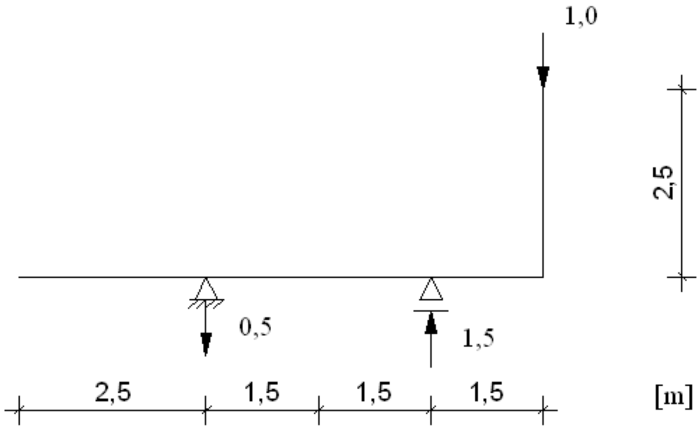
$\mathbf{X_1 = - 0,0675}$

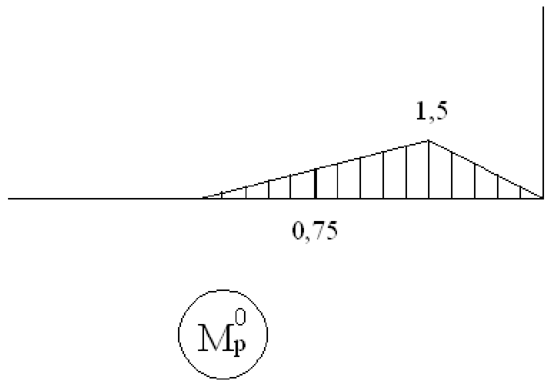




➢ Wyznaczenie $\mathbf{M_3^n}$:

- Stan $X_1 = 1$ – jak poprzednio (str. 3):
- Stan „P” - siła jednostkowa po kierunku q_3:



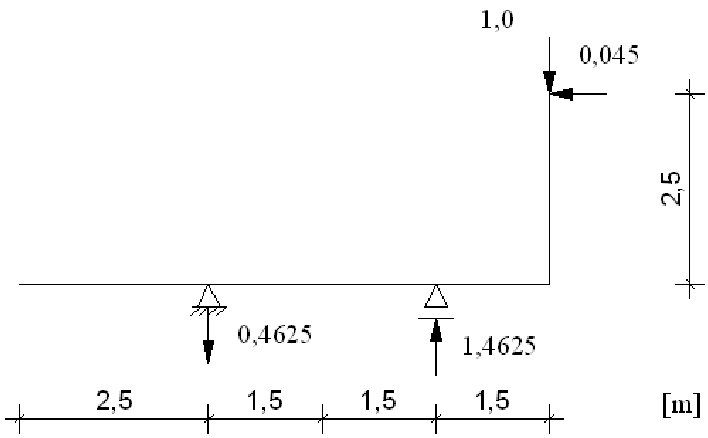


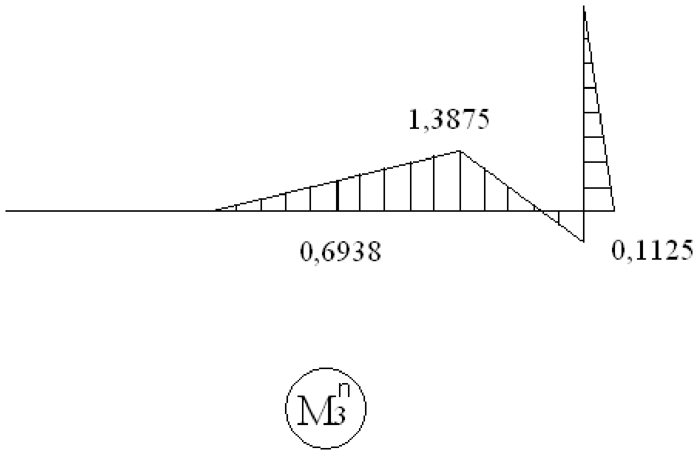
$\boldsymbol{\delta_{11}} = \frac{\mathbf{20{,}8333}}{\mathbf{EI}}$

$\delta_{1P} = \frac{1}{EI}\left[-\frac{1}{2}\cdot 1{,}5\cdot 3{,}0\cdot\frac{2}{3}\cdot 2{,}5 - \frac{1}{2}\cdot 1{,}5\cdot 1{,}5\cdot 2{,}5\right] = -\frac{\mathbf{0{,}9375}}{\mathbf{EI}}$

$X_1 = -\frac{\delta_{1P}}{\delta_{11}}$

$\mathbf{X_1 = 0{,}045}$





$\delta_{ik} = \sum \int \frac{M_i^n \cdot M_k^n}{EI} dx$

$\delta_{11} = \frac{1}{EI}\Big[\frac{1}{2} \cdot 2{,}5 \cdot 2{,}5 \cdot \frac{2}{3} \cdot 2{,}5 + \frac{1}{2} \cdot 2{,}5 \cdot 3{,}0 \cdot \left(\frac{2}{3} \cdot 2{,}5 - \frac{1}{3} \cdot 0{,}375\right) + \frac{1}{2} \cdot 0{,}375 \cdot 3{,}0 \cdot$

$\cdot \left(\frac{2}{3} \cdot 0{,}375 - \frac{1}{3} \cdot 2{,}5\right) + 0{,}375 \cdot 1{,}5 \cdot 0{,}375 + \frac{1}{2} \cdot 0{,}375 \cdot 2{,}5 \cdot \frac{2}{3} \cdot 0{,}375\Big]$

$\boldsymbol{\delta_{11} = \frac{10{,}99}{EI}}$

$\delta_{22} = \frac{1}{EI}\Big[\frac{1}{2} \cdot 1{,}5 \cdot 0{,}6656 \cdot \frac{2}{3} \cdot 0{,}6656 + \frac{1}{2} \cdot 0{,}6656 \cdot 1{,}5 \cdot \left(\frac{2}{3} \cdot 0{,}6656 - \frac{1}{3} \cdot 0{,}169\right) +$

$+ \frac{1}{2} \cdot 1{,}5 \cdot 0{,}169 \cdot \left(\frac{2}{3} \cdot 0{,}169 - \frac{1}{3} \cdot 0{,}6656\right) + 0{,}169 \cdot 1{,}5 \cdot 0{,}169 +$

$+ \frac{1}{2} \cdot 2{,}5 \cdot 0{,}169 \cdot \frac{2}{3} \cdot 0{,}169\Big]$

$\boldsymbol{\delta_{22} = \frac{0{,}4677}{EI}}$

$\delta_{33} = \frac{1}{EI}\Big[\frac{1}{2} \cdot 1{,}3875 \cdot 3{,}0 \cdot \frac{2}{3} \cdot 1{,}3875 + \frac{1}{2} \cdot 1{,}3875 \cdot 1{,}5 \cdot \left(\frac{2}{3} \cdot 1{,}3875 - \frac{1}{3} \cdot 0{,}1125\right) +$

$+ \frac{1}{2} \cdot 0{,}1125 \cdot 1{,}5 \cdot \left(\frac{2}{3} \cdot 0{,}1125 - \frac{1}{3} \cdot 1{,}3875\right) + \frac{1}{2} \cdot 0{,}1125 \cdot 2{,}5 \cdot \frac{2}{3} \cdot 0{,}1125\Big]$

$\boldsymbol{\delta_{33} = \frac{2{,}8266}{EI}}$

$\delta_{12} = \frac{1}{EI}\Big[-\frac{1}{2} \cdot 1{,}5 \cdot 0{,}6656 \cdot \left(\frac{2}{3} \cdot 1{,}0625 + \frac{1}{3} \cdot 2{,}5\right) + \frac{1}{2} \cdot 0{,}6656 \cdot 1{,}5 \cdot$

$\cdot \left(-\frac{2}{3} \cdot 1{,}0625 + \frac{1}{3} \cdot 0{,}375\right) + \frac{1}{2} \cdot 1{,}5 \cdot 0{,}169 \cdot \left(-\frac{2}{3} \cdot 0{,}375 + \frac{1}{3} \cdot 1{,}0625\right) -$

$- 0{,}169 \cdot 1{,}5 \cdot 0{,}375 - \frac{1}{2} \cdot 2{,}5 \cdot 0{,}169 \cdot \frac{2}{3} \cdot 0{,}375\Big]$

$\boldsymbol{\delta_{12} = -\frac{1{,}0}{EI}}$

$\delta_{13} = \frac{1}{EI}\Big[\frac{1}{2} \cdot 1{,}3875 \cdot 3{,}0 \cdot \left(-\frac{2}{3} \cdot 0{,}375 + \frac{1}{3} \cdot 2{,}5\right) - \frac{1}{2} \cdot 1{,}5 \cdot 1{,}3875 \cdot 0{,}375 +$

$+ \frac{1}{2} \cdot 0{,}1125 \cdot 1{,}5 \cdot 0{,}375 + \frac{1}{2} \cdot 0{,}1125 \cdot 2{,}5 \cdot \frac{2}{3} \cdot 0{,}375\Big]$

$\boldsymbol{\delta_{13} = \frac{0{,}8906}{EI}}$

$\delta_{23} = \frac{1}{EI}\Big[-\frac{1}{2} \cdot 0{,}6656 \cdot 1{,}5 \cdot \frac{2}{3} \cdot 0{,}69 - \frac{1}{2} \cdot 1{,}5 \cdot 0{,}6656 \cdot \left(\frac{2}{3} \cdot 0{,}69 + \frac{1}{3} \cdot 1{,}3875\right) +$

$$+\frac{1}{2}\cdot 0{,}169\cdot 1{,}5\cdot\left(\frac{2}{3}\cdot 1{,}3875+\frac{1}{3}\cdot 0{,}69\right)+\frac{1}{2}\cdot 1{,}3875\cdot 1{,}5\cdot 0{,}169-$$
$$\left.-\frac{1}{2}\cdot 0{,}1125\cdot 1{,}5\cdot 0{,}169-\frac{1}{2}\cdot 0{,}169\cdot 2{,}5\cdot\frac{2}{3}\cdot 0{,}1125\right]$$

$$\boldsymbol{\delta_{23} = -\frac{0{,}3984}{EI}}$$

$$\begin{cases} A_1\left(1-\frac{10{,}99}{EI}\cdot 320{,}0\cdot\omega^2\right)+\frac{1{,}0}{EI}\cdot 540{,}0\cdot A_2\cdot\omega^2-\frac{0{,}8906}{EI}\cdot(320{,}0+430{,}0)\cdot A_3\cdot\omega^2=0\\ \frac{1{,}0}{EI}\cdot 320{,}0\cdot A_1\cdot\omega^2+A_2(1-\frac{0{,}4677}{EI}\cdot 540{,}0\cdot\omega^2)+\frac{0{,}3984}{EI}\cdot(320{,}0+430{,}0)\cdot A_3\cdot\omega^2=0\\ -\frac{0{,}8906}{EI}\cdot 320{,}0\cdot A_1\cdot\omega^2+\frac{0{,}3984}{EI}\cdot 540{,}0\cdot A_2\cdot\omega^2+A_3(1-\frac{2{,}8266}{EI}\cdot(320{,}0+430{,}0)\cdot\omega^2)=0 \end{cases}$$

$$\lambda = \frac{\omega^2}{EI}$$

$$\begin{cases} A_1(1-3516{,}8\cdot\lambda)+540{,}0\cdot A_2\cdot\lambda-667{,}5\cdot A_3\cdot\lambda=0\\ 320{,}0\cdot A_1\cdot\lambda+A_2(1-252{,}558\cdot\lambda)+298{,}5\cdot A_3\cdot\lambda=0\\ -284{,}8\cdot A_1\cdot\lambda+214{,}92\cdot A_2\cdot\lambda+A_3(1-2119{,}95\cdot\lambda)=0 \end{cases}$$

- Wyznacznik:

$$\begin{vmatrix} (1-3516{,}8\cdot\lambda) & 540{,}0\cdot\lambda & -667{,}5\cdot\lambda\\ 320{,}0\cdot\lambda & (1-252{,}558\cdot\lambda) & 298{,}5\cdot\lambda\\ -284{,}8\cdot\lambda & 214{,}92\cdot\lambda & (1-2119{,}95\cdot\lambda) \end{vmatrix}=0$$

- Wartości obliczonych współczynników λ:

$\lambda_1 = 0{,}0002699$
$\lambda_2 = 0{,}0004990$
$\lambda_3 = 0{,}0055631$

3. Obliczenie częstości drgań własnych:

$$\omega = \sqrt{\lambda\cdot EI}$$

$$\omega_1 = \sqrt{0{,}0002699\cdot 3977000}$$
$$\boldsymbol{\omega_1 = 32{,}76\ \frac{rad}{s}}$$
$$\omega_2 = \sqrt{0{,}0004990\cdot 3977000}$$
$$\boldsymbol{\omega_2 = 44{,}55\ \frac{rad}{s}}$$
$$\omega_3 = \sqrt{0{,}0055631\cdot 3977000}$$
$$\boldsymbol{\omega_3 = 148{,}74\ \frac{rad}{s}}$$

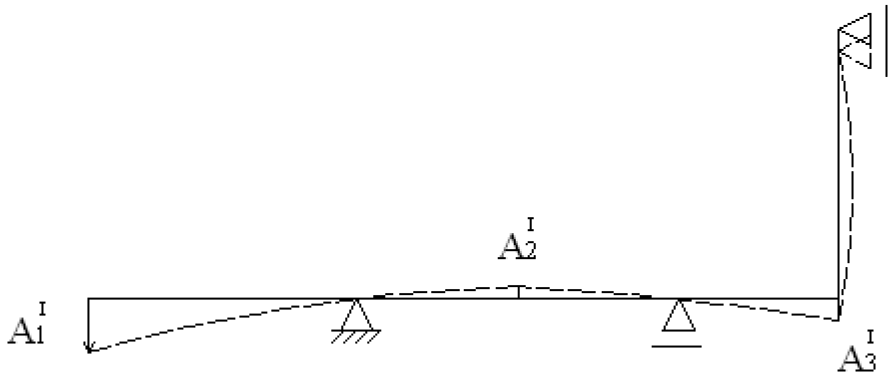
4. Obliczenie postaci drgań własnych:

- I postać drgań własnych:

  – $\lambda_1 = 0{,}0002699$
  – $A_1^I = 1{,}0$

$$\begin{cases} A_1(1 - 3516{,}8 \cdot \lambda) + 540{,}0 \cdot A_2 \cdot \lambda - 667{,}5 \cdot A_3 \cdot \lambda = 0 \\ 320{,}0 \cdot A_1 \cdot \lambda + A_2(1 - 252{,}558 \cdot \lambda) + 298{,}5 \cdot A_3 \cdot \lambda = 0 \end{cases}$$

$$\begin{cases} \mathbf{A_3^I = 0,1944} \\ \mathbf{A_2^I = -0,1094} \end{cases}$$

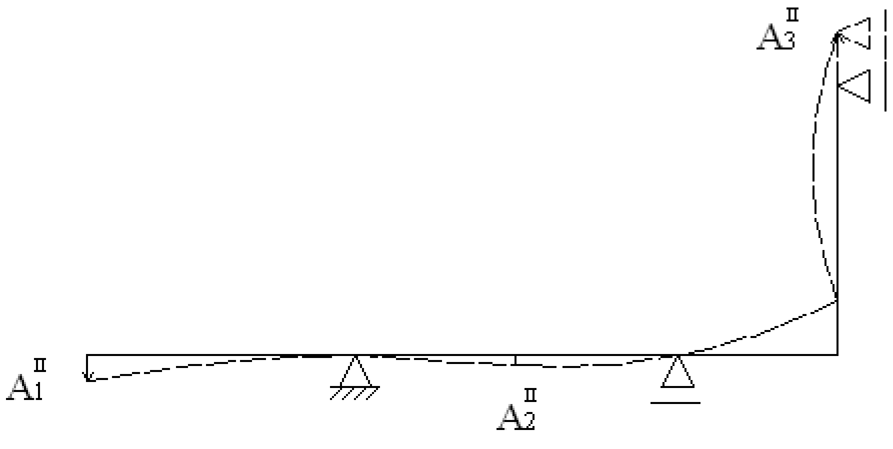


- II postać drgań własnych:

  – $\lambda_2 = 0{,}0004990$
  – $A_1^{II} = 1{,}0$

$$\begin{cases} A_1(1 - 3516{,}8 \cdot \lambda) + 540{,}0 \cdot A_2 \cdot \lambda - 667{,}5 \cdot A_3 \cdot \lambda = 0 \\ 320{,}0 \cdot A_1 \cdot \lambda + A_2(1 - 252{,}558 \cdot \lambda) + 298{,}5 \cdot A_3 \cdot \lambda = 0 \end{cases}$$

$$\begin{cases} \mathbf{A_3^{II} = -2,1218} \\ \mathbf{A_2^{II} = 0,17893} \end{cases}$$

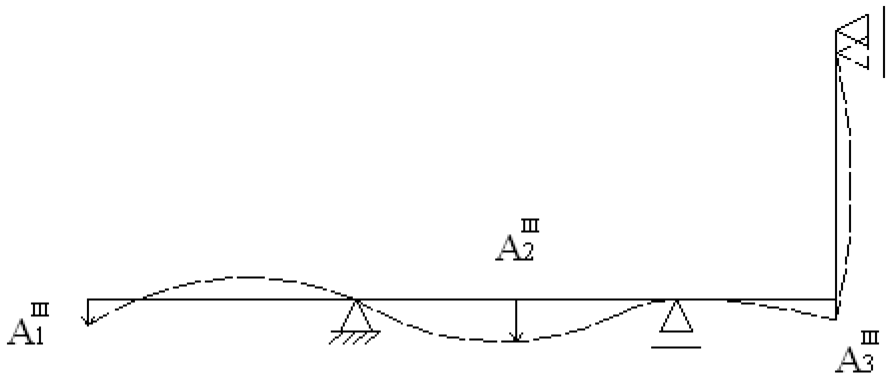


- III postać drgań własnych:

  – λ_3 = 0,005563
  – A_1^{III} = 1,0

$$\begin{cases} A_1(1-3516{,}8\cdot\lambda)+540{,}0\cdot A_2\cdot\lambda-667{,}5\cdot A_3\cdot\lambda=0 \\ 320{,}0\cdot A_1\cdot\lambda+A_2(1-252{,}558\cdot\lambda)+298{,}5\cdot A_3\cdot\lambda=0 \end{cases}$$

$$\begin{cases} \mathbf{A_3^{III}=0{,}6231} \\ \mathbf{A_2^{III}=\ 6{,}9499} \end{cases}$$



5. Sprawdzenie ortogonalności drgań:

- I para drgań:

$A_1^I \cdot A_1^{II} \cdot m_1 + A_2^I \cdot A_2^{II} \cdot m_3 + A_3^I \cdot A_3^{II} \cdot (m_1 + m_2) = 0$

1,0 · 1,0 · 320,0 + (-0,1094768) · 0,17893497 · 540,0 + 0,1944367 · (-2,12183426) · 750,0 = 0

- 0,000021676 ≅ 0

- II para drgań:

$A_1^{II} \cdot A_1^{III} \cdot m_1 + A_2^{II} \cdot A_2^{III} \cdot m_3 + A_3^{II} \cdot A_3^{III} \cdot (m_1 + m_2) = 0$

1,0 · 1,0 · 320,0 + 0,17893497 · 6,94989563 · 540,0 + (-2,12183426) · 0,62306651 · 750,0 = 0

- 0,00004268 ≅ 0

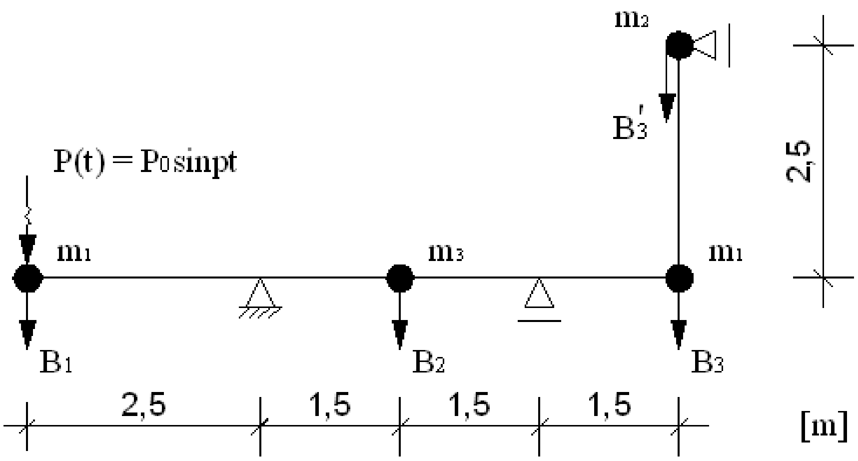
- III para drgań:

$A_1^I \cdot A_1^{III} \cdot m_1 + A_2^I \cdot A_2^{III} \cdot m_3 + A_3^I \cdot A_3^{III} \cdot (m_1 + m_2) = 0$

1,0 · 1,0 · 320,0 + (-0,1094768) · 6,94989563 · 540,0 + 0,1944367 · 0,62306651 · 750,0 = 0

- 0,000013236 ≅ 0

II. DRGANIA WYMUSZONE:



1. Dane:

- P_0 = 62,3 kN = 62300 N
- p = 28,0 Hz = 175,9292 $\frac{rad}{s}$
- najbliższa częstość drgań własnych: ω_3 = 148,75 $\frac{rad}{s}$
- $\eta = \frac{p}{\omega} = \frac{175,93}{148,74} = 1,18$ => strefa rezonansowa ($0,75 < \eta < 1,25$)

$q_1 = A_1 \cdot \sin(pt)$
$q_2 = A_2 \cdot \sin(pt)$
$q_3 = A_3 \cdot \sin(pt)$

$\ddot{q}_1 = - A_1 \cdot p^2 \cdot \sin(pt)$
$\ddot{q}_2 = - A_2 \cdot p^2 \cdot \sin(pt)$
$\ddot{q}_3 = - A_3 \cdot p^2 \cdot \sin(pt)$

$B_1 = - m_1 \cdot \ddot{q}_1$
$B_2 = - m_3 \cdot \ddot{q}_2$
$B_3 = - m_1 \cdot \ddot{q}_3$
$B_3' = - m_2 \cdot \ddot{q}_3$

$$\begin{cases} q_1 = \delta_{11} \cdot B_1 + \delta_{12} \cdot B_2 + \delta_{13} \cdot (B_3 + B_3') + \delta_{11} \cdot P(t) \\ q_2 = \delta_{21} \cdot B_1 + \delta_{22} \cdot B_2 + \delta_{23} \cdot (B_3 + B_3') + \delta_{21} \cdot P(t) \\ q_3 = \delta_{31} \cdot B_1 + \delta_{32} \cdot B_2 + \delta_{33} \cdot (B_3 + B_3') + \delta_{31} \cdot P(t) \end{cases}$$

$$\begin{cases} A_1 \sin(pt) = \delta_{11} \cdot 320,0 \cdot A_1 \cdot p^2 \sin(pt) + \delta_{12} \cdot 540,0 \cdot A_2 \cdot p^2 \sin(pt) + \delta_{13} \cdot (320,0 + 430,0) \cdot A_3 \cdot p^2 \sin(pt) + \delta_{11} \cdot P_0 \sin(pt) \\ A_2 \sin(pt) = \delta_{21} \cdot 320,0 \cdot A_1 \cdot p^2 \sin(pt) + \delta_{22} \cdot 540,0 \cdot A_2 \cdot p^2 \sin(pt) + \delta_{23} \cdot (320,0 + 430,0) \cdot A_3 \cdot p^2 \sin(pt) + \delta_{21} \cdot P_0 \sin(pt) \\ A_3 \sin(pt) = \delta_{31} \cdot 320,0 \cdot A_1 \cdot p^2 \sin(pt) + \delta_{32} \cdot 540,0 \cdot A_2 \cdot p^2 \sin(pt) + \delta_{33} \cdot (320,0 + 430,0) \cdot A_3 \cdot p^2 \sin(pt) + \delta_{31} \cdot P_0 \sin(pt) \end{cases}$$

$$\begin{cases} A_1 = \delta_{11} \cdot 320,0 \cdot A_1 \cdot p^2 + \delta_{12} \cdot 540,0 \cdot A_2 \cdot p^2 + \delta_{13} \cdot (320,0 + 430,0) \cdot A_3 \cdot p^2 + \delta_{11} \cdot P_0 \\ A_2 = \delta_{21} \cdot 320,0 \cdot A_1 \cdot p^2 + \delta_{22} \cdot 540,0 \cdot A_2 \cdot p^2 + \delta_{23} \cdot (320,0 + 430,0) \cdot A_3 \cdot p^2 + \delta_{21} \cdot P_0 \\ A_3 = \delta_{31} \cdot 320,0 \cdot A_1 \cdot p^2 + \delta_{32} \cdot 540,0 \cdot A_2 \cdot p^2 + \delta_{33} \cdot (320,0 + 430,0) \cdot A_3 \cdot p^2 + \delta_{31} \cdot P_0 \end{cases}$$

$$\begin{cases} A_1(1-\delta_{11}\cdot 320{,}0\cdot p^2)-\delta_{12}\cdot 540{,}0\cdot A_2\cdot p^2-\delta_{13}\cdot(320{,}0+430{,}0)\cdot A_3\cdot p^2-\delta_{11}\cdot P_0\ =0 \\ -\delta_{21}\cdot 320{,}0\cdot A_1\cdot p^2+A_2(1-\delta_{22}\cdot 540{,}0\cdot p^2)+\delta_{23}\cdot(320{,}0+430{,}0)\cdot A_3\cdot p^2-\delta_{21}\cdot P_0\ =0 \\ -\delta_{31}\cdot 320{,}0\cdot A_1\cdot p^2-\delta_{32}\cdot 540{,}0\cdot A_2\cdot p^2+A_3(1-\delta_{33}\cdot(320{,}0+430{,}0)\cdot p^2)-\delta_{31}\cdot P_0\ =0 \end{cases}$$

Współczynniki δ_{ik} są takie same jak dla drgań własnych (str.9)

$$\begin{cases} A_1\left(1-\frac{10{,}99}{EI}\cdot 320{,}0\cdot(175{,}9292)^2\right)+\frac{1{,}0}{EI}\cdot 540{,}0\cdot A_2\cdot(175{,}9292)^2-\frac{0{,}89}{EI}\cdot(320{,}0+430{,}0)\cdot A_3\cdot(175{,}9292)^2-\frac{10{,}99}{EI}\cdot 62300=0 \\ \frac{1{,}0}{EI}\cdot 320{,}0\cdot A_1\cdot(175{,}9292)^2+A_2\left(1-\frac{0{,}4677}{EI}\cdot 540{,}0\cdot(175{,}9292)^2\right)+\frac{0{,}398}{EI}\cdot(320{,}0+430{,}0)\cdot A_3\cdot(175{,}9292)^2+\frac{1{,}0}{EI}\cdot 62300=0 \\ -\frac{0{,}89}{EI}\cdot 320{,}0\cdot A_1\cdot(175{,}9292)\omega^2+\frac{0{,}398}{EI}\cdot 540{,}0\cdot A_2\cdot(175{,}9292)^2+A_3(1-\frac{2{,}8266}{EI}\cdot(320{,}0+430{,}0)\cdot(175{,}9292)^2-\frac{0{,}89}{EI}\cdot 62300=0 \end{cases}$$

$$\begin{cases} -26{,}3696\cdot A_1+4{,}2026\cdot A_2-5{,}1948\cdot A_3=0{,}1722 \\ 2{,}4904\cdot A_1-0{,}9655\cdot A_2+2{,}3231\cdot A_3=-\,0{,}01567 \\ -2{,}2165\cdot A_1+1{,}6726\cdot A_2-15{,}4986\cdot A_3=0{,}01394 \end{cases}$$

1. Rozwiązanie układu równań – metoda wyznaczników:

$$W=\begin{vmatrix} -26{,}3696 & 4{,}2026 & -5{,}1948 \\ 2{,}4904 & -0{,}9655 & 2{,}3231 \\ -2{,}2165 & 1{,}6726 & -15{,}4986 \end{vmatrix}=\mathbf{422{,}17916}$$

$$W_1=\begin{vmatrix} 0{,}1722 & 4{,}2026 & -5{,}1948 \\ -\,0{,}01567 & -0{,}9655 & 2{,}3231 \\ 0{,}01394 & 1{,}6726 & -15{,}4986 \end{vmatrix}=\mathbf{1{,}08936}$$

$$W_2=\begin{vmatrix} -26{,}3696 & 0{,}1722 & -5{,}1948 \\ 2{,}4904 & -\,0{,}01567 & 2{,}3231 \\ -2{,}2165 & 0{,}01394 & -15{,}4986 \end{vmatrix}=\mathbf{0{,}20968}$$

$$W_3=\begin{vmatrix} -26{,}3696 & 4{,}2026 & 0{,}1722 \\ 2{,}4904 & -0{,}9655 & -\,0{,}01567 \\ -2{,}2165 & 1{,}6726 & 0{,}01394 \end{vmatrix}=\mathbf{0{,}01262}$$

Obliczenie amplitud drgań wymuszonych:

$A_1=\frac{W_1}{W}=\frac{1{,}08936}{422{,}17916}$ **= 0,00258 m**

$A_2=\frac{W_2}{W}=\frac{0{,}20968}{422{,}17916}$ **= 0,000497 m**

$A_3=\frac{W_3}{W}=\frac{0{,}01262}{422{,}17916}$ **= 00002989 m**

2. Obliczenie sił bezwładności (dla sin(pt) = 1):

$B_1 = m_1\cdot A_1\cdot p^2\cdot \sin(pt)$

$B_1 = 320{,}0\cdot 0{,}00258\cdot(175{,}9292)^2\cdot 1{,}0 = 25550{,}0$ N

B_1 = 25,55 kN

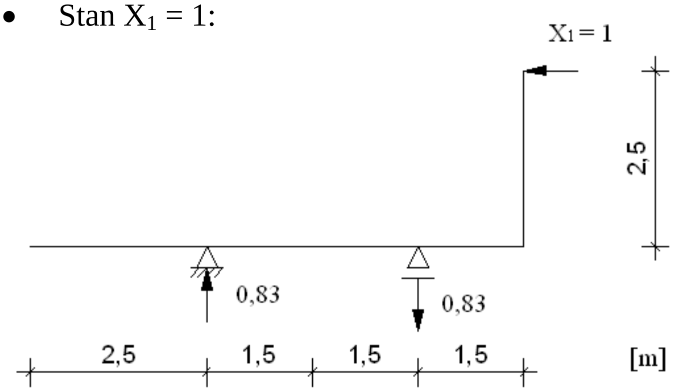
$B_2 = m_3 \cdot A_2 \cdot p^2 \cdot \sin(pt)$
$B_2 = 540{,}0 \cdot 0{,}000497 \cdot (175{,}9292)^2 \cdot 1{,}0 = 8310{,}0$ N
B_2 = 8,31kN

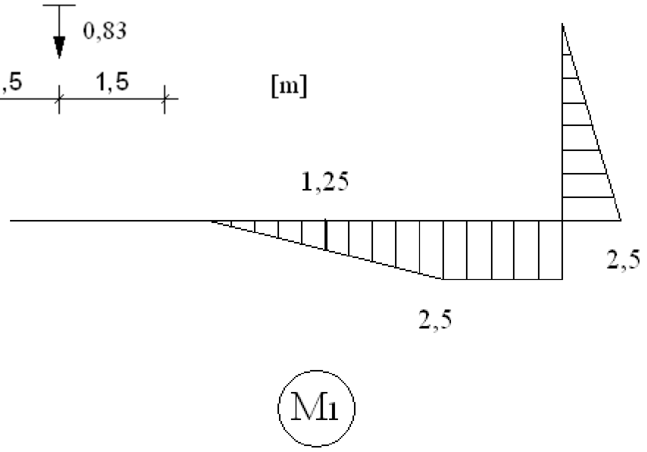
$B_3 = m_1 \cdot A_3 \cdot p^2 \cdot \sin(pt)$
$B_3 = 320{,}0 \cdot 0{,}00002989 \cdot (175{,}9292)^2 \cdot 1{,}0 = 296{,}0$ N
B_3 = 0,2960 kN

$B_3' = m_2 \cdot A_3 \cdot p^2 \cdot \sin(pt)$
$B_3' = 430{,}0 \cdot 0{,}00002989 \cdot (175{,}9292)^2 \cdot 1{,}0 = 397{,}8$ N
$B_3' =$ **0,3978 kN**

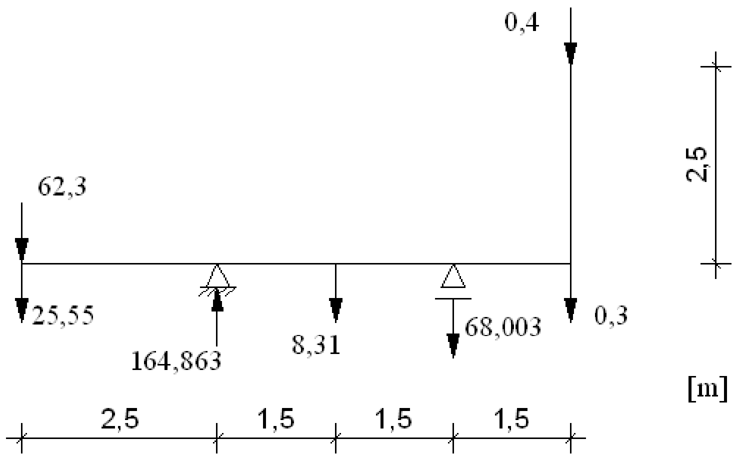
3. Obliczenie obwiedni momentów dynamicznych:

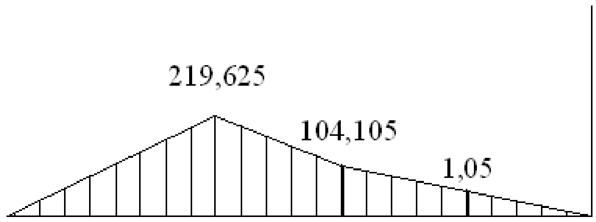
- Stan $X_1 = 1$:





- Stan „P”:



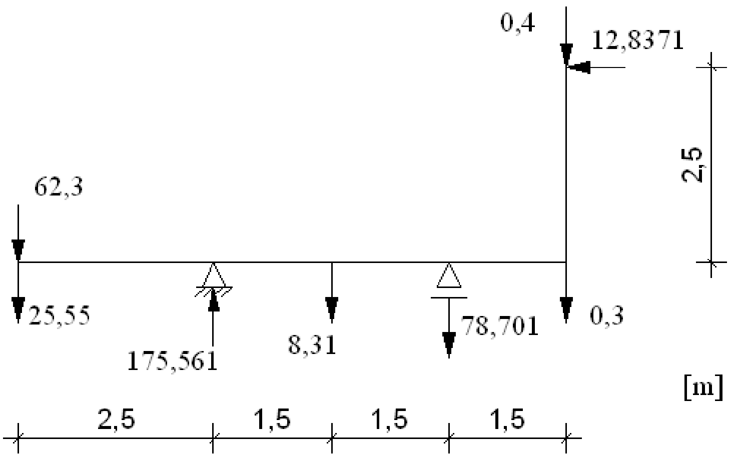


M_p^0

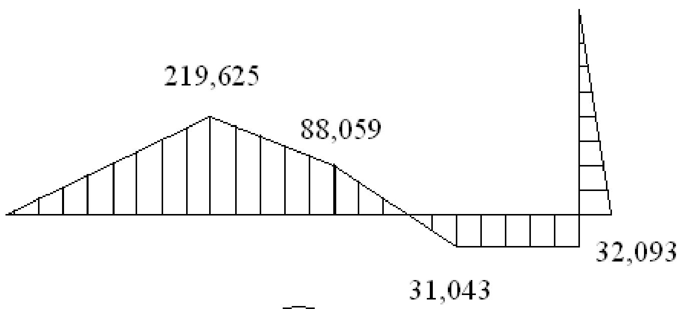
$$\delta_{11} = \frac{1}{EI}\left[\frac{1}{2}\cdot 2{,}5\cdot 2{,}5\cdot\frac{2}{3}\cdot 2{,}5 + \frac{1}{2}\cdot 3{,}0\cdot 2{,}5\cdot\frac{2}{3}\cdot 2{,}5 + 2{,}5\cdot 1{,}5\cdot 2{,}5\right] = \mathbf{\frac{20{,}8333}{EI}}$$

$$\delta_{1P} = \frac{1}{EI}\left[-\frac{1}{2}\cdot 1{,}25\cdot 1{,}5\cdot\left(\frac{2}{3}\cdot 104{,}105 + \frac{1}{3}\cdot 219{,}625\right) - \frac{1}{2}\cdot 1{,}25\cdot 1{,}5\cdot(\frac{2}{3}\cdot 104{,}105 + \right.$$
$$\left. + \frac{1}{3}\cdot 1{,}05) - \frac{1}{2}\cdot 2{,}5\cdot 1{,}5\cdot\left(\frac{2}{3}\cdot 1{,}05 + \frac{1}{3}\cdot 104{,}105\right) - \frac{1}{2}\cdot 1{,}05\cdot 1{,}5\cdot 2{,}5\right] = -\mathbf{\frac{267{,}4391}{EI}}$$

$X_1 = -\frac{\delta_{1P}}{\delta_{11}}$ **= 12,8371 kN**

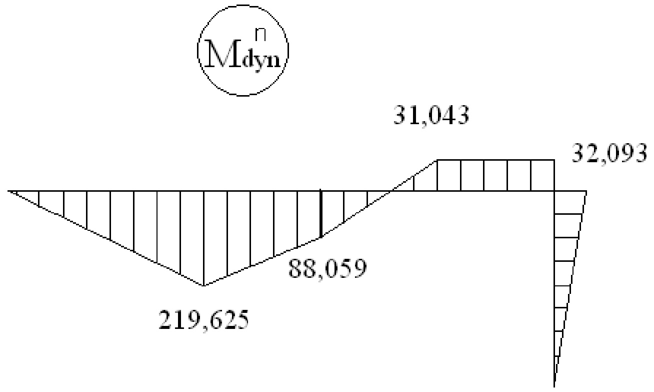


- Wykres momentów dynamicznych dla sin(pt) = 1



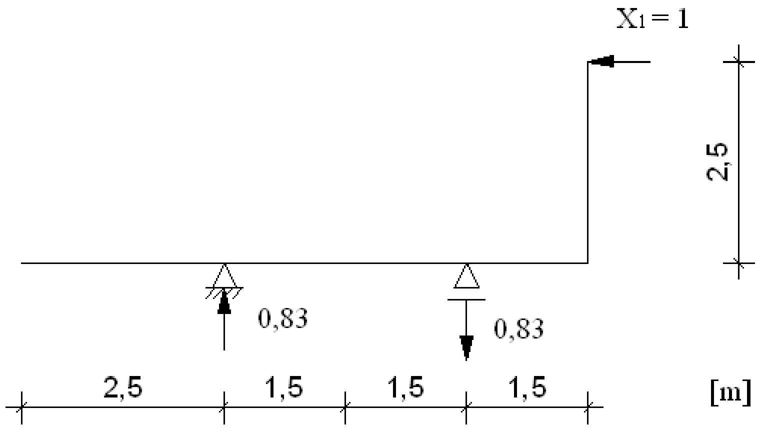
M_{dyn}^n

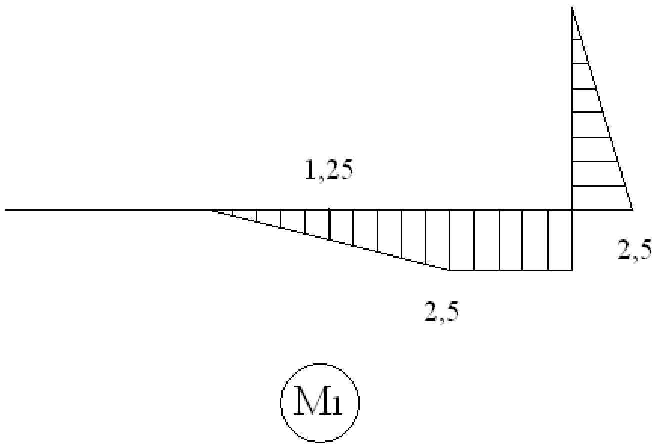
- Wykres momentów dynamicznych dla sin(pt) = - 1



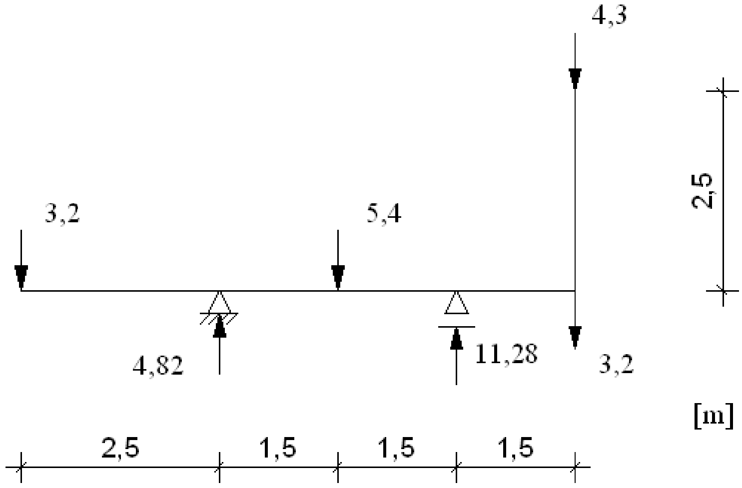
4. Obliczenie momentów statycznych od obciążenia ciężarem własnym (ciężar własny prętów pominięto):

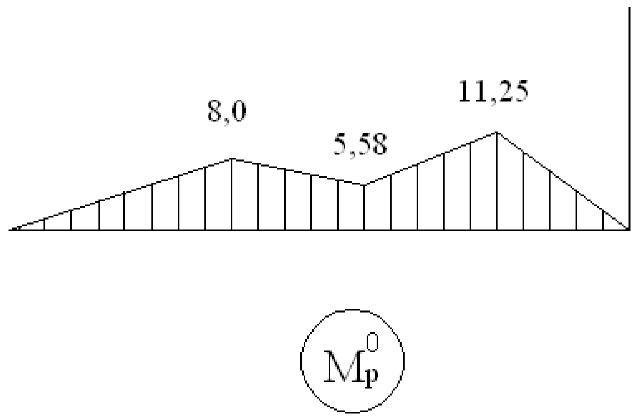
- Stan $X_1 = 1$:





- Stan „P”:

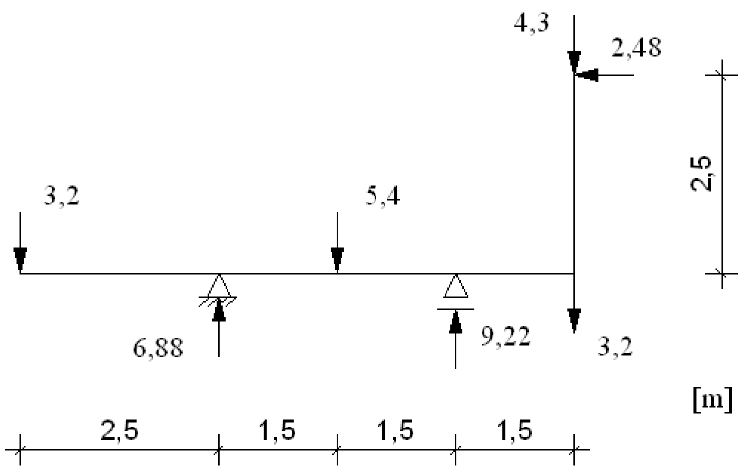


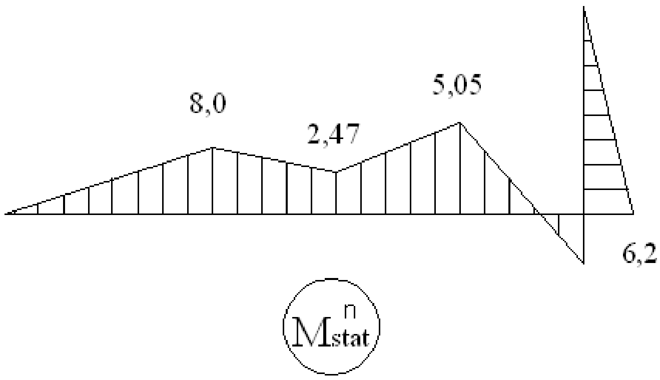


$\boldsymbol{\delta_{11}} = \frac{\mathbf{20{,}8333}}{\mathbf{EI}}$

$\delta_{1P} = \frac{1}{EI}\left[-\frac{1}{2}\cdot 1{,}25\cdot 1{,}5\cdot\left(\frac{2}{3}\cdot 5{,}58 + \frac{1}{3}\cdot 8{,}0\right) - \frac{1}{2}\cdot 1{,}25\cdot 1{,}5\cdot(\frac{2}{3}\cdot 5{,}58 + \right.$
$\left. + \frac{1}{3}\cdot 11{,}25) - \frac{1}{2}\cdot 2{,}5\cdot 1{,}5\cdot\left(\frac{2}{3}\cdot 11{,}25 + \frac{1}{3}\cdot 5{,}58\right) - \frac{1}{2}\cdot 11{,}25\cdot 1{,}5\cdot 2{,}5\right] = -\frac{\mathbf{51{,}6344}}{\mathbf{EI}}$

$X_1 = -\frac{\delta_{1P}}{\delta_{11}}$ **= 2,48 kN**





5. Sprawdzenie maksymalnych naprężeń:

$M_{max} = \eta_s \cdot M_{stat} + \eta_d \cdot M_{dyn}$

Przyjęto:

$\eta_s = 1{,}2$
$\eta_d = 5{,}0$

$M_{max} = 1{,}2 \cdot 8{,}0 + 5{,}0 \cdot 219{,}625$

M_{max} = 1107,725 kNm = 110772,5 kNcm

$\sigma = \frac{M_{max}}{W}$

$W = 194{,}0\ cm^3$

$\sigma = \frac{110772{,}5}{194{,}0}$

$\sigma = 570{,}9923\ \frac{kN}{cm^2}$
σ = 5709,923 MPa > σ_{dop} = 215,0 MPa

Przekrój nie spełnia warunku wyrzymałościowego.